# Importance of meson-meson and of diquark-antidiquark creation operators for a $\bar{b}\bar{b}ud$ tetraquark

"Experimental and theoretical status of and perspectives for $XYZ$ states"


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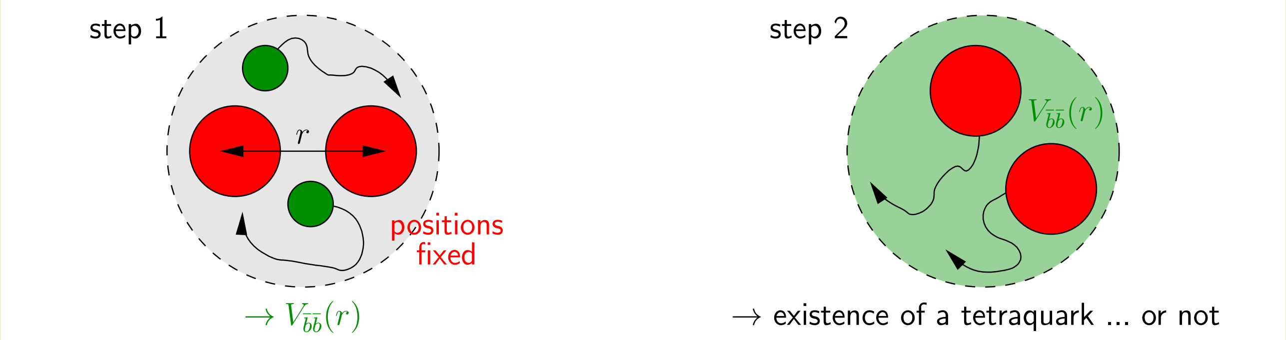
# Basic idea: lattice QCD + BO

- Study heavy-heavy-light-light tetraquarks $\bar{b}\bar{b}qq$ in two steps.

  (1) **Compute potentials of two static quarks $\bar{b}\bar{b}$ in the presence of two lighter quarks $qq$ ($q \in \{u, d, s, c\}$) using lattice QCD.**

  (2) **Check, whether these potentials are sufficiently attractive to host bound states or resonances** ($\rightarrow$ tetraquarks) **by using techniques from quantum mechanics and scattering theory.**

  ((1) + (2) $\rightarrow$ Born-Oppenheimer approximation).



# Previous work on $\bar{b}\bar{b}qq$ tetraquarks

- Lattice QCD static potentials and Born-Oppenheimer approximation.
  [W. Detmold, K. Orginos, M. J. Savage, Phys. Rev. D **76**, 114503 (2007) [arXiv:hep-lat/0703009]]
  [M.W., PoS **LATTICE2010**, 162 (2010) [arXiv:1008.1538]]
  [G. Bali, M. Hetzenegger, PoS **LATTICE2010**, 142 (2010) [arXiv:1011.0571]]
  [P. Bicudo, M.W., Phys. Rev. D **87**, 114511 (2013) [arXiv:1209.6274]]
  [Z. S. Brown, K. Orginos, Phys. Rev. D **86**, 114506 (2012) [arXiv:1210.1953]]
  [P. Bicudo, K. Cichy, A. Peters, B. Wagenbach, M.W., Phys. Rev. D **92**, 014507 (2015) [arXiv:1505.00613]]
  [P. Bicudo, K. Cichy, A. Peters, M.W., Phys. Rev. D **93**, 034501 (2016) [arXiv:1510.03441]]
  [P. Bicudo, J. Scheunert, M.W., Phys. Rev. D **95**, 034502 (2017) [arXiv:1612.02758]]
  [P. Bicudo, M. Cardoso, A. Peters, M. Pflaumer, M.W., Phys. Rev. D **96**, 054510 (2017) [arXiv:1704.02383]]
  [P. Bicudo, A. Peters, S. Velten, M.W., arXiv:2101.00723]

