Studying mesons by means of lattice QCD

HIC for FAIR – Meeting of the PAC, Giessen

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Lattice activities of Expert Group 1

- I will speak about a specific topic within the lattice activities of the HIC for FAIR Expert Group 1

  “spectroscopy of mesons, studying the structure of mesons”

  (and related projects).

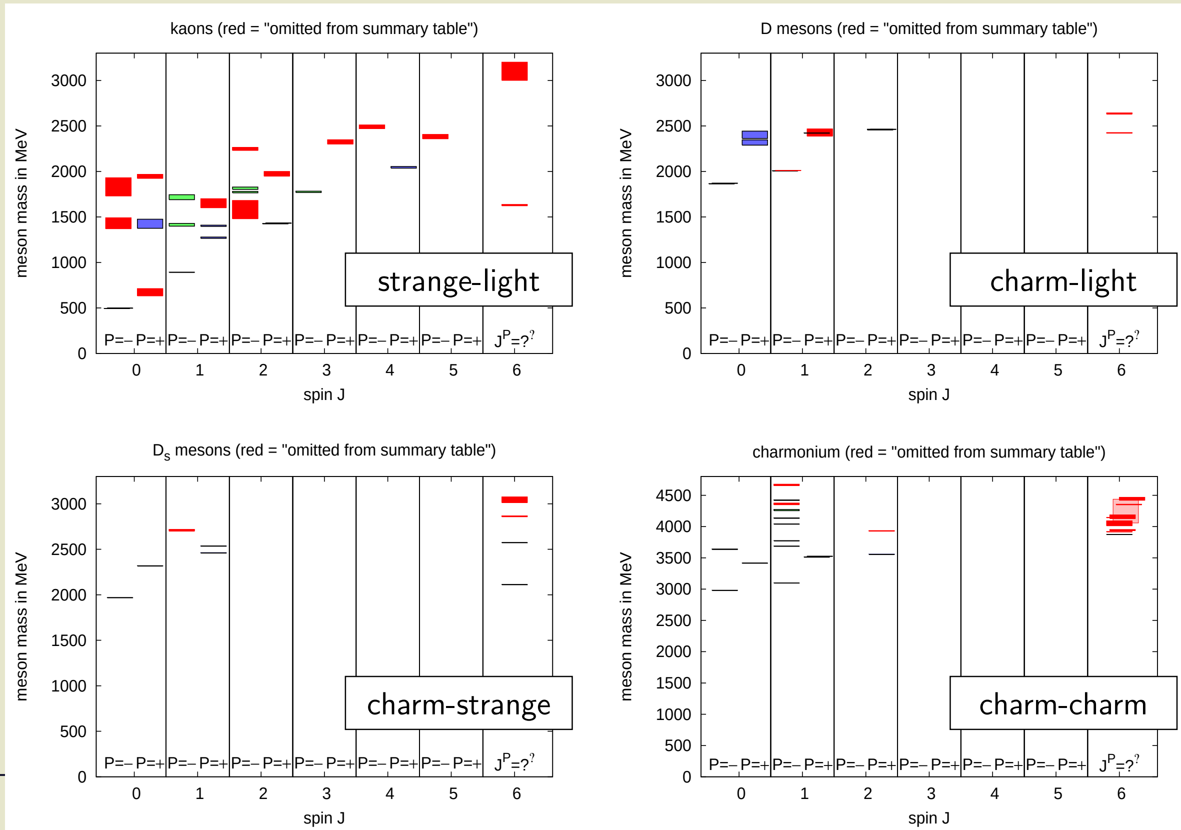
- There are other topics as well, which I will not discuss today; in particular also a lot of lattice work has been done and there are many impressive and nice results regarding “QCD at finite temperature and density” (cf. e.g. talks by Owe Philipsen at previous PAC meetings).

Goals, motivation (1)

- **Compute the meson spectrum as fully as possible using lattice QCD:**
  - In particular consider all mesons with at least one s or c quark, i.e.
    - kaons (strange-light mesons, K, $\kappa = K_0^*(800)$, ...),
    - D mesons (charm-light mesons, D, D^*, $D^{**} = \{D_0^*, D_1, D_2^*\}$, ...),
    - D_s mesons (charm-strange mesons, D_s, D_s^*, D_{s0}^*, D_{s1}, D_{s2}^*, ...),
    - charmonium (charm-charm mesons, η_c, J/ψ, ...),
    - "strangeonium" (strange-strange mesons, $a_0(980)$, $f_0(980)$, ...).
  - Consider parity $\pm$, charge conjugation $\pm$, radial and orbital excitations.
  - Lattice QCD setup:
    - Lattice QCD $\equiv$ from first principles (QCD), (ideally) no systematic errors.
    - 2+1+1 flavors of dynamical quarks (we [ETMC = European Twisted Mass Collaboration] are among the first to simulate the four lightest quark flavors [$\rightarrow$ very realistic setup]).

