The static-light meson spectrum from twisted mass lattice QCD

Karl Jansen, Chris Michael, Andrea Shindler, Marc Wagner

Humboldt-Universität zu Berlin, Institut für Physik

mcwagner@physik.hu-berlin.de

http://people.physik.hu-berlin.de/∼mcwagner/

June 26, 2008

European Twisted Mass Collaboration

- **Cyprus**: University of Nikosia.
- **France**: University of Paris Sud, LPSC Grenoble.
- **Germany**: Humboldt University Berlin, University of Münster, DESY Hamburg, DESY Zeuthen.
- **Great Britain**: University of Glasgow, University of Liverpool.
- **Italy**: University of Rome I, University of Rome II, University of Rome III, ECT* Trento.
- **Netherlands**: University of Groningen.
- **Spain**: University of Valencia.
- **Switzerland**: University of Bern.

Introduction

- **Static-light meson**: a bound state of an infinitely heavy quark and a light quark ("a B-meson in leading order").
- Static-light mesons can be classified according to certain quantum numbers:
  - Total angular momentum $F = 0, 1, 2, 3, \ldots$
  - Parity $P = \pm$.
- **Goal**: compute static-light meson masses for low lying states (ground state, first excited state) for different quantum numbers F and P.

Outline

- Basic principle.
- Twisted mass lattice QCD.
- Static-light meson creation operators on the lattice.
- Simulation setup and numerical results.
- Summary and outlook.

Basic principle (1)

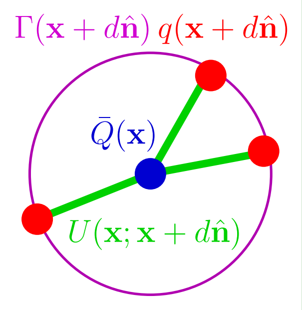
- Let $\mathcal{O}(\mathbf{x})$ be a suitable "static-light meson creation operator", i.e. an operator such that $\mathcal{O}(\mathbf{x})|\Omega\rangle$ is a state containing a static-light meson at position $\mathbf{x}$ ($|\Omega\rangle$: vacuum).
- Determine the mass of the ground state of the corresponding static-light meson from the exponential behavior of the corresponding correlation function $\mathcal{C}$ at large Euclidean times T:

$$\begin{aligned} \mathcal{C}(T) \quad &= \quad \langle\Omega|\Big(\mathcal{O}(\mathbf{x},T)\Big)^\dagger \mathcal{O}(\mathbf{x},0)|\Omega\rangle \quad = \\ &= \quad \langle\Omega|e^{+HT}\Big(\mathcal{O}(\mathbf{x},0)\Big)^\dagger e^{-HT}\mathcal{O}(\mathbf{x},0)|\Omega\rangle \quad = \\ &= \quad \sum_n \Big|\langle n|\mathcal{O}(\mathbf{x},0)|\Omega\rangle\Big|^2 \exp\Big(-(E_n - E_\Omega)T\Big) \quad \approx \quad \text{(for } T \gg 1\text{)} \\ &\approx \quad \Big|\langle 0|\mathcal{O}(\mathbf{x},0)|\Omega\rangle\Big|^2 \exp\Big(-\underbrace{(E_0 - E_\Omega)}_{\text{meson mass}} T\Big). \end{aligned}$$

Basic principle (2)

- To compute the static-light spectrum, i.e. meson masses for different quantum numbers, consider extended meson creation operators with different spatial structure and different spin structure yielding well defined total angular momentum F.
- Static-light meson masses are degenerate with respect to the static spin.
- Therefore, it is more appropriate to label static-light mesons by $J = L \pm 1/2$, where L is the angular momentum quantum number and $\pm$ describes the coupling of the light spin.
- Parity P is also a good quantum number.
- Since static-light mesons are made from non-identical quarks, charge conjugation is not a useful quantum number (static-light meson masses are degenerate with respect to charge conjugation).

