Hybrid spin-dependent and hybrid-quarkonium mixing potentials

"QWG 2025 - 17th International Workshop on Heavy Quarkonium" – CERN

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Main goals / literature

(1) Compute heavy hybrid potentials (static limit as well as $1/m_Q$ corrections), i.e. potentials of a heavy quark antiquark pair ($\bar{b}b$ or $\bar{c}c$), where the flux tubes are excited with quantum numbers different from the ground state.
$\rightarrow$ SU(3) lattice gauge theory

(2) Use these potentials to compute the spectra of $\bar{b}b$ and $\bar{c}c$ hybrid mesons and to study their mixing with ordinary quarkonium.
$\rightarrow$ Effective field theory, Born-Oppenheimer approximation, quantum mechanics
[Talk by Paul Pütz, "Spin dependent potentials for hybrids", Monday 17. Nov 2025, 14:20]

- Our work mostly focuses on (1)
  [C. Schlosser, M.W., Phys. Rev. D **105**, no. 5, 054503 (2022) [arXiv:2111.00741]]
  [C. Schlosser, M.W., Phys. Rev. D **111**, no. 7, 074504 (2025) [arXiv:2501.08844]]
  but the talk is more about (2).
  (my goal in this talk is to explain, why we compute heavy hybrid potentials, what one can study with them and how ... rather than explaining less interesting lattice technicalities)

- Selected further references on (1) and (2):
  [K. J. Juge, J. Kuti, C. Morningstar, Phys. Rev. Lett. **90**, 161601 (2003) [hep-lat/0207004]
  [S. Capitani, O. Philipsen, C. Reisinger, C. Riehl. M.W., Phys. Rev. D **99**, 034502 (2019) [arXiv:1811.11046 [hep-lat]]]
  [R. Oncala, J. Soto, Phys. Rev. D **96**, no. 1, 014004 (2017) [arXiv:1702.03900]]
  [J. Soto, S. T. Valls, Phys. Rev. D **108**, no. 1, 014025 (2023) [arXiv:2302.01765]]
  [M. Berwein, N. Brambilla, A. Mohapatra, A. Vairo, Phys. Rev. D **110**, no. 9, 094040 (2024) [arXiv:2408.04719]]

$\bar{b}b$ and $\bar{c}c$ hybrid meson masses: BO

- Born-Oppenheimer approximation: Compute $\bar{b}b$ and $\bar{c}c$ hybrid meson masses in two steps.

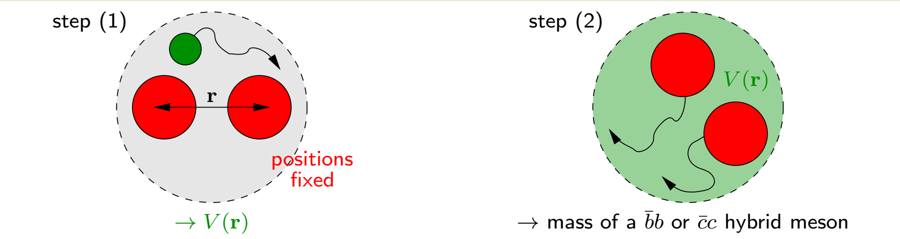
  (1) **Compute potentials of two static quarks ($\bar{b}b$ or $\bar{c}c$) and corrections proportional to $1/m_Q$, $(1/m_Q)^2$, ... (m_Q is the heavy quark mass) in the presence of excited gluons using lattice gauge theory.**

  (2) **Solve the Schrödinger equation for the relative coordinate of $\bar{b}b$ or $\bar{c}c$ using the potentials from (1),**

$$\bigg(-\frac{1}{m_Q}\triangle + V^{(0)}(\mathbf{r}) + \frac{1}{m_Q}V^{(1)}(\mathbf{r}) + \ldots\bigg)\Psi(\mathbf{r}) \quad = \quad E\Psi(\mathbf{r})$$

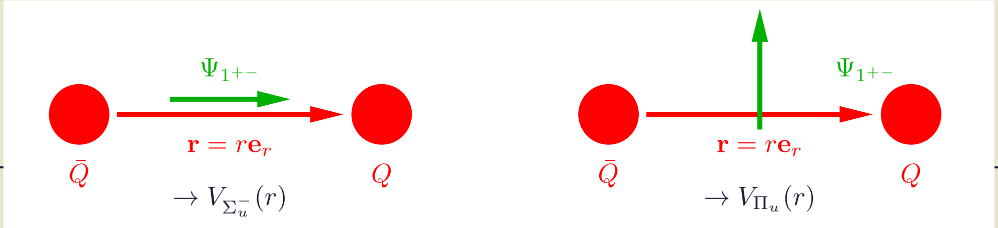
  (Ψ has 16 components, $V^{(n)}$ is a 16×16 matrix). Energy eigenvalues E correspond to masses of $\bar{b}b$ and $\bar{c}c$ hybrid mesons.