Goals, motivation (2)

- Experimental status (Particle Data Group): 73 known states.

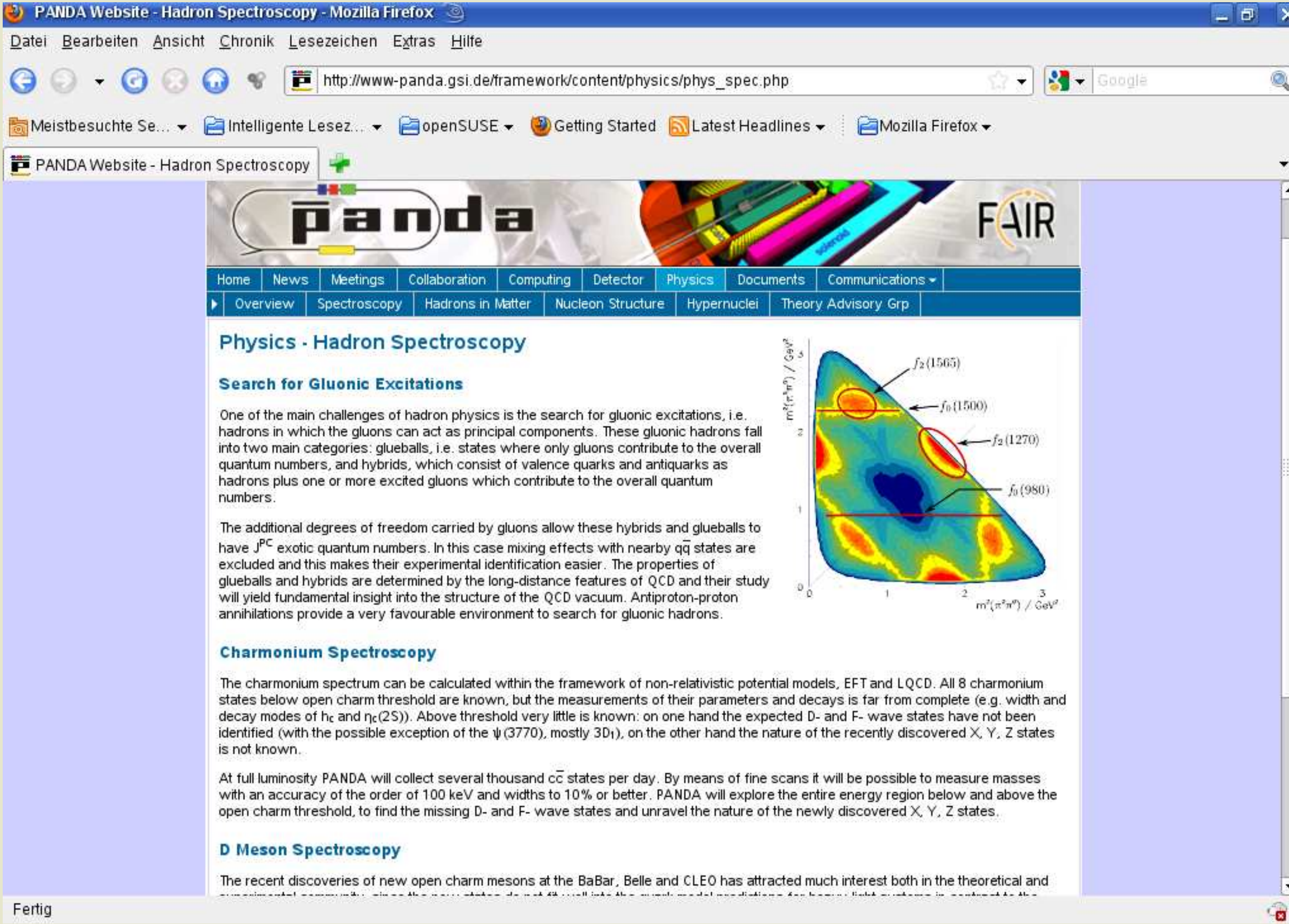


Goals, motivation (3)

- Why is a lattice computation of the s and c meson spectrum important?
  - Some mesons, e.g. D_s, η_c, J/ψ, have been measured experimentally with high precision and can also be computed on the lattice very accurately $\longrightarrow$ ideal candidates to test QCD by means of lattice QCD.
  - Some mesons are only poorly understood $\longrightarrow$ lattice QCD is the perfect tool to clarify the situation:
    - 31 meson states labeled with "omitted from summary table" (states colored red), i.e. vague experimental signals, experimental contradictions, states not well established.
    - Example $D_{s0}^*(2317)$, $D_{s1}(2460)$: masses significantly lower than expected from quark models, almost equal or even lower than the corresponding D mesons; could be tetraquarks, ...
    - Example $\kappa = K_0^*(800)$ ($I = 1/2 \ \rightarrow \ \bar{s}u$): part of a light scalar SU(3) nonet ... Why is the $a_0(980)$ ($I = 1 \ \rightarrow \ \bar{d}u$, belongs to the same nonet) heavier? Not understandable in a quark-antiquark picture.