Basic principle (3)



- **General form of a static-light meson creation operator**:

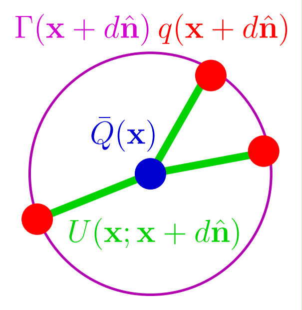
$$\mathcal{O}(\mathbf{x}) \quad = \quad \bar{Q}(\mathbf{x}) \int d\hat{\mathbf{n}}\, \Gamma(\hat{\mathbf{n}}) U(\mathbf{x}; \mathbf{x} + d\hat{\mathbf{n}}) q(\mathbf{x} + d\hat{\mathbf{n}}).$$

  - $\bar{Q}(\mathbf{x})$ creates an infinitely heavy i.e. static antiquark at position $\mathbf{x}$.
  - $q(\mathbf{x} + d\hat{\mathbf{n}})$ creates a light quark at position $\mathbf{x} + d\hat{\mathbf{n}}$ separated by a distance d from the static antiquark.
  - The spatial parallel transporter

$$U(\mathbf{x}; \mathbf{x} + d\hat{\mathbf{n}}) \quad = \quad P\left\{\exp\left(+i \int_{\mathbf{x}}^{\mathbf{x}+d\hat{\mathbf{n}}} dz_j\, A_j(\mathbf{z})\right)\right\}$$

    connects the antiquark and the quark in a gauge invariant way via gluons.
  - The integration over the unit sphere $\int d\hat{\mathbf{n}}$ combined with a suitable weight factor $\Gamma(\hat{\mathbf{n}})$ yields well defined total angular momentum J and parity P ($\Gamma(\hat{\mathbf{n}})$ is a combination of spherical harmonics [$\rightarrow$ angular momentum] and γ-matrices [$\rightarrow$ spin]; Wigner-Eckart theorem).

Basic principle (4)



- **General form of a static-light meson creation operator**:

$$\mathcal{O}(\mathbf{x}) \quad = \quad \bar{Q}(\mathbf{x}) \int d\hat{\mathbf{n}}\, \Gamma(\hat{\mathbf{n}}) U(\mathbf{x}; \mathbf{x} + d\hat{\mathbf{n}}) q(\mathbf{x} + d\hat{\mathbf{n}}).$$

- **List of operators** (L: angular momentum; S: total spin; F: total angular momentum; J: angular momentum and light spin; P: parity):

common notation	$\Gamma(\mathbf{x})$	L^P	S^P	F^P	J^P
S	γ_5 $\gamma_5 \gamma_j x_j$	0^+ 1^-	0^- 1^+	0^-	$(1/2)^-$
P_-	1 $\gamma_j x_j$	0^+ 1^-	0^+ 1^-	0^+	$(1/2)^+$
P_+	$\gamma_1 x_1 - \gamma_2 x_2$	1^-	1^-	2^+	$(3/2)^+$
D_-	$\gamma_5(\gamma_1 x_1 - \gamma_2 x_2)$	1^-	1^+	2^-	$(3/2)^-$
D_+	$\gamma_1 x_2 x_3 + \gamma_2 x_3 x_1 + \gamma_3 x_1 x_2$	2^+	1^-	3^-	$(5/2)^-$
F_-	$\gamma_5(\gamma_1 x_2 x_3 + \gamma_2 x_3 x_1 + \gamma_3 x_1 x_2)$	2^+	1^+	3^+	$(5/2)^+$

Twisted mass lattice QCD

- Twisted mass action (two degenerate flavors, "continuum version"):

$$S_{\text{fermionic}} = \int d^4x\,\bar{\chi}\Big(\gamma_\mu D_\mu + m + \underbrace{i\mu\gamma_5\tau_3}_{\text{twisted mass term}} - \underbrace{\frac{a}{2}\Box}_{\text{Wilson term}}\Big)\chi$$

$$\psi = e^{i\omega\gamma_5\tau_3/2}\chi$$

(ψ: physical basis quark fields; χ: twisted basis quark fields; μ: twisted mass; τ_3: third Pauli matrix acting in flavor space; a: lattice spacing).

  - Wilson term: removes fermionic doublers.
  - Twisted mass term: automatic $\mathcal{O}(a)$ improvement, when tuned to maximal twist ($\omega = \pi/2$).

\+ Automatic $\mathcal{O}(a)$ improvement.

\+ Numerically cheap, i.e. large lattices and small lattice spacings possible.

– Explicit breaking of parity and flavor symmetry.