  [E. Braaten, C. Langmack, D. H. Smith, Phys. Rev. D **90**, 014044 (2014) [arXiv:1402.0438]]
  ...
  [R. Oncala, J. Soto, Phys. Rev. D **96**, no. 1, 014004 (2017) [arXiv:1702.03900]]
  [J. Soto, S. T. Valls, Phys. Rev. D **108**, no. 1, 014025 (2023) [arXiv:2302.01765]]
  [C. Schlosser, M.W., Phys. Rev. D **111**, no. 7, 074504 (2025) [arXiv:2501.08844]]



Potentials in the static limit, $(1/m_Q)^0$

- Heavy hybrid meson: $\bar{Q}Q$ + (valence) gluon.
- The gluon has spin 1, represented by a 3-vector,
  - a chromomagnetic field operator $\mathbf{B}$ (in the lattice computation of hybrid potentials),
  - a 3-component wave function $\Psi_{1^{+-}}$ (in the Schrödinger equation; 1^{+-} denotes the quantum numbers of a gluon, i.e. indicates that $\Psi_{1^{+-}}$ is a hybrid wave function).
- In the static approximation (infinitely heavy quarks), quark spins are irrelevant.
- There is another 3-vector, the relative coordinate of the quarks, $\mathbf{r} = \mathbf{r}_Q - \mathbf{r}_{\bar{Q}}$.
- Two possibilities:
  - $\mathbf{B}$ (or equivalently $\Psi_{1^{+-}}$) and $\mathbf{r}$ are parallel.
    $\rightarrow$ Corresponding hybrid static potential is denoted as $V_{\Sigma_u^-}(r)$.
  - $\mathbf{B}$ (or equivalently $\Psi_{1^{+-}}$) and $\mathbf{r}$ are perpendicular.
    $\rightarrow$ Corresponding hybrid static potential is denoted as $V_{\Pi_u}(r)$.

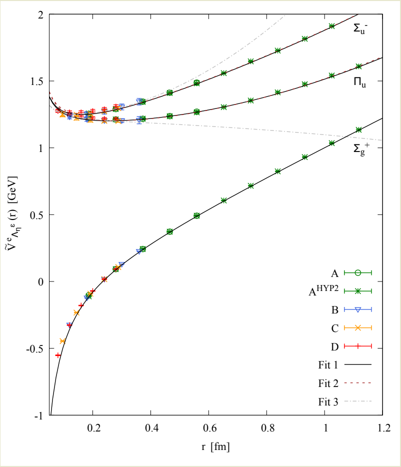


Lattice computation of $V_{\Sigma_u^-}(r)$, $V_{\Pi_u}(r)$ (1)

- Two possibilities:
  - $\mathbf{B}$ and $\mathbf{r}$ are parallel.
    $\rightarrow$ Corresponding hybrid static potential is denoted as $V_{\Sigma_u^-}(r)$.
  - $\mathbf{B}$ and $\mathbf{r}$ are perpendicular.
    $\rightarrow$ Corresponding hybrid static potential is denoted as $V_{\Pi_u}(r)$.

- To determine static potentials use lattice gauge theory to compute temporal correlation functions of suitable operators $\mathcal{O}(r)$

$$\lim_{t\rightarrow\infty} \langle\Omega|\mathcal{O}^\dagger(r,t)\mathcal{O}(r,0)|\Omega\rangle \quad \propto \quad \exp\Big(-V(r)t\Big).$$



- Suitable operators are:

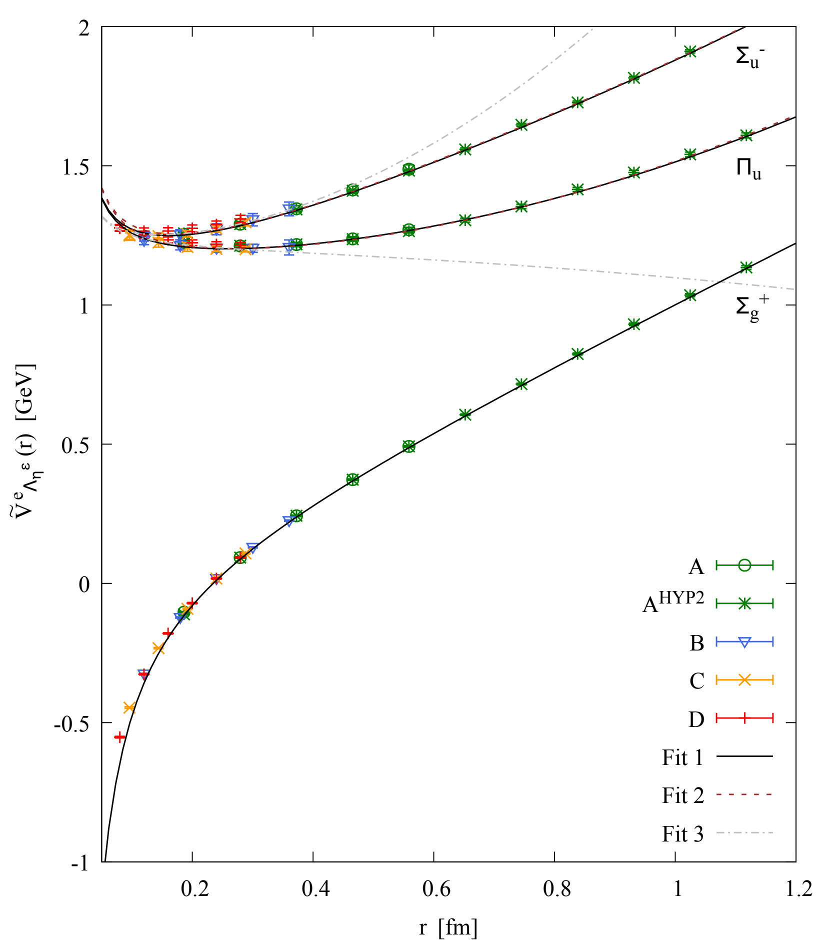
$$\mathcal{O}_{\Sigma_u^-}(r) = \bar{Q}(-r/2)U(-r/2;0)B_z(0)U(0;+r/2)Q(+r/2)$$

$$\mathcal{O}_{\Pi_u}(r) = \left\{ \begin{array}{l} \bar{Q}(-r/2)U(-r/2;0)B_x(0)U(0;+r/2)Q(+r/2) \\ \bar{Q}(-r/2)U(-r/2;0)B_y(0)U(0;+r/2)Q(+r/2) \end{array} \right. .$$

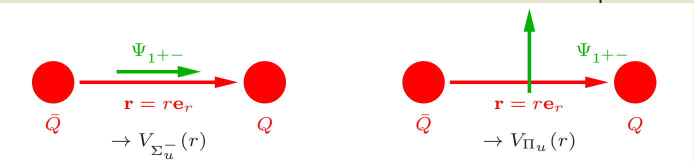
  - Q and $\bar{Q}$ (spinless quarks) are separated in z direction; only z coordinates are written.
  - U denote straight gluonic parallel transporters (necessary for gauge invariance).

Lattice computation of $V_{\Sigma_u^-}(r)$, $V_{\Pi_u}(r)$ (2)

- Σ_g^+: Ordinary static potential (→ ordinary quarkonium).
- Σ_u^-, Π_u: Hybrid static potentials (→ hybrid quarkonium).
- Computation on 4 ensembles with lattice spacings $a = 0.040\,\text{fm}, \ldots, 0.096\,\text{fm}$.
  [C. Schlosser, M.W., Phys. Rev. D **105**, no. 5, 054503 (2022) [arXiv:2111.00741]]



$(1/m_Q)^0$: Potential matrix, SE



- Two possibilities:
  - Ψ_{1+-} and $\mathbf{r}$ are parallel.
    $\rightarrow$ Corresponding hybrid static potential is denoted as $V_{\Sigma_u^-}(r)$.
  - Ψ_{1+-} and $\mathbf{r}$ are perpendicular.
    $\rightarrow$ Corresponding hybrid static potential is denoted as $V_{\Pi_u}(r)$.
- Schrödinger equation:

$$\left(-\frac{1}{m_Q}\triangle + V_{1+-}^{(0)}(\mathbf{r})\right)\Psi_{1+-}(\mathbf{r}) \quad = \quad E\Psi_{1+-}(\mathbf{r}).$$