Goals, motivation (4)

- Why is a lattice computation of the s and c meson spectrum important?
  - Lattice QCD predictions of meson masses could give valuable input for future experiments.
  - Comprehensive new information expected from FAIR (PANDA) and other existing and new facilities (BABAR, Belle, CEBAF, CLEO, upgraded BES, CDF, D0, LHC, ...) ... i.e. a "hot topic".
  - Lattice results for the s and c meson spectrum exist, but no comprehensive picture available at the moment (different discretizations, scale setting methods, numbers of quark flavors, sometimes rather coarse lattice spacings, unphysically heavy u/d quarks, no extrapolations).



Physics - Hadron Spectroscopy

Search for Gluonic Excitations

One of the main challenges of hadron physics is the search for gluonic excitations, i.e. hadrons in which the gluons can act as principal components. These gluonic hadrons fall into two main categories: glueballs, i.e. states where only gluons contribute to the overall quantum numbers, and hybrids, which consist of valence quarks and antiquarks as hadrons plus one or more excited gluons which contribute to the overall quantum numbers.

The additional degrees of freedom carried by gluons allow these hybrids and glueballs to have J^{PC} exotic quantum numbers. In this case mixing effects with nearby $q\bar{q}$ states are excluded and this makes their experimental identification easier. The properties of glueballs and hybrids are determined by the long-distance features of QCD and their study will yield fundamental insight into the structure of the QCD vacuum. Antiproton-proton annihilations provide a very favourable environment to search for gluonic hadrons.

Charmonium Spectroscopy

The charmonium spectrum can be calculated within the framework of non-relativistic potential models, EFT and LQCD. All 8 charmonium states below open charm threshold are known, but the measurements of their parameters and decays is far from complete (e.g. width and decay modes of h_c and $\eta_c(2S)$). Above threshold very little is known: on one hand the expected D- and F- wave states have not been identified (with the possible exception of the $\psi(3770)$, mostly $3D_1$), on the other hand the nature of the recently discovered X, Y, Z states is not known.

At full luminosity PANDA will collect several thousand $c\bar{c}$ states per day. By means of fine scans it will be possible to measure masses with an accuracy of the order of 100 keV and widths to 10% or better. PANDA will explore the entire energy region below and above the open charm threshold, to find the missing D- and F- wave states and unravel the nature of the newly discovered X, Y, Z states.

D Meson Spectroscopy

The recent discoveries of new open charm mesons at the BaBar, Belle and CLEO has attracted much interest both in the theoretical and

Outline

- A brief introduction into lattice QCD hadron spectroscopy.
  - QCD (quantum chromodynamics).
  - Hadron spectroscopy.
  - Lattice QCD.
- Three project examples:
  (1) Masses of D and D_s mesons.
  (2) Investigating tetraquark candidates.
  (3) Heavy-heavy-light-light tetraquarks.
- List of further ongoing/planned projects.

QCD (quantum chromodynamics)

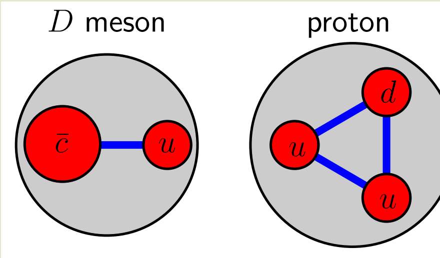
- Quantum field theory of quarks (six flavors u, d, s, c, t, b, which differ in mass) and gluons.

- Part of the standard model explaining the formation of hadrons (usually mesons $= q\bar{q}$ and baryons $= qqq/\bar{q}\bar{q}\bar{q}$) and their masses; essential for decays involving hadrons.

- Definition of QCD simple:

$$S = \int d^4x \left(\sum_{f \in \{u,d,s,c,t,b\}} \bar{\psi}^{(f)} \Big(\gamma_\mu \Big(\partial_\mu - i A_\mu \Big) + m^{(f)} \Big) \psi^{(f)} + \frac{1}{2g^2} \mathrm{Tr} \Big(F_{\mu\nu} F_{\mu\nu} \Big) \right)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu].$$



- However, no analytical solutions for low energy QCD observables, e.g. hadron masses, known, because of the absence of any small parameter (i.e. perturbation theory not applicable)
$\rightarrow$ solve QCD numerically by means of lattice QCD.