Meson operators on the lattice (1)

- **Static-light meson creation operators in the continuum**:

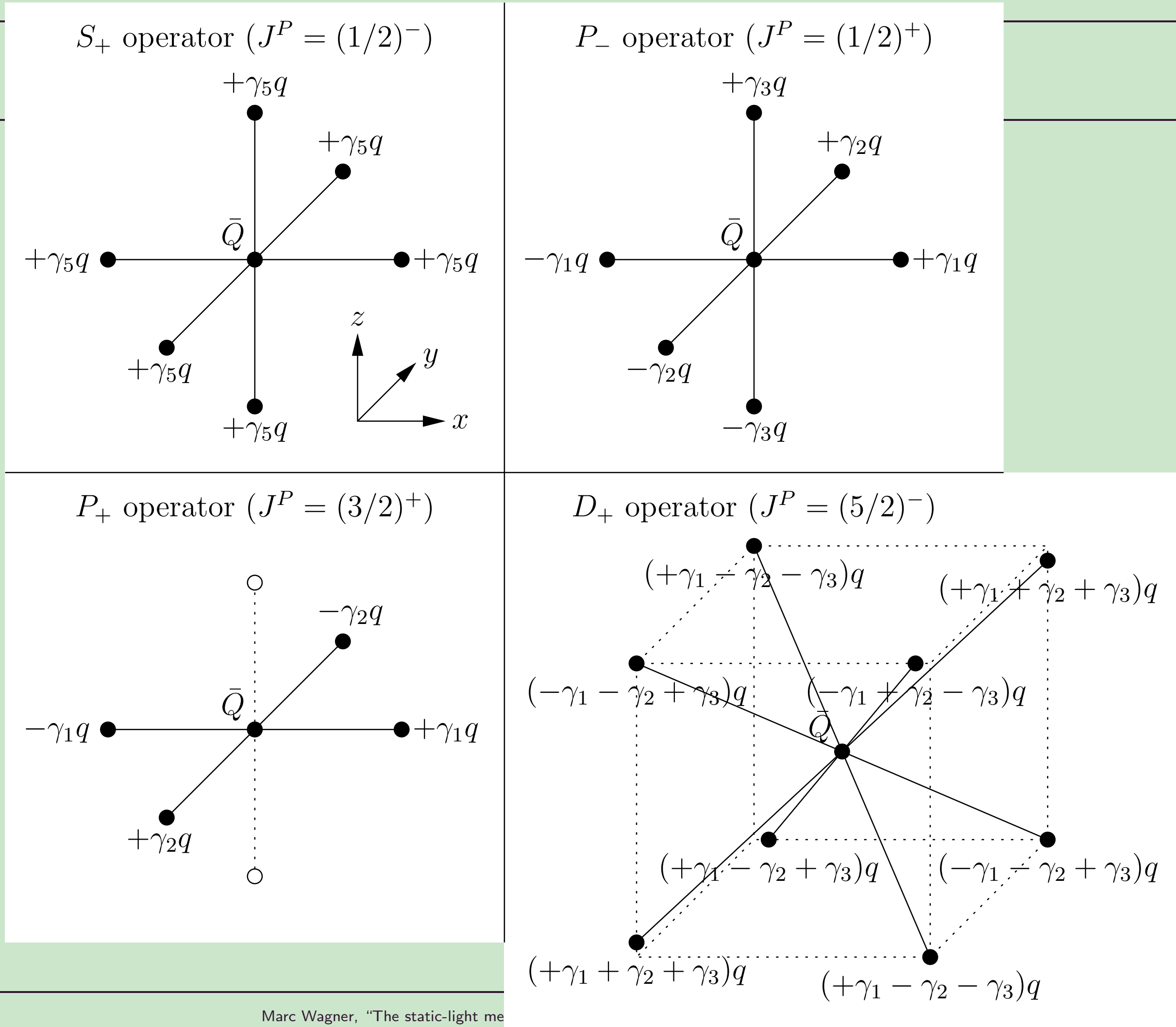
$$\mathcal{O}(\mathbf{x}) \quad = \quad \bar{Q}(\mathbf{x}) \int d\hat{\mathbf{n}} \, \Gamma(\hat{\mathbf{n}}) U(\mathbf{x}; \mathbf{x} + d\hat{\mathbf{n}}) q(\mathbf{x} + d\hat{\mathbf{n}}).$$