- Ψ_{1+-} is a 3-component wave function (the components represent the gluon spin)
  $\rightarrow$ The Hamilton operator is a 3×3 matrix.
  $\rightarrow$ 3 coupled PDEs.
- The 3×3 potential matrix is

$$V_{1+-}^{(0)}(\mathbf{r}) \quad = \quad V_{\Sigma_u^-}(r)\Big(\mathbf{e}_r \otimes \mathbf{e}_r\Big) + V_{\Pi_u}(r)\Big(1 - \mathbf{e}_r \otimes \mathbf{e}_r\Big).$$

[M. Berwein, N. Brambilla, J. Tarrús Castellà, A. Vairo, Phys. Rev. D **92**, no. 11, 114019 (2015) [arXiv:1510.04299]]
[C. Schlosser, M.W., Phys. Rev. D **111**, no. 7, 074504 (2025) [arXiv:2501.08844]]

Radial SEs (1)

- The Schrödinger equation

$$\left(-\frac{1}{m_Q}\triangle + V_{1^{+-}}^{(0)}(\mathbf{r})\right)\Psi_{1^{+-}}(\mathbf{r}) \quad = \quad E\Psi_{1^{+-}}(\mathbf{r})$$

(a PDE) can be decoupled into infinitely many radial equations (ODEs).

- Orbital angular momentum $L^{PC} = 0^{++}, 1^{--}, 2^{++}, \ldots$ (spherical harmonics Y_{LM} in $\Psi_{1^{+-}}$)
- Gluon spin 1^{+-}. (3 components of $\Psi_{1^{+-}}$)
- Heavy spins are irrelevant/neglected in the static approximation.

$\rightarrow$ Total angular momentum (with heavy spins excluded): $J'^{PC} = L^{PC} \otimes 1^{+-}$.

- $J'^{PC} = 1^{-+} = 1^{--} \otimes 1^{+-}$ (H_2 multiplet)
  $J'^{PC} = 2^{+-} = 2^{++} \otimes 1^{+-}$ (H_5 multiplet)
  ...
- $J'^{PC} = 0^{-+} = 1^{--} \otimes 1^{+-}$ (H_3 multiplet)
  $J'^{PC} = 1^{+-} = 0^{++} \otimes 1^{+-} \;/\; 2^{++} \otimes 1^{+-}$ (H_1 multiplet)
  $J'^{PC} = 2^{-+} = 1^{--} \otimes 1^{+-} \;/\; 3^{--} \otimes 1^{+-}$ (H_4 multiplet)
  ...

Radial SEs (2)

- Hybrid quantum numbers: $J'^{PC} = 1^{-+}, 2^{+-}, \ldots, 0^{-+}, 1^{+-}, 2^{-+}, \ldots$
- No mixing with ordinary quarkonium, which can have quantum numbers $J'^{PC} = L^{PC} = 0^{++}, 1^{--}, 2^{++}, \ldots$
- Example: Radial Schrödinger equation for $J'^{PC} = 0^{-+} = 1^{--} \otimes 1^{+-}$ (H_3 multiplet).

$$\Psi_{1^{+-}}(\mathbf{r}) = \bigg(\underbrace{Y_{1x}(\Omega)}_{\propto x}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \underbrace{Y_{1y}(\Omega)}_{\propto y}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \underbrace{Y_{1z}(\Omega)}_{\propto z}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\bigg) R(r) \propto \mathbf{e}_r R(r).$$