Hadron spectroscopy (1)

- Let $\mathcal{O}$ be a suitable "hadron creation operator", i.e. an operator such that $\mathcal{O}|\Omega\rangle$ is a state containing the hadron of interest ($|\Omega\rangle$: QCD vacuum).
- More precisely: ... an operator such that $\mathcal{O}|\Omega\rangle$ has the same quantum numbers (J^{PC}, flavor) as the hadron of interest.
- Examples:
  - Kaon ($K \quad \rightarrow \quad J^P = 0^-$) creation operator:

$$\mathcal{O}_K \quad = \quad \int d^3x \, \bar{s}(\mathbf{x})\gamma_5 u(\mathbf{x}).$$

  - D meson ($D \quad \rightarrow \quad J^P = 0^-$) creation operator:

$$\mathcal{O}_D \quad = \quad \int d^3x \, \bar{c}(\mathbf{x})\gamma_5 u(\mathbf{x}).$$

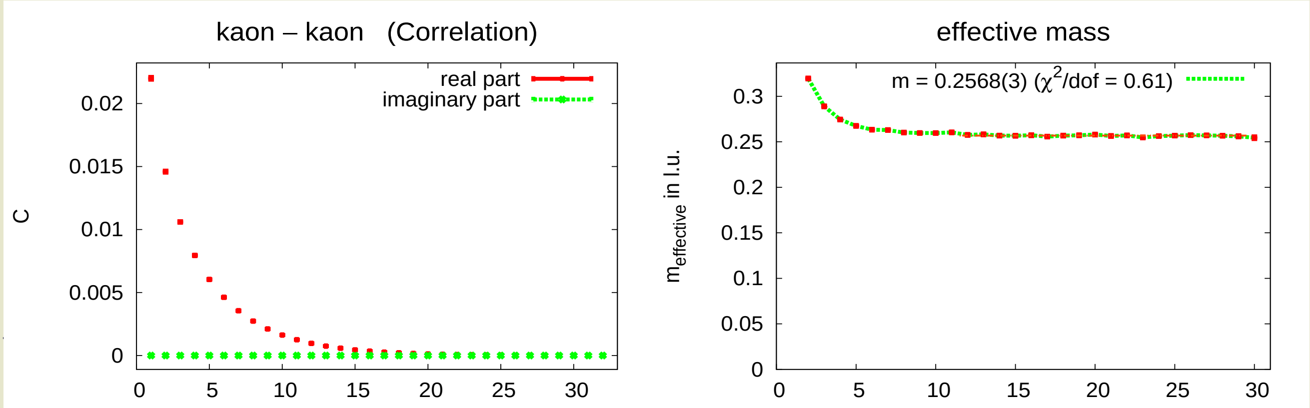
Hadron spectroscopy (2)

- Determine the mass of the ground state of the hadron of interest from the exponential behavior of the corresponding correlation function $\mathcal{C}$ at large Euclidean times T:

$$\mathcal{C}(t) \equiv \langle\Omega|\Big(\mathcal{O}(t)\Big)^{\dagger}\mathcal{O}(0)|\Omega\rangle = \langle\Omega|e^{+Ht}\Big(\mathcal{O}(0)\Big)^{\dagger}e^{-Ht}\mathcal{O}(0)|\Omega\rangle =$$

$$= \sum_n \Big|\langle n|\mathcal{O}(0)|\Omega\rangle\Big|^2 \exp\Big(-(E_n - E_\Omega)t\Big) \approx \quad (\text{for } t \gg 1)$$

$$\approx \Big|\langle 0|\mathcal{O}(0)|\Omega\rangle\Big|^2 \exp\Big(-\underbrace{(E_0 - E_\Omega)}_{m(\text{hadron})}t\Big) \propto \exp\Big(-m(\text{hadron})t\Big).$$



Lattice QCD (1)

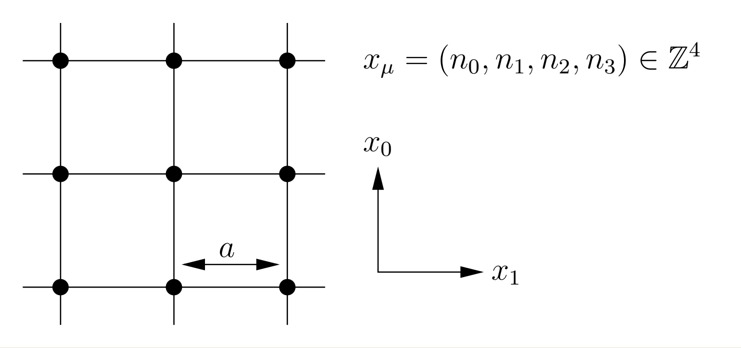
- Goal: compute QCD correlation functions $\mathcal{C}$ of meson creation operators numerically (corresponding hadron masses can be read off from their exponential behavior).
- Use the path integral formulation of QCD,

$$\mathcal{C}(t) = \langle\Omega|\Big(\mathcal{O}(t)\Big)^\dagger \mathcal{O}(0)|\Omega\rangle =$$
$$= \frac{1}{Z}\int \Big(\prod_f D\psi^{(f)}\, D\bar{\psi}^{(f)}\Big) DA_\mu \Big(\mathcal{O}(t)\Big)^\dagger \mathcal{O}(0) e^{-S[\psi^{(f)},\bar{\psi}^{(f)},A_\mu]}.$$