- **Static-light meson creation operators on the lattice**:

$$\mathcal{O}^{\text{6-path}}(\mathbf{x}) \quad = \quad \bar{Q}(\mathbf{x}) \sum_{\hat{\mathbf{n}} = \pm \mathbf{e}_1, \pm \mathbf{e}_2, \pm \mathbf{e}_3} \Gamma(\hat{\mathbf{n}}) U(\mathbf{x}; \mathbf{x} + d\hat{\mathbf{n}}) q(\mathbf{x} + d\hat{\mathbf{n}}) \quad , \quad d \in \mathbb{N}_+$$

$$\mathcal{O}^{\text{8-path}}(\mathbf{x}) \quad = \quad \bar{Q}(\mathbf{x}) \sum_{\hat{\mathbf{n}} = \pm \mathbf{e}_1 \pm \mathbf{e}_2 \pm \mathbf{e}_3} \Gamma(\hat{\mathbf{n}}) U(\mathbf{x}; \mathbf{x} + d\hat{\mathbf{n}}) q(\mathbf{x} + d\hat{\mathbf{n}}) \quad , \quad d \in \mathbb{N}_+.$$

- **Main difference**:
  - The integrations over spheres $\int d\hat{\mathbf{n}}$ are replaced by finite sums $\sum_{\hat{\mathbf{n}}}$.
  - Spherical harmonics contained in Γ are approximated by six or eight points respectively.



S_+ operator ($J^P = (1/2)^-$)
$+\gamma_5 q$
$+\gamma_5 q$
$\bar{Q}$
$+\gamma_5 q$
$+\gamma_5 q$
$+\gamma_5 q$
$+\gamma_5 q$
z
y
x
P_- operator ($J^P = (1/2)^+$)
$+\gamma_3 q$
$+\gamma_2 q$
$\bar{Q}$
$-\gamma_1 q$
$+\gamma_1 q$
$-\gamma_2 q$
$-\gamma_3 q$
P_+ operator ($J^P = (3/2)^+$)
$-\gamma_2 q$
$\bar{Q}$
$-\gamma_1 q$
$+\gamma_1 q$
$+\gamma_2 q$
D_+ operator ($J^P = (5/2)^-$)
$(+\gamma_1 - \gamma_2 - \gamma_3)q$
$(+\gamma_1 + \gamma_2 + \gamma_3)q$
$(-\gamma_1 - \gamma_2 + \gamma_3)q$
$(-\gamma_1 + \gamma_2 - \gamma_3)q$
$\bar{Q}$
$(+\gamma_1 - \gamma_2 + \gamma_3)q$
$(-\gamma_1 - \gamma_2 + \gamma_3)q$
$(+\gamma_1 + \gamma_2 + \gamma_3)q$
$(+\gamma_1 - \gamma_2 - \gamma_3)q$

Meson operators on the lattice (2)

- To determine the total angular momentum quantum numbers of lattice meson creation operators, expand them in terms of spherical harmonics:
  - Expansions are infinite sums.
  - Lattice operators have no well defined total angular momentum; they always create an infinite superposition of total angular momentum eigenstates.
  - In contrast to the continuum, where there is an infinite number of fixed angular momentum representations (continuous rotation group SO(3)), on the lattice there are only five different representations (discrete rotation group $\mathrm{O_h}$):

$$
\begin{aligned}
A_1 &\rightarrow L = 0, 4, 6, 8, \ldots \\
A_2 &\rightarrow L = 3, 6, 7, 9, \ldots \\
E &\rightarrow L = 2, 4, 5, 6, \ldots \\
T_1 &\rightarrow L = 1, 3, 4, 5(2\times), \ldots \\
T_2 &\rightarrow L = 2, 3, 4, 5, \ldots
\end{aligned}
$$

Further lattice techniques

- **Stochastic propagators**:
  - Statistical noise is significantly reduced.
  - Spatial smearing is easy.
- **Smearing techniques**:
  - HYP2 smearing of links in time direction to reduce the self energy of the static quark ($\rightarrow$ statistical noise is reduced).
  - Jacobi smearing of light quark operators and APE smearing of spatial links to increase ground state overlaps ($\rightarrow$ allows to extract static-light meson masses at smaller temporal separations, where the signal quality is better).
- **Correlation matrices**:
  - Increase ground state overlaps.
  - Extract excited states.

Simulation setup

- $24^3 \times 48$ lattices.

- Twisted mass Dirac operator with two degenerate flavors,

$$Q^{(\chi)} = \gamma_\mu D_\mu + m + i\mu\gamma_5 + \frac{a}{2}\Box \quad , \quad m + 4 = \frac{1}{2\kappa}$$

  with $\kappa = 0.160856$.

- Tree-level Symanzik improved gauge action with $\beta = 3.9$.

