Fermions in the pseudoparticle approach

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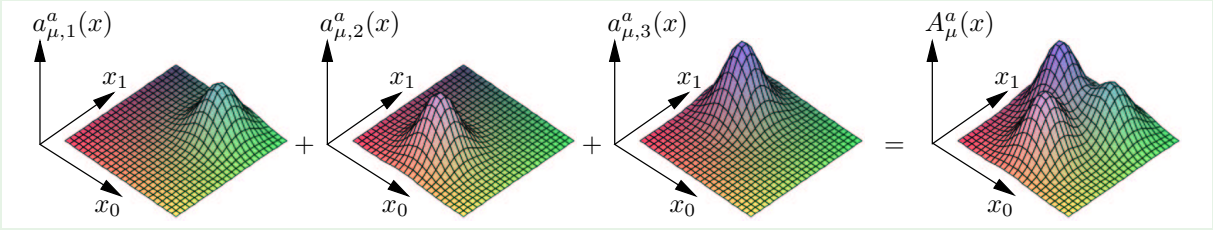
Introduction (1)

- Models for SU(2) Yang-Mills theory with a small number of degrees of freedom:
  - Regular gauge instanton models and meron models (F. Lenz, J. W. Negele, M. Thies, 2003).
  - Pseudoparticle approach (F. Lenz, M.W., 2005).
  - Calorons with non-trivial holonomy (P. Gerhold, E.-M. Ilgenfritz, M. Müller-Preussker, 2006).
- Basic principle: restrict the path integral to those gauge field configurations, which can be represented as a linear superposition of a small number of localized building blocks (instantons, merons, akyrons, calorons, ...).



Introduction (2)

- Successes of these models:
  - Linear potential between two static charges at large separations (confinement).
  - Confinement-deconfinement phase transition.
  - String tension, topological susceptibility and critical temperature in qualitative agreement with lattice results.
- Intention: get a better understanding of confining gauge field configurations and the mechanism of confinement.
- How can fermions be included in such models?
- First steps in this direction will be discussed in this talk:
  - Basic principle of the PP approach in fermionic theories.
  - Testing ground: the $1+1$-dimensional Gross-Neveu model in the large-N-limit (phase diagram, chiral condensate).

Basic principle

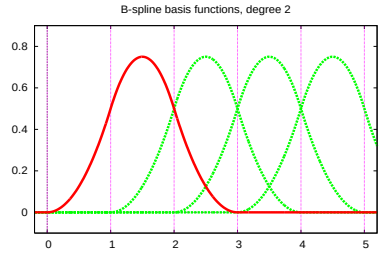
- Consider fermionic field configurations ψ, which can be represented as a linear superposition of a fixed number of localized building blocks (PPs):

$$\psi(x) \;=\; \sum_j \underbrace{\phi_j G_j(x)}_{j\text{-th PP}} \quad , \quad \int D\psi\, D\bar{\psi}\, \ldots \;=\; \int \left(\prod_j d\phi_j\, d\bar{\phi}_j \right) \ldots$$

(ϕ_j: Grassmann-valued spinors; G_j: localized functions).

- In this talk: PPs G_j are uniformly distributed "hat functions" (B-spline basis functions of degree 2).
  - "Sensible set of field configurations" (any not too heavily oscillating field configuration can be approximated)
    $\rightarrow$ we can expect to reproduce correct Gross-Neveu results.
  - Piecewise polynomial functions
    $\rightarrow$ certain integrals can be calculated analytically.

PPs G_j as functions of x



The Gross-Neveu model

- Action and partition function of the $1+1$-dimensional Gross-Neveu model:

$$
\begin{aligned}
S &= \int d^2x \left(\sum_{n=1}^{N} \bar{\psi}^{(n)} \Big(\gamma_0 (\partial_0 + \mu) + \gamma_1 \partial_1 \Big) \psi^{(n)} - \frac{g^2}{2} \left(\sum_{n=1}^{N} \bar{\psi}^{(n)} \psi^{(n)} \right)^2 \right) \\
Z &= \int \left(\prod_{n=1}^{N} D\psi^{(n)} \, D\bar{\psi}^{(n)} \right) e^{-S} \qquad \text{or equivalently} \\
S_{\text{effective}} &= N \left(\frac{1}{2\lambda} \int d^2x \, \sigma^2 - \ln \Big(\det \Big(\gamma_0 (\partial_0 + \mu) + \gamma_1 \partial_1 + \sigma \Big) \Big) \right) \\
Z &\propto \int D\sigma \, e^{-S_{\text{effective}}}
\end{aligned}
$$