- Full lattice QCD ($b$ quarks with Non Relativistic QCD) [list not complete]:
  [A. Francis, R. J. Hudspith, R. Lewis, K. Maltman, Phys. Rev. Lett. **118**, 142001 (2017) [arXiv:1607.05214 [hep-lat]]]
  [P. Junnarkar, N. Mathur, M. Padmanath, Phys. Rev. D **99**, 034507 (2019) [arXiv:1810.12285 [hep-lat]]]
  [L. Leskovec, S. Meinel, M. Pflaumer, M.W., Phys. Rev. D **100**, 014503 (2019) [arXiv:1904.04197] [hep-lat]]]

- Other approches: quark models, effective field theories, QCD sum rules ... [list not complete]:
  [M. Karliner, J. L. Rosner, Phys. Rev. Lett. **119**, 202001 (2017) [arXiv:1707.07666]]
  [E. J. Eichten, C. Quigg, Phys. Rev. Lett. **119**, 202002 (2017) [arXiv:1707.09575]]
  [Z. G. Wang, Acta Phys. Polon. B **49**, 1781 (2018) [arXiv:1708.04545]]
  [W. Park, S. Noh, S. H. Lee, Acta Phys. Polon. B **50**, 1151-1157 (2019) [arXiv:1809.05257]]
  [B. Wang, Z. W. Liu, X. Liu, Phys. Rev. D **99**, 036007 (2019) [arXiv:1812.04457]]
  [M. Z. Liu, T. W. Wu, M. Pavon Valderrama, J. J. Xie, L. S. Geng, Phys. Rev. D **99**, 094018 (2019) [arXiv:1902.03044]]

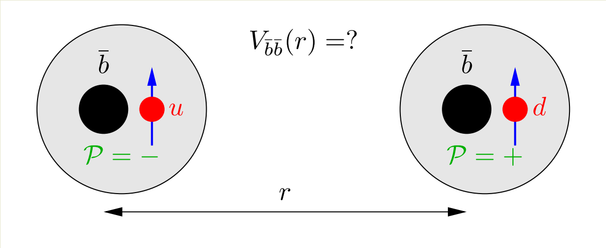
# Outline

- $\bar{b}\bar{b}qq$ / $BB$ potentials.
- Stable $\bar{b}\bar{b}qq$ tetraquarks.
- Structure of a $\bar{b}\bar{b}qq$ tetraquark with quantum numbers $I(J^P) = 0(1^+)$ (meson-meson versus diquark-antidiquark structure).

# $\bar{b}\bar{b}qq$ / $BB$ potentials (1)

- At large $\bar{b}\bar{b}$ separation $r$, the four quarks will form two static-light mesons $\bar{b}q$ and $\bar{b}q$.
- Spins of static antiquarks $\bar{b}\bar{b}$ are irrelevant (they do not appear in the Hamiltonian).
- Compute and study the dependence of $\bar{b}\bar{b}$ potentials in the presence of $qq$ on
  - the "light" quark flavors $q \in \{u, d, s, c\}$ (isospin, flavor),
  - the "light" quark spin (the static quark spin is irrelevant),
  - the type of the meson $B$, $B^*$ and/or $B_0^*$, $B_1^*$ (parity).

$\rightarrow$ Many different channels: attractive as well as repulsive, different asymptotic values ...

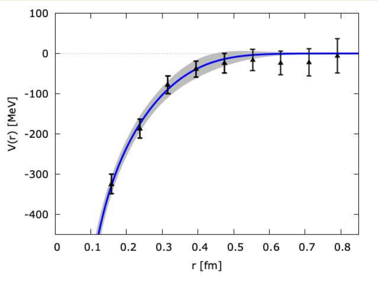


# $\bar{b}\bar{b}qq$ / $BB$ potentials (2)

- To determine potentials, compute temporal correlation functions of operators

$$\mathcal{O}_{BB,\Gamma} \quad = \quad 2N_{BB}(\mathcal{C}\Gamma)_{AB}(\mathcal{C}\tilde{\Gamma})_{CD}\Big(\bar{Q}^a_C(-\mathbf{r}/2)\psi^{(f)a}_A(-\mathbf{r}/2)\Big)\Big(\bar{Q}^b_D(+\mathbf{r}/2)\psi^{(f')b}_B(+\mathbf{r}/2)\Big).$$

- The most attractive potential of a $B^{(*)}B^*$ meson pair has $(I, |j_z|, P, P_x) = (0, 0, +, -)$:
  - $C = \gamma_0\gamma_2$ (charge conjugation matrix).
  - $\psi^{(f)}\psi^{(f')} = ud - du$, $\Gamma \in \{(1+\gamma_0)\gamma_5\,,\ (1-\gamma_0)\gamma_5\}$.
  - $\bar{Q}\bar{Q} = \bar{b}\bar{b}$, $\tilde{\Gamma} \in \{(1+\gamma_0)\gamma_5\,,\ (1+\gamma_0)\gamma_j\}$ (irrelevant).