- Lattice spacing $a \approx 0.0855(5)\,\mathrm{fm}$, spatial lattice extension $24 \times a \approx 2.05\,\mathrm{fm}$.

μ	m_π in MeV	number of gauges
0.0040	$314(2)$	1400
0.0064	$391(1)$	1450
0.0085	$448(1)$	1350
0.0100	$485(1)$	0 (≈ 1000 planned)
0.0150	$597(2)$	250 (≈ 1000 planned)

Results (1)

- To compute ground states and excited states, consider 6×6 correlation matrices

$$\mathcal{C}_{jk}(T) \quad = \quad \langle\Omega|\Big(\mathcal{O}_j(\mathbf{x},T)\Big)^\dagger \mathcal{O}_k(\mathbf{x},0)|\Omega\rangle.$$

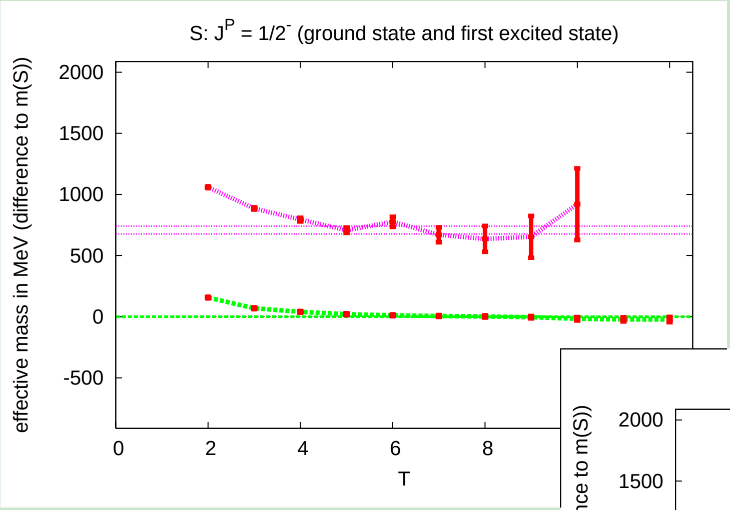
  - Different smearing levels, i.e. different meson extensions.
  - Operators with parity $P = +$ and $P = -$ in the same correlation matrix, because of parity mixing induced by the twisted mass Dirac operator.
  - Fixed total angular momentum J for each correlation matrix.

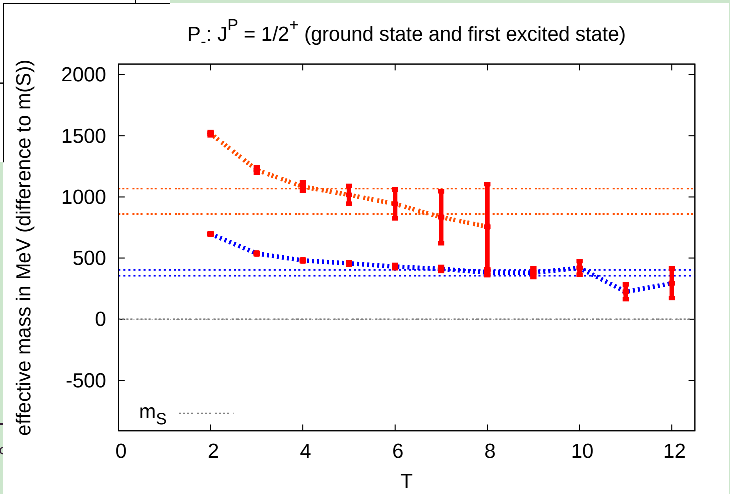
- Two approaches:
  - Effective masses by solving a generalized eigenvalue problem (visualization of static-light meson masses and their statistical accuracy).
  - χ^2 fitting of an ansatz of exponentials to the correlation matrices (numerical values and statistical errors for static-light meson masses).
  - Both approaches yield consistent results.

Results (2)

state	J^P	$m - m_S$ in MeV
S^*	$(1/2)^-$	$709(32)$
P_- P_-^*	$(1/2)^+$	$379(24)$ $965(104)$
P_+ P_+^*	$(3/2)^+$	
D_-	$(3/2)^-$	
D_+	$(5/2)^-$	
F_-	$(5/2)^+$	

- $\mu = 0.0040$, $J = 1/2$: S $(P = -)$ and P_- $(P = +)$.