  (N: number of flavors; μ: chemical potential; g: coupling constant; $\lambda = Ng^2$; $\sigma = \sum_{n=1}^{N} \bar{\psi}^{(n)} \psi^{(n)}$: chiral condensate [a real scalar field]).

- Large-N-limit: $N \to \infty$, $\lambda = Ng^2 = \text{constant}$ (there is no need to compute the σ-path integral; it is sufficient to minimize $S_{\text{effective}}$ with respect to σ).

PP results (1)

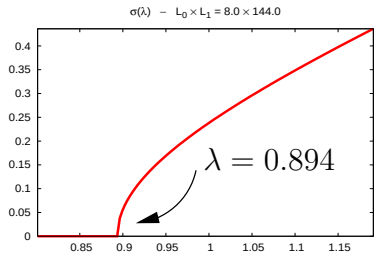
Phase diagram for $\sigma = \text{constant}$ ("old phase diagram") (1)

- PP effective action:

$$\frac{S_{\text{effective}}}{N} = \frac{L_0 L_1}{2\lambda}\sigma^2 - \ln\Big(\det\langle G_j|\Big(\gamma_0(\partial_0+\mu)+\gamma_1\partial_1+\sigma\Big)|G_{j'}\rangle\Big).$$

- Spacetime volumes: $L_0 \times L_1 = 8 \times 144$ ... $L_0 \times L_1 = 48 \times 144$.
- One PP per unit volume.
- To set the scale: finite temperature computations at $L_0 = 8$ and $\mu = 0$
  $\rightarrow \quad \sigma(\lambda) \quad \rightarrow \quad \Big\{\lambda = 0.894 \quad \leftrightarrow \quad T_{\text{critical}} = 1/8\Big\}.$
- "Zero temperature computation" at $\lambda = 0.894$, $L_0 = 48$ and $\mu = 0$
  $\rightarrow \quad \sigma_0 = 0.282.$

σ as a function of λ

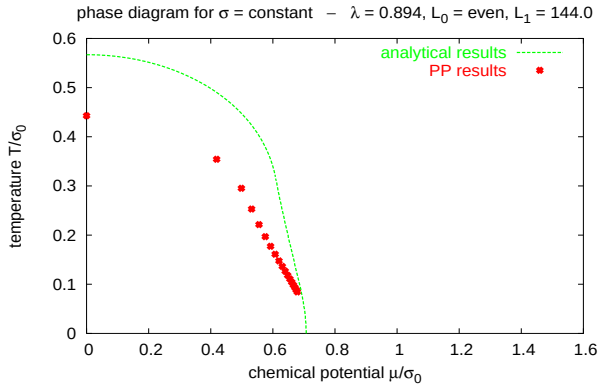


PP results (2)

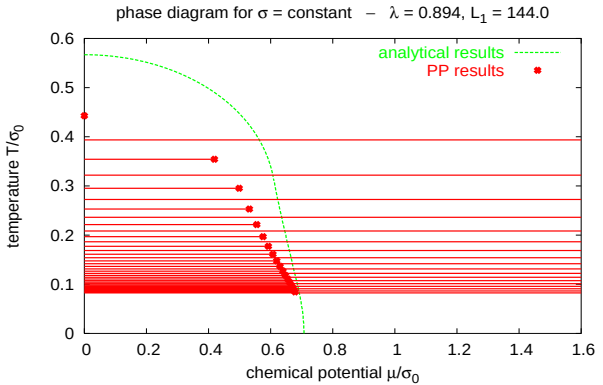
Phase diagram for $\sigma = \text{constant}$ ("old phase diagram") (2)

- Computations of the chiral condensate σ at $\lambda = 0.894$ and at different temperature $T = 1/L_0$ and chemical potential μ yield the phase diagram.
- Results are not even in qualitative agreement with analytical results, i.e. these results are completely useless!!!
- It can be shown that such "bad results" will be obtained for any choice of localized and uniformly distributed PPs!!!

even numbers of PPs in x_0-direction



even and odd numbers of PPs in x_0-direction



What is the reason for these bad results?