- Parameterize lattice results by

$$V_{qq,j_z,\mathcal{P},\mathcal{P}_x}(r) \quad = \quad -\frac{\alpha}{r}\exp\bigg(-\Big(\frac{r}{d}\Big)^p\bigg) + V_0$$

(1-gluon exchange at small $r$; color screening at large $r$).
[P. Bicudo, K. Cichy, A. Peters, M.W., Phys. Rev. D **93**, 034501 (2016) [arXiv:1510.03441]]

# Stable $\bar{b}\bar{b}qq$ tetraquarks

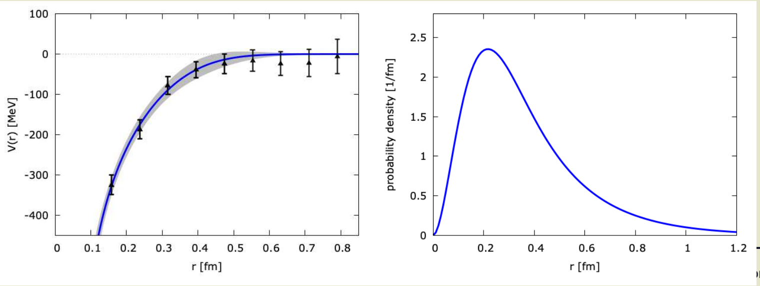
- Solve the Schrödinger equation for the relative coordinate of the heavy quarks $\bar{b}\bar{b}$ using the previously computed $\bar{b}\bar{b}qq$ / $BB$ potentials,

$$\left(\frac{1}{m_b}\left(-\frac{d^2}{dr^2}+\frac{L(L+1)}{r^2}\right)+V_{qq,j_z,\mathcal{P},\mathcal{P}_x}(r)-2m_{\mathrm{sl}}\right)R(r) \quad = \quad ER(r).$$

- Possibly existing bound states, i.e. $E < 0$, indicate stable $\bar{b}\bar{b}qq$ tetraquarks.

- There is a bound state for orbital angular momentum $L = 0$ of $\bar{b}\bar{b}$:
  - Binding energy $-E = 38(18)$ MeV with respect to the $BB^*$ threshold.
  - Quantum numbers: $I(J^P) = 0(1^+)$.

- No further bound states.
  [P. Bicudo, M.W., Phys. Rev. D **87**, 114511 (2013) [arXiv:1209.6274]]



# Structure of the $\bar{b}\bar{b}qq$ tetraquark (1)

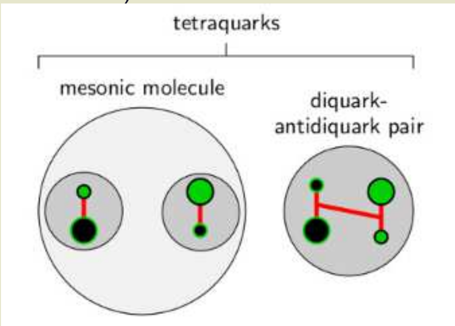
- Two types of operators, which probe the same sector:
  [P. Bicudo, A. Peters, S. Velten, M.W., arXiv:2101.00723]
  - **Meson-meson operator** $(BB)$:

    $$\mathcal{O}_{BB,\Gamma} \quad = \quad 2N_{BB}(\mathcal{C}\Gamma)_{AB}(\mathcal{C}\tilde{\Gamma})_{CD}\Big(\bar{Q}^a_C(-\mathbf{r}/2)\psi^{(f)a}_A(-\mathbf{r}/2)\Big)\Big(\bar{Q}^b_D(+\mathbf{r}/2)\psi^{(f')b}_B(+\mathbf{r}/2)\Big)$$