  - $|\Omega\rangle$: ground state/vacuum.
  - $(\mathcal{O}(t))^\dagger, \mathcal{O}(0)$: functions of the quark and gluon fields (cf. previous slides).
  - $\int(\prod_f D\psi^{(f)}\, D\bar{\psi}^{(f)}) DA_\mu$: integral over all possible quark and gluon field configurations $\psi^{(f)}(\mathbf{x}, t)$ and $A_\mu(\mathbf{x}, t)$.
  - $e^{-S[\psi^{(f)},\bar{\psi}^{(f)},A_\mu]}$: weight factor containing the QCD action.

Lattice QCD (2)

- Numerical implementation of the path integral formalism in QCD:
  - Discretize spacetime with sufficiently small lattice spacing $a \approx 0.05\,\text{fm} \ldots 0.10\,\text{fm}$
    $\rightarrow$ "continuum physics".
  - "Make spacetime periodic" with sufficiently large extension $L \approx 2.0\,\text{fm} \ldots 4.0\,\text{fm}$ (4-dimensional torus)
    $\rightarrow$ "no finite size effects".



Lattice QCD (3)

- Numerical implementation of the path integral formalism in QCD:
  - After discretization the path integral becomes an ordinary multidimensional integral:

$$\int D\psi\, D\bar{\psi}\, DA \ldots \quad \rightarrow \quad \prod_{x_\mu} \bigg(\int d\psi(x_\mu)\, d\bar{\psi}(x_\mu)\, dU(x_\mu) \bigg) \ldots$$

  - Typical present-day dimensionality of a discretized QCD path integral:
    * x_μ: $32^4 \approx 10^6$ lattice sites.
    * $\psi = \psi_A^{a,(f)}$: 24 quark degrees of freedom for every flavor ($\times 2$ particle/antiparticle, $\times 3$ color, $\times 4$ spin), 2 flavors.
    * $U = U_\mu^{ab}$: 32 gluon degrees of freedom ($\times 8$ color, $\times 4$ spin).
    * In total: $32^4 \times (2 \times 24 + 32) \approx \mathbf{83} \times \mathbf{10^6}$ dimensional integral.

  $\longrightarrow$ standard approaches for numerical integration not applicable

  $\longrightarrow$ sophisticated algorithms mandatory (stochastic integration techniques, so-called Monte-Carlo algorithms).

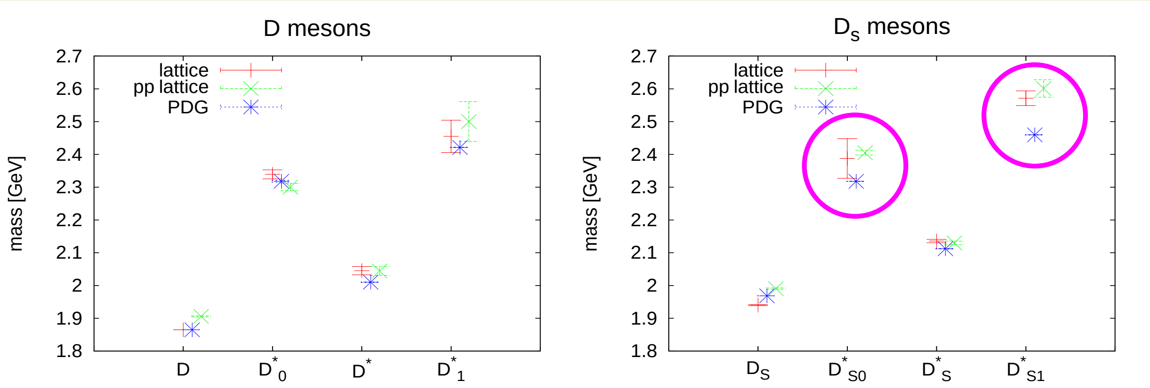
Masses of D and D_s mesons (1)

- First preliminary results of a large scale project.