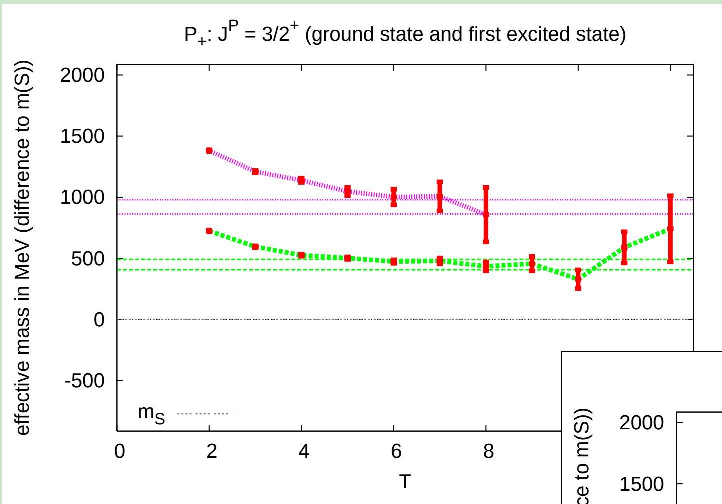


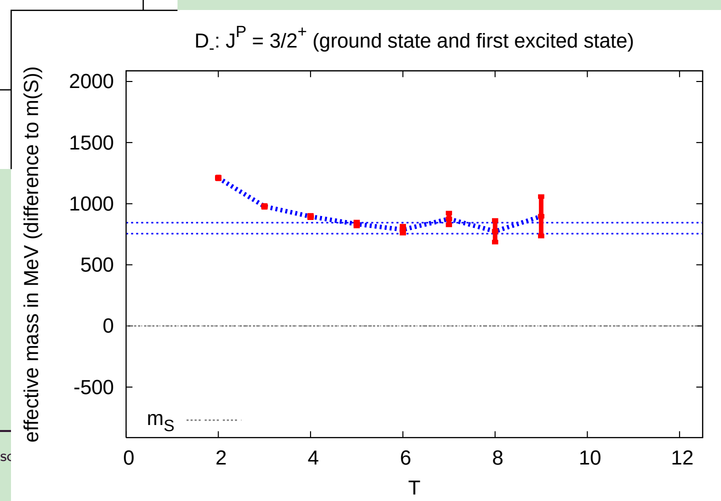


Results (3)

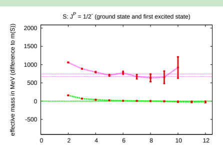
- $\mu = 0.0040$, $J = 3/2$: P_+ $(P = +)$ and D_- $(P = -)$.

state	J^P	$m - m_S$ in MeV
S^*	$(1/2)^-$	$709(32)$
P_- P_-^*	$(1/2)^+$	$379(24)$ $965(104)$
P_+ P_+^*	$(3/2)^+$	$449(43)$ $921(58)$
D_-	$(3/2)^-$	$800(45)$
D_+	$(5/2)^-$	
F_-	$(5/2)^+$	