    with $\Gamma \in \{(1+\gamma_0)\gamma_5\,,\,\gamma_5\}$ $(\rightarrow (j_z, \mathcal{P}, \mathcal{P}_x) = (0, -, +))$.
  - **Diquark-antidiquark operator** $(Dd)$:

    $$\mathcal{O}_{Dd,\Gamma} = -N_{Dd}\epsilon^{abc}\Big(\psi^{(f)b}_A(\mathbf{z})(\mathcal{C}\Gamma)_{AB}\psi^{(f')c}_B(\mathbf{z})\Big)$$
    $$\epsilon^{ade}\Big(\bar{Q}^f_C(-\mathbf{r}/2)U^{fd}(-\mathbf{r}/2;\mathbf{z})(\mathcal{C}\tilde{\Gamma})_{CD}\bar{Q}^g_D(+\mathbf{r}/2)U^{ge}(+\mathbf{r}/2;\mathbf{z})\Big)$$

    with $\Gamma \in \{(1+\gamma_0)\gamma_5\,,\,\gamma_5\}$ $(\rightarrow (j_z, \mathcal{P}, \mathcal{P}_x) = (0, -, +))$.




- $\psi^{(f)}\psi^{(f')} = ud - du$ $(\rightarrow I = 0)$.
- $\tilde{\Gamma} = (1+\gamma_0)\gamma_3$ (essentially irrelevant).
- Compute the $4 \times 4$ correlation matrix

  $$C_{jk}(t) = \langle\Omega|\mathcal{O}^\dagger_j(t)\mathcal{O}_k(0)|\Omega\rangle.$$

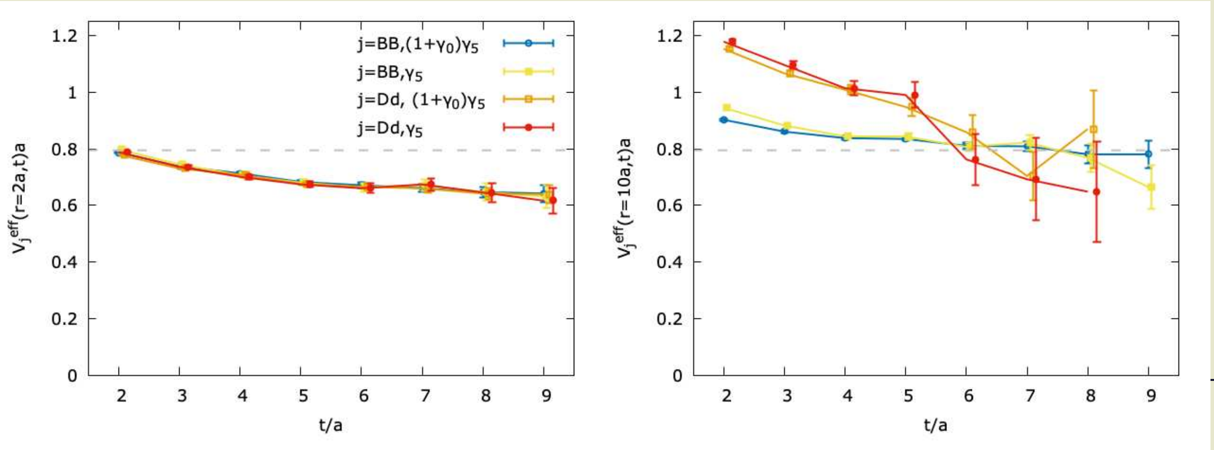
# Structure of the $\bar{b}\bar{b}qq$ tetraquark (2)

- Effective energies corresponding to diagonal elements of the correlation matrix,

$$V_j^{\text{eff}}(r,t) \quad = \quad -\frac{1}{a}\log\left(\frac{C_{jj}(t)}{C_{jj}(t-a)}\right) \quad \text{(no sum over } j\text{).}$$

- For large $\bar{b}\bar{b}$ separations (right plot $r \approx 0.79\,\text{fm}$), $BB$ effective energies reach plateaus at smaller $t$ separations than $Dd$ effective energies.
  $\rightarrow$ $BB$ dominates at large $r$, $Dd$ not important (energetically disfavored due to flux tube).