- PP effective action:

$$\frac{S_{\text{effective}}}{N} = \frac{L_0 L_1}{2\lambda}\sigma^2 - \ln\Big(\det\langle G_j|\underbrace{\Big(\gamma_0(\partial_0+\mu)+\gamma_1\partial_1+\sigma\Big)}_{=Q}|G_{j'}\rangle\Big).$$

- To expose the problem it is convenient to diagonalize the matrix:

$$\frac{S_{\text{effective}}}{N} = \frac{L_0 L_1}{2\lambda}\sigma^2 - \ln\Big(\det\underbrace{\langle\tilde{G}_j|Q|\tilde{G}_{j'}\rangle}_{=\text{diag}(\lambda_1,\lambda_2,\ldots)}\Big) \quad , \quad |\tilde{G}_j\rangle = U_{jk}|G_k\rangle.$$

- Problem (most extreme case): $Q|\tilde{G}_j\rangle \neq 0$ but $\lambda_j = \langle\tilde{G}_j|Q|\tilde{G}_j\rangle = 0$ because $Q|\tilde{G}_j\rangle \perp \text{span}\{|\tilde{G}_k\rangle\}$.
- These unphysical zero modes (low lying modes) spoil the results.
- In other words: applying the operator Q to the PPs $|\tilde{G}_j\rangle$ yields results, which are (partially) outside the PP function space.

Solution

- Use eigenfunctions of Q as pseudoparticles?
  - "Perfect solution" ... but in general, i.e. for non constant chiral condensate σ, too time consuming (eigenfunctions depend on σ).
- Since $\det(Q)$ is real and positive $\det(Q) = \sqrt{\det(Q^\dagger Q)}$.
- Avoid the above mentioned problem by applying the PP approach to $\sqrt{\det(Q^\dagger Q)}$ instead of $\det(Q)$:
  - Effective action:

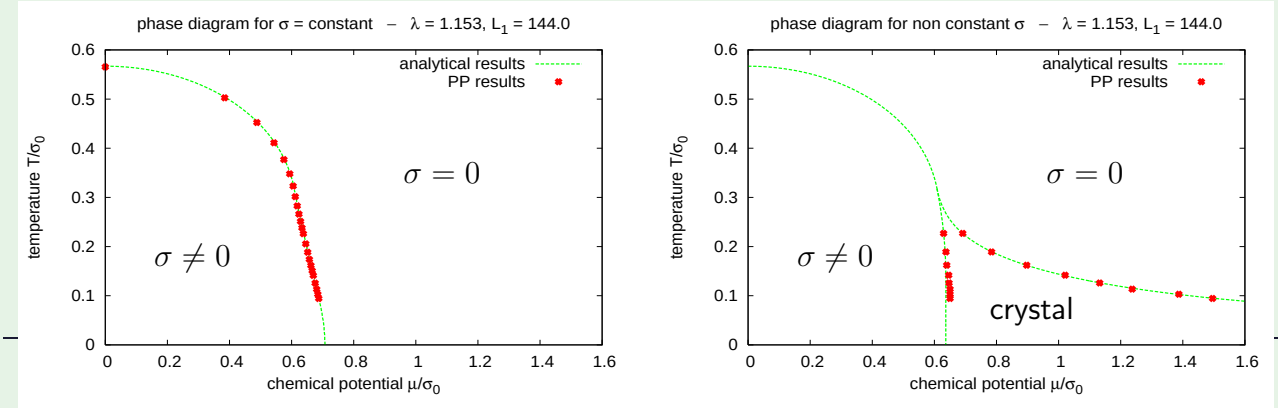
$$\frac{S_{\text{effective}}}{N} = \frac{L_0 L_1}{2\lambda}\sigma^2 - \ln\sqrt{\det\langle G_j|Q^\dagger Q|G_{j'}\rangle}.$$

  - Both $\langle G_j|Q^\dagger$ and $Q|G_{j'}\rangle$ are (partially) outside the PP function space but their overlap is computed in the same space.