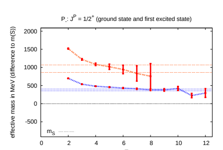




S



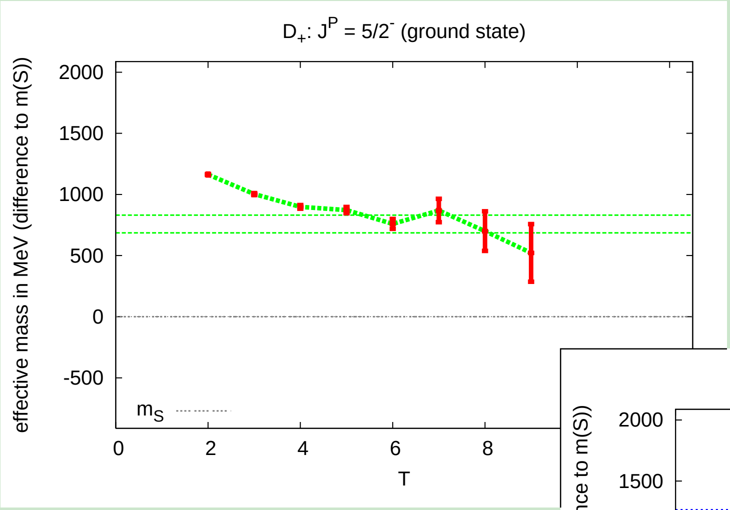
P_-

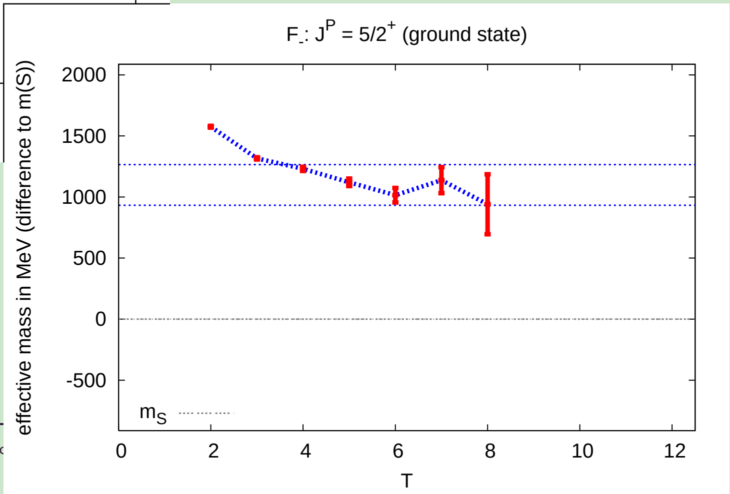


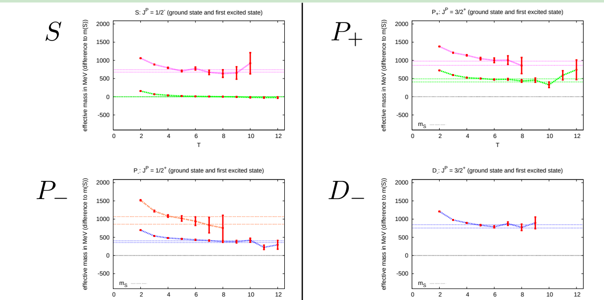
Results (4)

state	J^P	$m - m_S$ in MeV
S^*	$(1/2)^-$	$709(32)$
P_- P_-^*	$(1/2)^+$	$379(24)$ $965(104)$
P_+ P_+^*	$(3/2)^+$	$449(43)$ $921(58)$
D_-	$(3/2)^-$	$800(45)$
D_+	$(5/2)^-$	$758(73)$
F_-	$(5/2)^+$	$1099(166)$

- $\mu = 0.0040$, $J = 5/2$: D_+ $(P = -)$ and F_- $(P = +)$.

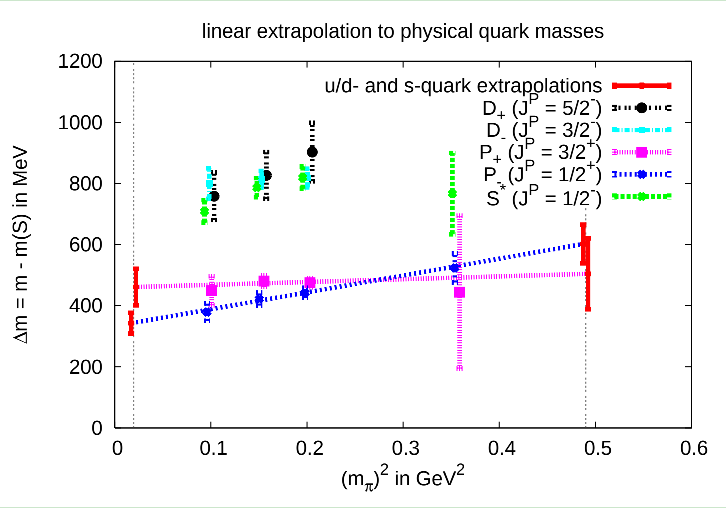






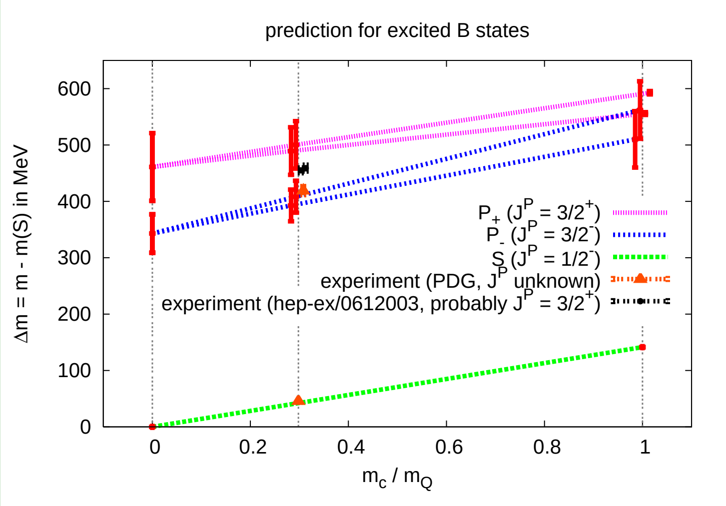
Results (5)