- For small $\bar{b}\bar{b}$ separations (left plot $r \approx 0.16\,\text{fm}$), $BB$ and $Dd$ effective energies similar.
  $\rightarrow$ More detailed investigation at small $r$ necessary.



# Structure of the $\bar{b}\bar{b}qq$ tetraquark (3)

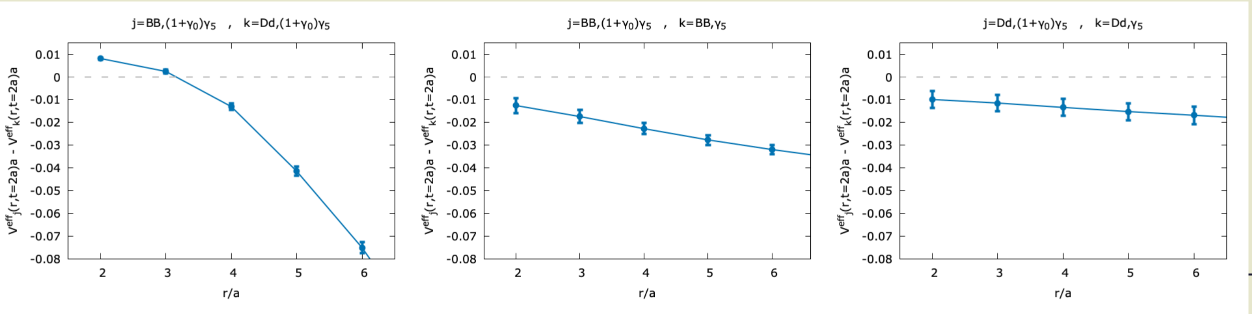
- Differences of effective energies corresponding to diagonal elements of the correlation matrix at small temporal separation $t = 2a$ as functions of the $\bar{b}\bar{b}$ separation $r$,

  $$V_j^{\text{eff}}(r, t = 2a) - V_k^{\text{eff}}(r, t = 2a).$$

- $BB$ versus $Dd$ (left): $Dd$ dominates for $r \lesssim 3.15\,a \approx 0.25\,\text{fm}$, while $BB$ dominates for $r \gtrsim 3.15\,a \approx 0.25\,\text{fm}$.

- $BB$ operators (center): $\Gamma = (1+\gamma_0)\gamma_5$ leads to larger ground state overlap than $\Gamma = \gamma_5$. (Expected. Via a Fierz transformation one can show that $\Gamma = (1+\gamma_0)\gamma_5$ generates exclusively ground state mesons, while $\gamma_5$ also generates parity excitations.)

- $Dd$ operators (right): $\Gamma = (1+\gamma_0)\gamma_5$ leads to larger ground state overlap than $\Gamma = \gamma_5$. (Interesting. In the literature mostly $\gamma_5$ is discussed.)



# Structure of the $\bar{b}\bar{b}qq$ tetraquark (4)

- Optimize trial states

$$|\Phi_{b,d}\rangle \quad = \quad b|\Phi_{BB,(1+\gamma_0)\gamma_5}\rangle + d|\Phi_{Dd,(1+\gamma_0)\gamma_5}\rangle$$

  ($|\Phi_j\rangle = \mathcal{O}_j|\Omega\rangle$) by minimizing effective energies

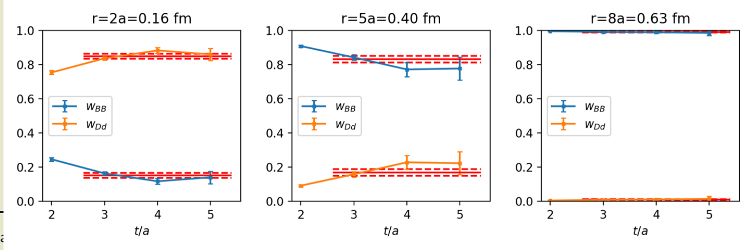
$$V^{\text{eff}}_{b,d}(r,t) \quad = \quad -\frac{1}{a}\log\left(\frac{C_{[b,d][b,d]}(t)}{C_{[b,d][b,d]}(t-a)}\right) \quad , \quad C_{[b,d][b,d]}(t) = \left(\begin{array}{c} b \\ d \end{array}\right)^\dagger_j C_{jk}(t) \left(\begin{array}{c} b \\ d \end{array}\right)_k .$$

  with respect to $b, d \in \mathbb{C}$.