- Inserting this wave function into the Schrödinger equation

$$\bigg(-\frac{1}{m_Q}\triangle + V^{(0)}_{1^{+-}}(\mathbf{r})\bigg)\Psi_{1^{+-}}(\mathbf{r}) = E\Psi_{1^{+-}}(\mathbf{r}) \qquad \text{(3 coupled PDEs containing all } J'^{PC}\text{)}$$

$$V^{(0)}_{1^{+-}}(\mathbf{r}) = V_{\Sigma_u^-}(r)\Big(\mathbf{e}_r \otimes \mathbf{e}_r\Big) + V_{\Pi_u}(r)\Big(1 - \mathbf{e}_r \otimes \mathbf{e}_r\Big)$$

leads to the radial Schrödinger equation

$$\bigg(-\frac{1}{m_Q}\frac{d^2}{dr^2} + \frac{2}{m_Q r^2} + V_{\Sigma_u^-}(r)\bigg)R(r) = ER(r) \qquad \text{(1 coupled ODE for } J'^{PC} = 0^{-+}\text{).}$$

Spin-dependent and mixing potentials (1)

- Currently ongoing research: **inclusion of the heavy quark spins** and **mixing with ordinary quarkonium** order by order in $1/m_Q$ (at the moment just order $1/m_Q$).
  - The $\bar{Q}Q$ pair can have spin $S = 0$ ($S_z = 0$ [$\rightarrow$ 1 possible configuration]) or $S = 1$ ($S_z = -1, 0, +1$ [$\rightarrow 3$ possible configurations]), i.e. 4 spin configurations in total.
  - On previous slides: $\Psi_{1+-}(\mathbf{r})$ has 3 components (gluon spin 1).
  - From now on: $\Psi_{1+-}(\mathbf{r})$ has $3 \times 4 = 12$ components (gluon spin 1, quark spin 0 or 1).
  - Moreover, there can be mixing with ordinary quarkonium (represented by a 4-component wave function $\Psi_{0++}(\mathbf{r})$).

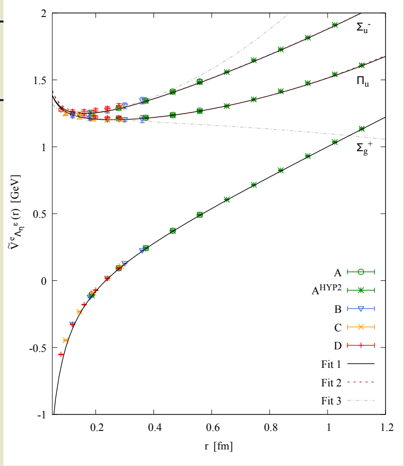
$\rightarrow$ Including both heavy quark spins and mixing leads to

  - the 16-component wave function $\Psi(\mathbf{r}) = (\Psi_{0++}(\mathbf{r}), \Psi_{1+-}(\mathbf{r}))$,
  - a 16×16 Hamilton operator and potential matrix $V(\mathbf{r}) = V^{(0)}(\mathbf{r}) + (1/m_Q)V^{(1)}(\mathbf{r})$,
  - the Schrödinger equation

$$\left(-\frac{1}{m_Q}\triangle + V^{(0)}(\mathbf{r}) + \frac{1}{m_Q}V^{(1)}(\mathbf{r})\right)\Psi(\mathbf{r}) \quad = \quad E\Psi(\mathbf{r})$$

  (16 coupled PDEs).

Spin-dependent and mixing potentials (2)



- Schrödinger equation:

$$\left(-\frac{1}{m_Q}\triangle + V^{(0)}(\mathbf{r}) + \frac{1}{m_Q}V^{(1)}(\mathbf{r})\right)\Psi(\mathbf{r}) \quad = \quad E\Psi(\mathbf{r})$$

  with $\Psi(\mathbf{r}) = (\Psi_{0^{++}}(\mathbf{r}), \Psi_{1^{+-}}(\mathbf{r}))$.

- $V^{(0)}(\mathbf{r})$:
  - 16×16 matrix, block diagonal.
  - Upper left 4×4 block: ordinary static potential $V_{\Sigma_g^+}(r)$ ("quarkonium channels").
  - Lower right 12×12 block: hybrid static potentials $V_{\Sigma_u^-}(r)$, $V_{\Pi_u}(r)$ ("hybrid channels").

- $V^{(1)}(\mathbf{r})$:
  - 16×16 matrix, introduces heavy spin effects, couples quarkonium and hybrid channels.
  - Contains 2 hybrid spin-dependent potentials $V_{11}^{sa}(r)$, $V_{10}^{sb}(r)$.
  - Contains 2 hybrid-quarkonium mixing potentials $V_{\Sigma_u^-}^{\text{mix}}(r)$, $V_{\Pi_u}^{\text{mix}}(r)$.