PP results (3)

Phase diagram for $\sigma = \text{constant}$ and for non constant σ

- Spacetime volumes: $L_0 \times L_1 = 8 \times 144$ ... $L_0 \times L_1 = 48 \times 144$.
- One PP per unit volume for $\psi^{(n)}$, one PP per (3×3)-volume for σ.
- To set the scale: finite temperature computations at $L_0 = 8$ and $\mu = 0$ $\rightarrow \quad \sigma(\lambda) \quad \rightarrow \quad \Big\{\lambda = 1.153 \quad \leftrightarrow \quad T_{\text{critical}} = 1/8\Big\}$.
- Computations of the chiral condensate σ at $\lambda = 1.153$ and at different temperature $T = 1/L_0$ and chemical potential μ yield the phase diagram.



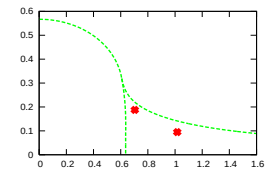
PP results (4)

Spatially non constant chiral condensate $\sigma(x_1)$

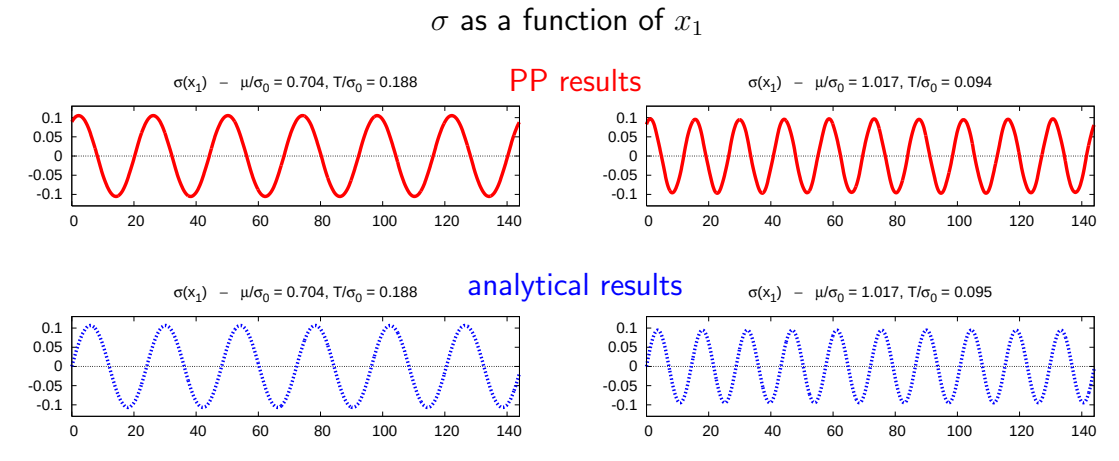
- Analytical result:

$$\sigma(x_1) \quad = \quad A\kappa^2 \frac{\mathrm{sn}(Ax_1,\kappa)\mathrm{cn}(Ax_1,\kappa)}{\mathrm{dn}(Ax_1,\kappa)}$$

$(A = A(\mu, T),\ \kappa = \kappa(\mu, T))$.



- PP and analytical results at two arbitrarily chosen points in the crystal phase:



Conclusions and outlook

- The PP approach has been applied to compute the phase diagram of the $1+1$-dimensional Gross-Neveu model in the large-N-limit (both for $\sigma = \text{constant}$ and for spatially non constant σ): results are in excellent agreement with analytical results.
- The next step is to apply the PP approach to QCD:
  - Current research: chiral symmetry breaking by computing the low lying eigenmodes of the Dirac operator in the quenched approximation (Banks-Casher relation).
  - Goal: obtain a model with a small number of degrees of freedom, which exhibits chiral symmetry breaking and a confinement deconfinement phase transition at the same time.
  - Compute further observables: pion masses, ...