- Since norm and phase of $b$ and $d$ are irrelevant, consider relative weights of $BB$ and $Dd$,

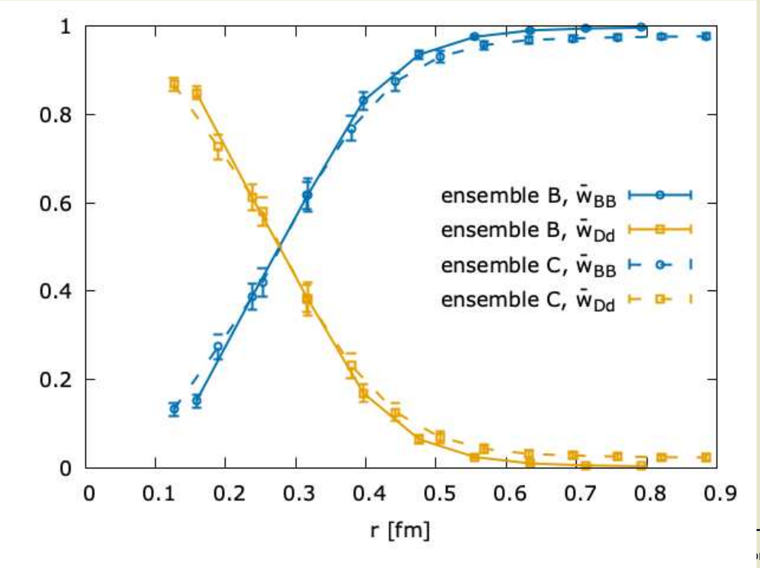
$$w_{BB} \quad = \quad \frac{|b|^2}{|b|^2 + |d|^2} \quad , \quad w_{Dd} \quad = \quad \frac{|d|^2}{|b|^2 + |d|^2} \quad = \quad 1 - w_{BB} .$$

- For fixed $\bar{b}\bar{b}$ separation $r$, $w_{BB}$ and $w_{Dd}$ depend only weakly on $t$.
  $\rightarrow$ $w_{BB}$ and $w_{Dd}$ estimate the percentage of $BB$ and of $Dd$.



# Structure of the $\bar{b}\bar{b}qq$ tetraquark (5)

- $w_{BB}$ and $w_{Dd}$ as functions of the $\bar{b}\bar{b}$ separation $r$ (for two ensembles, $a \approx 0.079\,\text{fm}$ and $a \approx 0.063\,\text{fm}$).
- $r \lesssim 0.2\,\text{fm}$: Clear diquark-antidiquark dominance.
- $0.2\,\text{fm} \lesssim r \lesssim 0.3\,\text{fm}$: Diquark-antidiquark dominance turns into meson-meson dominance.
- $0.5\,\text{fm} \lesssim r$: Essentially a meson-meson system.



# Structure of the $\bar{b}\bar{b}qq$ tetraquark (6)

- Generalized eigenvalue problem (GEVP)

  $$C_{jk}(t) v_k^{(n)}(t) \quad = \quad \lambda^{(n)}(t) C_{jk}(t_0) v_k^{(n)}(t) \quad , \quad n \quad = \quad 0, \ldots, N-1$$

  for $t_0/a \geq 1$ and $t/a > t_0/a$ with corresponding effective energies

  $$V^{\textrm{eff},(n)}(r,t) \quad = \quad -\frac{1}{a} \log\left(\frac{\lambda^{(n)}(t)}{\lambda^{(n)}(t-a)}\right).$$

- Eigenvector components $v_j^{(n)}(t)$ (which we always normalize according to $\sum_j |v_j^{(n)}(t)|^2 = 1$) contain information about the relative importance of the operators. For large $t$ and $t_0$,

  $$|n\rangle \approx \sum_j v_j^{(n)}(t) |\Phi_j\rangle,$$

  where $\approx$ denotes an approximate expansion of the energy eigenstate $|n\rangle$ in terms of the trial states $|\Phi_j\rangle$.