    [R. Oncala, J. Soto, Phys. Rev. D **96**, no. 1, 014004 (2017) [arXiv:1702.03900]]
    [J. Soto, S. T. Valls, Phys. Rev. D **108**, no. 1, 014025 (2023) [arXiv:2302.01765]]
  - **The main focus of our work is to carry out the first lattice gauge theory computation of these four $1/m_Q$ potentials.**

Spin-dependent and mixing potentials (3)

- All J^{PC} possible and included in the Schrödinger equation. (exception 0^{--})
- For each J: three PC sectors, where ordinary quarkonium and hybrids mix, one pure hybrid PC sector (0^{+-}, 1^{-+}, 2^{+-}, ...).
- For each J: suitable angular momentum projections lead to radial coupled-channel Schrödinger equations with 6, 2, 4, and 4 channels, respectively (for $J < 2$ a smaller number of channels).
  [R. Oncala, J. Soto, Phys. Rev. D **96**, no. 1, 014004 (2017) [arXiv:1702.03900]]
  [J. Soto, S. T. Valls, Phys. Rev. D **108**, no. 1, 014025 (2023) [arXiv:2302.01765]]

L^{PC}	gluon	J'^{PC}	multiplet	S^{PC}	J^{PC}	type
0^{++}		0^{++}		0^{-+}	0^{-+}	ordinary
0^{++}	1^{+-}	1^{+-}	H_1	1^{--}	0^{-+}	hybrid
2^{++}	1^{+-}	1^{+-}	H_1	1^{--}	0^{-+}	hybrid
1^{--}		1^{--}		1^{--}	0^{++}	ordinary
1^{--}	1^{+-}	0^{-+}	H_3	0^{-+}	0^{++}	hybrid
1^{--}	1^{+-}	1^{-+}	H_2	1^{--}	0^{+-}	hybrid
1^{--}		1^{--}		0^{-+}	1^{+-}	ordinary
1^{--}	1^{+-}	0^{-+}	H_3	1^{--}	1^{+-}	hybrid
1^{--}	1^{+-}	1^{-+}	H_2	1^{--}	1^{+-}	hybrid
1^{--}	1^{+-}	2^{-+}	H_4	1^{--}	1^{+-}	hybrid
3^{--}	1^{+-}	2^{-+}	H_4	1^{--}	1^{+-}	hybrid
1^{--}		1^{--}		1^{--}	1^{++}	ordinary
1^{--}	1^{+-}	1^{-+}	H_2	0^{-+}	1^{++}	hybrid
0^{++}		0^{++}		1^{--}	1^{--}	ordinary
2^{++}		2^{++}		1^{--}	1^{--}	ordinary
0^{++}	1^{+-}	1^{+-}	H_1	0^{-+}	1^{--}	hybrid
2^{++}	1^{+-}	1^{+-}	H_1	0^{-+}	1^{--}	hybrid
0^{++}	1^{+-}	1^{+-}	H_1	1^{--}	1^{-+}	hybrid
2^{++}	1^{+-}	1^{+-}	H_1	1^{--}	1^{-+}	hybrid
2^{++}	1^{+-}	2^{+-}	H_5	1^{--}	1^{-+}	hybrid
2^{++}		2^{++}		0^{-+}	2^{-+}	ordinary
0^{++}	1^{+-}	1^{+-}	H_1	1^{--}	2^{-+}	hybrid
2^{++}	1^{+-}	1^{+-}	H_1	1^{--}	2^{-+}	hybrid
2^{++}	1^{+-}	2^{+-}	H_5	1^{--}	2^{-+}	hybrid
2^{++}	1^{+-}	3^{+-}		1^{--}	2^{-+}	hybrid
4^{++}	1^{+-}	3^{+-}		1^{--}	2^{-+}	hybrid
2^{++}		2^{++}		1^{--}	2^{--}	ordinary
2^{++}	1^{+-}	2^{+-}	H_5	0^{-+}	2^{--}	hybrid

Lattice computation of $V_{11}^{sa}(r)$, $V_{10}^{sb}(r)$, $V_{\Sigma_u^-}^{\textsf{mix}}(r)$, $V_{\Pi_u}^{\textsf{mix}}(r)$ (1)

- The $1/m_Q$ potentials $V_{11}^{sa}(r)$, $V_{10}^{sb}(r)$, $V_{\Sigma_u^-}^{\textsf{mix}}(r)$, $V_{\Pi_u}^{\textsf{mix}}(r)$ are related to chromomagnetic expectation values,