  [M. Kalinowski, M.W., poster at "Quark Confinement and the Hadron Spectrum X", Munich (2012)]

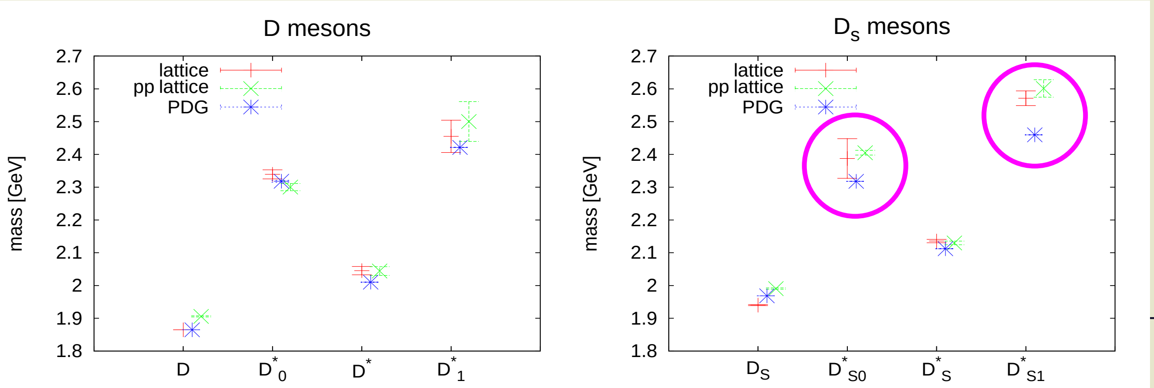
- D and D_s meson states computed (in the plots from left to right):
  - $J^P = 0^-$: D, D_s.
  - $J^P = 0^+$: D_0^*, D_{s0}^*.
  - $J^P = 1^-$: D^*, D_s^*.
  - $J^P = 1^+$: D_1, D_{s1}.

- **FAIR (PANDA)**: D/D_s meson spectroscopy, ...



Masses of D and D_s mesons (2)

- Different lattice discretizations (red points and green points) indicate that discretization errors are $\lesssim 2\%$ (will be removed in the near future).
- Experimental meson masses: blue points.
  - Lattice results in agreement with experimental results for most states.
  - Disagreement only for D^*_{s0} and D_{s1}:
    - Similar disagreement, when using $q\bar{q}$ phenomenological models.
    - Indication that an ordinary quark-antiquark picture is not valid for D^*_{s0} and D_{s1} (could be DK molecules, diquark-antidiquark pairs, ...).



Investigating tetraquark candidates (1)

- D_{s0}^* and D_{s1} do not seem to be ordinary quark-antiquark states ... could be four quark states, i.e. tetraquarks ...?
- Another prominent tetraquark candidate is the nonet of light scalar mesons,
  - $\sigma \equiv f_0(500)$, $I = 0$, $400 \ldots 550$ MeV,
  - $\kappa \equiv K_0^*(800)$, $I = 1/2$, 682 ± 29 MeV,
  - $f_0(980)$, $a_0(980)$, $I = 0, 1$, 990 ± 20 MeV, 980 ± 20 MeV

  ($J^P = 0^+$), which is poorly understood:
  - All nine states are unexpectedly light (should rather be close to the corresponding $J^P = 1^+, 2^+$ states around $1200 \ldots 1500$ MeV).
  - The ordering of states is inverted compared to expectation:
    - E.g. in a $q\bar{q}$ picture the $I = 1$ $a_0(980)$ states must necessarily be formed by two u/d quarks (e.g. $a_0(980) \equiv \bar{d}u$), while the $I = 1/2$ κ states are e.g. $\kappa \equiv \bar{s}u$; since $m_s > m_{u/d}$ one would expect $m(\kappa) > m(a_0(980))$ ... which is not the case.

Investigating tetraquark candidates (2)

  * In a tetraquark picture the quark content could be the following: $\kappa \equiv \bar{s}l\bar{l}l$, while $a_0(980) \equiv \bar{s}l\bar{l}s$; this would naturally explain the observed ordering.
- Certain decays also support a tetraquark interpretation: e.g. $a_0(980)$ readily decays to $K + \bar{K}$, which indicates that besides the two light quarks $\bar{d}u$ required by $I = 1$ also an $s\bar{s}$ pair is present.

$\rightarrow$ Study these states by means of lattice QCD to confirm or to rule out their interpretation in terms of tetraquarks.

- Further examples of heavy mesons, which are tetraquark candidates: charmonium states $X(3872)$, $Z(4430)^{\pm}$, $Z(4050)^{\pm}$, $Z(4250)^{\pm}$, ...
- **FAIR (PANDA)**: charmonium spectroscopy, D/D_s meson spectroscopy, search for gluonic excitations (light scalar mesons), ...