# Structure of the $\bar{b}\bar{b}qq$ tetraquark (7)

- One can show: For $t_0 = t - a$, optimizing trial states by minimizing effective energies (as on previous slides) is equivalent to solving a GEVP, i.e.

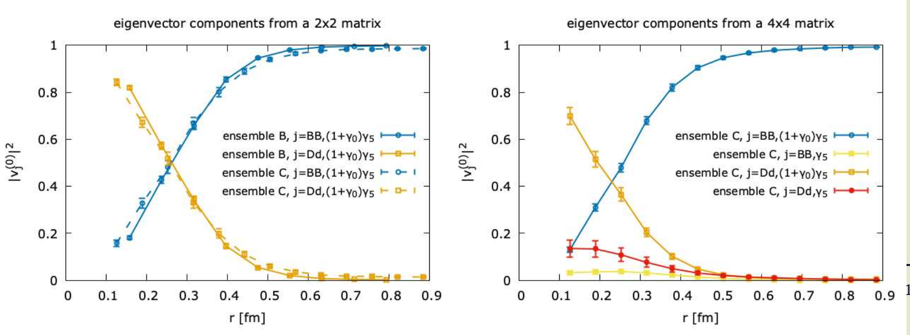
$$(w_{BB}, w_{Dd}) \quad = \quad (|v^{(0)}_{BB,(1+\gamma_0)\gamma_5}|^2, |v^{(0)}_{Dd,(1+\gamma_0)\gamma_5}|^2)$$

  (might offer another perspective on GEVP eigenvector components).
  $\rightarrow$ Results for $w_{BB}$ and $w_{Dd}$ can also be interpreted as GEVP results.
  [P. Bicudo, A. Peters, S. Velten, M.W., arXiv:2101.00723]

- In the literature typically small values for $t_0$ are used, e.g. $t_0/a = 1$ (instead of $t_0 = t - a$ as used to obtain $w_{BB}$ and $w_{Dd}$ on previous slides).

- Similar results also for $t_0/a = 1$, when using a $2 \times 2$ correlation matrix (left plot).

- Consistent results, when using a $4 \times 4$ correlation matrix (right plot).



# Structure of the $\bar{b}\bar{b}qq$ tetraquark (8)

- Define the $r$ dependent $BB$ and $Dd$ percentages,

$$p_{BB}(r) = w_{BB} \quad , \quad p_{Dd}(r) = w_{Dd}$$

and use the probability density of the $\bar{b}\bar{b}$ separation

$$p_r(r) \quad = \quad 4\pi |R(r)|^2$$

obtained from the BO wave function $R(r)/r$, to estimate the total $BB$ and $Dd$ percentages of the $\bar{b}\bar{b}ud$ tetraquark with quantum numbers $I(J^P) = 0(1^+)$:

$$\%BB = \int dr\, p_r(r) p_{BB}(r) \quad , \quad \%Dd = \int dr\, p_r(r) p_{Dd}(r) = 1 - \%BB.$$

- We find $\%BB = 0.58$, $\%Dd = 0.42$.
- Using $|v^{(0)}_{BB,(1+\gamma_0)\gamma_5}|^2, |v^{(0)}_{Dd,(1+\gamma_0)\gamma_5}|^2$ instead of $w_{BB}, w_{Dd}$ we find $\%BB = 0.60$, $\%Dd = 0.40$.
- Results are in agreement with a GEVP result we obtained in a full lattice QCD computation, where the $\bar{b}$ quarks are treated within NRQCD.
  [L. Leskovec, S. Meinel, M. Pflaumer and M.W., Phys. Rev. D **100**, 014503 (2019) [arXiv:1904.04197]]
  [M. Pflaumer, private communications]

# Summary

- The hadronically stable $\bar{b}\bar{b}ud$ tetraquark with quantum numbers $I(J^P) = 0(1^+)$ is neither exclusively a meson-meson system nor a diquark-antdiquark pair.
- $r \lesssim 0.2\,\text{fm}$: Clear diquark-antidiquark dominance.
- $r \gtrsim 0.3\,\text{fm}$: Clear meson-meson dominance.
- Total $BB$ and $Dd$ percentages: $\% BB \approx 0.60$, $\% Dd \approx 0.40$.