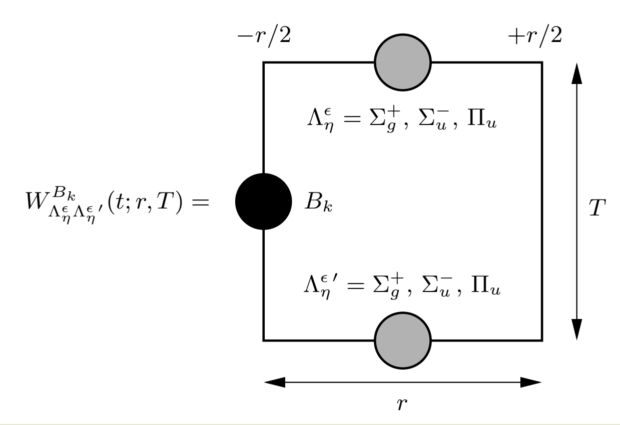
$$V_{11}^{sa}(r) = +igc_F \langle 0, \Pi_u^- | B_z(-r/2) | 0, \Pi_u^+ \rangle (r)$$

$$V_{10}^{sb}(r) = +igc_F \langle 0, \Sigma_u^- | B_y(-r/2) | 0, \Pi_u^+ \rangle (r)$$

$$V_{\Pi_u}^{\textsf{mix}}(r) = +\frac{igc_F}{2m_Q} \langle 0, \Sigma_g^+ | B_x(-r/2) | 0, \Pi_u^+ \rangle (r)$$

$$V_{\Sigma_u^-}^{\textsf{mix}}(r) = +\frac{igc_F}{2m_Q} \langle 0, \Sigma_g^+ | B_z(-r/2) | 0, \Sigma_u^- \rangle (r)$$

(for heavy quarks separated along the z axis).



- Such matrix elements can be extracted from generalized Wilson loops,

$$\langle 0, \Lambda_\eta^\epsilon | B_k(-r/2) | 0, \Lambda_\eta^\epsilon{}' \rangle (r) = \lim_{T \to \infty} \frac{i}{g} R^{B_k}_{\Lambda_\eta^\epsilon \Lambda_\eta^\epsilon{}'}(t;r,T)$$

$$R^{B_k}_{\Lambda_\eta^\epsilon \Lambda_\eta^\epsilon{}'}(t;r,T) =$$

$$= W^{B_k}_{\Lambda_\eta^\epsilon \Lambda_\eta^\epsilon{}'}(t;r,T) \left(\frac{1}{W_{\Lambda_\eta^\epsilon}(r,T) W_{\Lambda_\eta^\epsilon{}'}(r,T)}\right)^{1/2} \left(\frac{W_{\Lambda_\eta^\epsilon{}'}(r,T/2-t) W_{\Lambda_\eta^\epsilon}(r,T/2+t)}{W_{\Lambda_\eta^\epsilon}(r,T/2-t) W_{\Lambda_\eta^\epsilon{}'}(r,T/2+t)}\right)^{1/2} .$$