- Linear extrapolation in $(m_\pi)^2$ to physical light quark masses:
  - B mesons: u/d quark extrapolation ($m_\pi = 139.6\,\mathrm{MeV}$).
  - B_s mesons: s quark extrapolation ("$m_\pi = 700.0\,\mathrm{MeV}$").
    - ∗ However: sea of two degenerate s quarks.



Results (6)

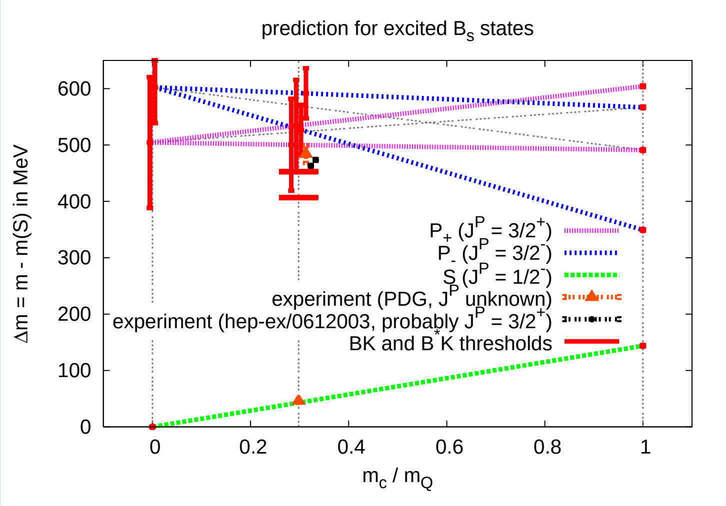
- Prediction for excited B states B_0^*, B_1^*, B_1 and B_2^* (P wave states):
  - Linear interpolation in m_c/m_Q to physical b quark mass (input: u/d extrapolated lattice data for $m_Q = \infty$, experimental data for $m_Q = m_c$).
- Status of experimental results:
  - PDG: one excited state, J^P unknown.
  - CDF and CØ collaborations (hep-ex/0612003): two excited states, B_1 and B_2^*.



	$m - m(S)$ in MeV		
state	lattice	PDG	hep-ex/...
B_0^*	$393(28)$		-
B_1^*	$408(28)$	↑	-
B_1	$489(42)$	$419(8)$	$455(5)$
B_2^*	$500(42)$	↓	$459(6)$

Results (7)

- Prediction for excited B_s states B^*_{s0}, B^*_{s1}, B_{s1} and B^*_{s2} (P wave states):
  - Linear interpolation in m_c/m_Q to physical b quark mass (input: s extrapolated lattice data for $m_Q = \infty$, experimental data for $m_Q = m_c$).
- Status of experimental results:
  - PDG: one excited state, J^P unknown.
  - CDF and CØ collaborations (hep-ex/0612003): two excited states, B_1 and B^*_2.



	$m - m(S)$ in MeV		
state	lattice	PDG	hep-ex/...
B^*_{s0}	$527(44)$		-
B^*_{s1}	$592(44)$	↑	-
B_{s1}	$500(81)$	$484(16)$	$463(1)$
B^*_{s2}	$534(81)$	↓	$474(2)$

Summary

- Static-light meson masses have been computed via twisted mass lattice QCD at a small value of the lattice spacing ($a = 0.0855\,\mathrm{fm}$) and at small values of the pion mass ($m_\pi = 314\,\mathrm{MeV}\,, \ldots\,, 597\,\mathrm{MeV}$):
  - Total angular momentum $J = 1/2\,, 3/2\,, 5/2$.
  - Parity $P = +\,, -$.
  - Ground states and first excited states.
- Interpolation/extrapolation to physical quark masses allow predictions for the spectrum of B mesons and B_s mesons. Results are in agreement with currently available experimental results within statistical errors.

Outlook

- Extrapolate to the continuum by considering other values for the lattice spacing.
- Include a sea of u/d quarks for B_s computations by using 2+1+1 flavor twisted mass lattice QCD.
- Compute static light decay constants f_B and f_{B_s}.