Investigating tetraquark candidates (3)

- Tetraquark operators for $a_0(980)$ (quantum numbers $I(J^P) = 1(0^+)$):
  - Needs two light quarks due to $I = 1$, e.g. $u\bar{d}$.
  - $a_0(980)$ decays to $K\bar{K}$ ... suggests an $s\bar{s}$ component.
  - Molecule type (models a bound $K\bar{K}$ state):

$$\mathcal{O}_{a_0(980)}^{K\bar{K}\text{ molecule}} = \int d^3x \, \Big(\bar{s}(\mathbf{x})\gamma_5 u(\mathbf{x})\Big)\Big(\bar{d}(\mathbf{x})\gamma_5 s(\mathbf{x})\Big).$$

  - Diquark type (models a bound diquark-antidiquark):

$$\mathcal{O}_{a_0(980)}^{\text{diquark}} = \int d^3x \, \Big(\epsilon^{abc}\bar{s}^b(\mathbf{x})C\gamma_5\bar{d}^{c,T}(\mathbf{x})\Big)\Big(\epsilon^{ade}u^{d,T}(\mathbf{x})C\gamma_5 s^e(\mathbf{x})\Big).$$

Investigating tetraquark candidates (4)

- Note that there are also two-particle states, which have the same quantum numbers as $a_0(980)$, $I(J^P) = 1(0^+)$,
  - $K + \bar{K}$ ($m(K) \approx 500\,\text{MeV}$),
  - $\eta + \pi$ ($m(\eta) \approx 700\,\text{MeV}$, $m(\pi) \approx 300\,\text{MeV}$ in our lattice setup),

  which are both around the expected $a_0(980)$ mass $980 \pm 20\,\text{MeV}$
  $\rightarrow$ appropriate two-particle operators are needed:
  - Two particle $K + \bar{K}$ type:

$$\mathcal{O}^{K+\bar{K}\ \text{two-particle}}_{a_0(980)\ \text{quantum numbers}} = \bigg(\int d^3x\, \bar{s}(\mathbf{x})\gamma_5 u(\mathbf{x})\bigg)\bigg(\int d^3y\, \bar{d}(\mathbf{y})\gamma_5 s(\mathbf{y})\bigg).$$

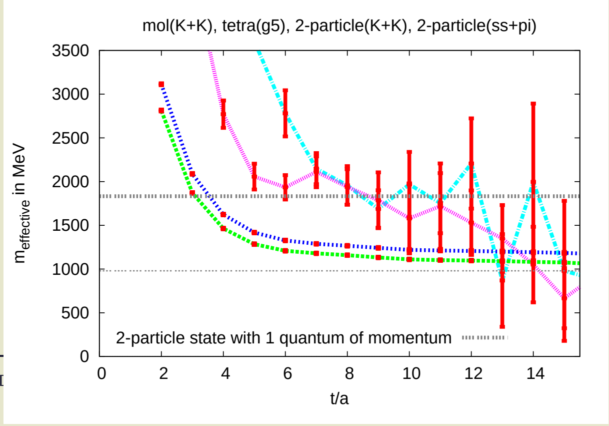
  - Two particle $\eta + \pi$ type:

$$\mathcal{O}^{\eta+\pi\ \text{two-particle}}_{a_0(980)\ \text{quantum numbers}} = \bigg(\int d^3x\, \bar{s}(\mathbf{x})\gamma_5 s(\mathbf{x})\bigg)\bigg(\int d^3y\, \bar{d}(\mathbf{y})\gamma_5 u(\mathbf{y})\bigg).$$

Investigating tetraquark candidates (5)

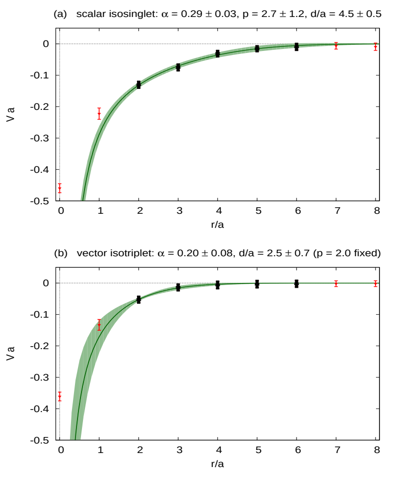
- Current status of our computations:
  - There are two states around the expected $a_0(980)$ mass 980 ± 20 MeV:
    - one has $\gtrsim 95\%$ operator content "two-particle $\eta + \pi$",
    - the other has $\gtrsim 95\%$ operator content "two-particle $K + \bar{K}$".
  - Higher states have energies $\gtrsim 1700\,\mathrm{MeV}$ (consistent with two-particle $\eta + \pi$ and $K + \bar{K}$ excitations with one relative quantum of momentum).
- **Conclusions: $a_0(980)$ is not a strongly bound four-quark state … probably a rather unstable resonance.**
- Similar results for κ.

  [M.W., C. Alexandrou, J. O. Daldrop, M. Dalla Brida, M. Gravina, L. Scorzato, C. Urbach, C. Wiese, talk at "Quark Confinement and the Hadron Spectrum X", Munich (2012)]
- Investigation of $a_0(980)$ as a resonance ongoing (very challenging in lattice QCD).