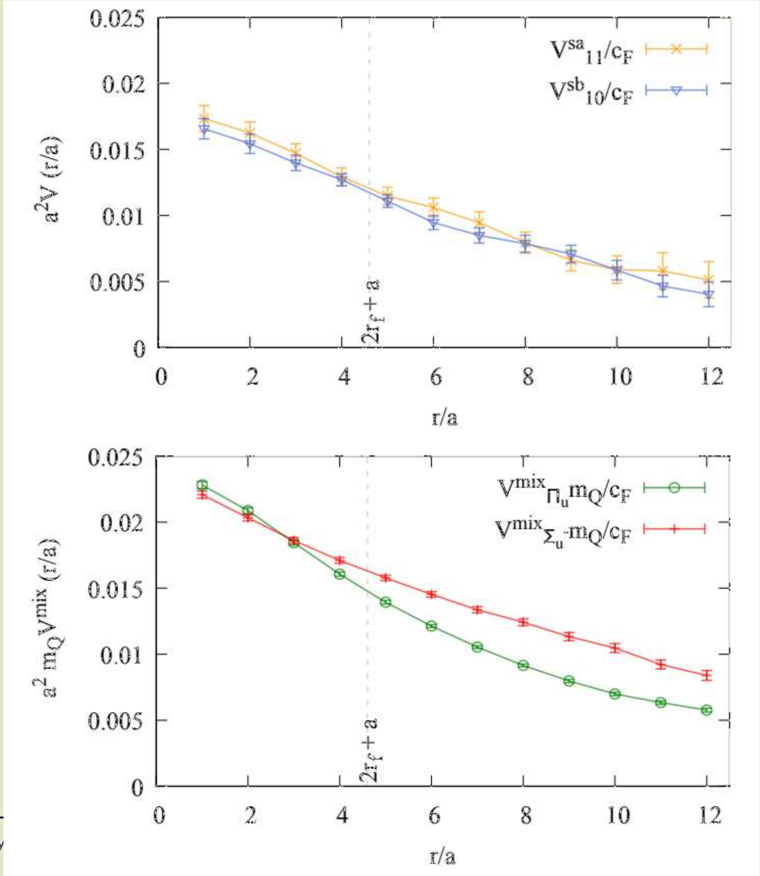
Lattice computation of $V_{11}^{sa}(r)$, $V_{10}^{sb}(r)$, $V_{\Sigma_u^-}^{\text{mix}}(r)$, $V_{\Pi_u}^{\text{mix}}(r)$ (2)

- Technical difficulties (compared to the computation of static [i.e. $(1/m_Q)^0$] potentials]):
  - Twice as large T extents for generalized Wilson loops needed for comparable ground state domination.
    - $(-)$ Loops with large temporal extents are needed (poor signal-to-noise ratio).
  - $(-)$ Chromomagnetic field insertions introduce large discretization errors.
  - Chromomagnetic field insertions need renormalization.
    - $(-)$ Corresponding renormalization constants ($=$ matching factors) c_F are know for the gradient flow scheme, but only perturbatively to order α_s.
      [N. Brambilla, X. P. Wang, JHEP **06**, 210 (2024) [arXiv:2312.05032]]
  - $\rightarrow$ Apply **gradient flow**, which is a technique to smear gauge link configurations in a mathematically controlled way via a single parameter called gradient flow time t_f.
    - $(+)$ Gradient Flow reduces (certain) discretization errors as well as statistical errors.
    - $(-)$ An extrapolation to gradient flow time t_f is necessary. This requires independent computations at several different t_f. Computing time increases accordingly.
    - $(-)$ "Gradient flow smearing" introduces systematic errors for small $\bar{Q}Q$ separations.
      $\rightarrow$ Discard $\bar{Q}Q$ separations $r \lesssim 2\sqrt{8t_f} + a$ (in our computation $r \leq 4a$).
      $\rightarrow$ Time consuming computations at rather small values of the lattice spacing a with many lattice sites required.

Lattice computation of $V_{11}^{sa}(r)$, $V_{10}^{sb}(r)$, $V_{\Sigma_u^-}^{\textbf{mix}}(r)$, $V_{\Pi_u}^{\textbf{mix}}(r)$ (3)

- Results shown have been obtained at a single lattice spacing $a = 0.060\,\text{fm}$ and a single gradient flow time t_f.
  - (−) No continuum extrapolation.
  - (−) No zero flow time extrapolation, no renormalization.

  → **Proof of principle**.
  [C. Schlosser, M.W., Phys. Rev. D **111**, no. 7, 074504 (2025) [arXiv:2501.08844]]



Summary and outlook

- **First lattice gauge theory computation of $\bar{Q}Q$ potentials describing heavy spin corrections for hybrid quarkonium and mixing with ordinary quarkonium.**
- At the moment only results for a single lattice spacing a and a single flow time t_f.
  $\rightarrow$ No continuum extrapolation.
  $\rightarrow$ No zero flow time extrapolation, no renormalization.
  "Proof of principle".
- Both smaller and larger separations are needed (lattices with many sites required).
- Smaller statistical errors via longer simulations and computations.

$\rightarrow$ Corresponding large scale computations are about to be started.

- Use these potentials in coupled-channel Schrödinger equations to predict masses of $\bar{b}b$ and $\bar{c}c$ hybrid mesons and their composition (i.e. the amount of mixing with ordinary quarkonium).
  [Talk by Paul Pütz, "Spin dependent potentials for hybrids", Monday 17. Nov 2025, 14:20]