Heavy-heavy-light-light tetraquarks (1)

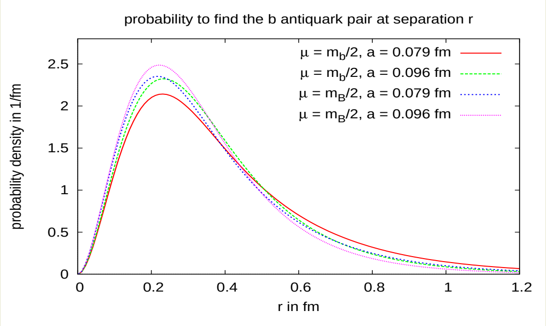
- Study possibly existing $QQ\bar{q}\bar{q}$ (heavy-heavy-light-light) tetraquark states:
  - Use the static approximation for the heavy quarks QQ (reduces the necessary computation time significantly).
  - Most appropriate for $QQ \equiv bb$.
  - Could also yield information about $QQ \equiv cc$.
- Proceed in two steps:
  (1) Compute the potential of two heavy quarks QQ in the background of two light antiquarks $\bar{q}\bar{q}$ by means of lattice QCD
  $\longrightarrow$ many different channels/quantum numbers.
  [M.W., PoS LATTICE **2010**, 162 (2010) [arXiv:1008.1538 [hep-lat]]]
  [M.W., Acta Phys. Polon. Supp. **4**, 747 (2011) [arXiv:1103.5147 [hep-lat]]]
  (2) Solve the non-relativistic Schrödinger equation for the relative coordinate of the heavy quarks QQ.



Heavy-heavy-light-light tetraquarks (2)

- Clear indication for a bound state for $QQ \equiv bb$ in a specific channel:
  - Quantum numbers: $I(J^P) = 0(0^+)\, ,\, 0(1^+)$ (degeneracy with respect to the heavy quark spin).
  - Binding energy: $E \approx -50\,\mathrm{MeV}$.

  [P. Bicudo, M.W., arXiv:1209.6274 [hep-ph]]



- No four-quark binding in other channels.
- Next steps:
  - Extend these investigations to the experimentally more interesting case of $Q\bar{Q}$ (instead of QQ).
  - Statements about $QQ = cc$ and $Q\bar{Q} = c\bar{c}$ (instead of $QQ = bb$ and $Q\bar{Q} = b\bar{b}$).
  - **FAIR (PANDA)**: charmonium spectroscopy, ...

Further ongoing/planned projects (1)

- Develop/optimize numerical methods, to increase statistical precision of "lattice meson results".

  [C. Wiese, M.W. "Efficient computation of meson and four-quark-correlation functions," in preparation]

- Determine resonance parameters of unstable mesons (e.g. the previously discussed $a_0(980)$ and κ).

  [Ongoing: C. Alexandrou, J. O. Daldrop, M. Dalla Brida, M. Gravina, L. Scorzato, C. Urbach, M.W., C. Wiese]

- Study glueballs and their mixing with light scalar mesons.

  [A. Uenver, "Numerische Bestimmung von Glueball-Massen mit SU(2) Gittereichtheorie," bachelor thesis GU Frankfurt am Main (2012)]

- Study mesons with gluonic excitations (hybrids).

Further ongoing/planned projects (2)

- Combine lattice methods with model calculations for mesons: computation of inhomogeneous condensates in the parity doublet model (cf. talk of F. Giacosa at 17:30)
  $\longrightarrow$ joint project together with Expert Group 2.

  [Ongoing: F. Giacosa, A. Heinz, D. Rischke, J. Scheunert, M.W.]

- Study the strong force between a quark and an antiquark.

  [K. Jansen, F. Karbstein, A. Nagy, M.W., "$\Lambda_{\bar{MS}}$ from the static potential for QCD with $n_f = 2$ dynamical quark flavors," JHEP **1201**, 025 (2012) [arXiv:1110.6859 [hep-ph]]]

  [F. Bruckmann, S. Dinter, E. -M. Ilgenfritz, B. Maier, M. Muller-Preussker, M.W., "Confining dyon gas with finite-volume effects under control," Phys. Rev. D **85**, 034502 (2012) [arXiv:1111.3158 [hep-ph]]]

  [M.W., O. Philipsen, "Definitions of a static SU(2) color triplet potential," talk at "Quark Confinement and the Hadron Spectrum X", Munich (2012)]

Conclusions

- The lattice results for mesons and tetraquark candidates presented are:
  - Preliminary

    $\longrightarrow$ certain systematic errors need to be studied and quantified, e.g. lattice discretization errors, unphysically heavy u/d quark masses

    $\longrightarrow$ statistical errors need to be reduced.

    (The projects discussed have been started only around one year ago; lattice projects typically need a lot of human and HPC resources, i.e. are long-term projects taking several years.)
  - Promising

    $\longrightarrow$ contact to experimental results established (e.g. D, D_s spectrum)

    $\longrightarrow$ first statements about states, which are presently not well understood (tetraquark candidates, $a_0(980)$, κ, ...).
- **Long-term goal: meson spectroscopy/structure from first principles (QCD) free of systematic errors, which is relevant and important in the context of FAIR/PANDA.**