Open flavor four-quark states from the lattice

"PANDA Collaboration Meeting" – GSI

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June 25, 2024

Exotic systems discussed in this talk

- This talk summarizes our **investigations of heavy exotic mesons with lattice QCD**:
  - mainly open flavor four-quark states,
    tetraquarks $\bar{Q}\bar{Q}qq$, (heavy quarks $Q \in \{b, c\}$, light quarks $q \in \{u, d, s\}$)
  - but also
    tetraquarks $\bar{Q}Q\bar{q}q$ and **hybrid mesons** $\bar{Q}Q +$ gluons (very brief).
- Why b quarks? Why not only c quarks, which are more relevant in the context of PANDA?
  - b quarks are technically simpler, e.g. Heavy Quark Effective Theory or Non Relativistic QCD applicable/more accurate.
  - Systems with $Q = b$ are physically simpler, e.g. certain tetraquarks are QCD-stable.
  - **Long-term goal**: accurate predictions for exotic mesons with c quarks.
  - **Important intermediate steps**: computations with b quarks (or even static quarks).

Two types of approaches

- Two types of approaches, when studying **heavy-light exotic mesons** with lattice QCD:
  - **Born-Oppenheimer approximation** (a 2-step procedure):
    - **The focus of this talk.**

    (1) Compute the potential $V(r)$ of the two heavy quarks (approximated as static quarks) in the presence of two light quarks and/or gluons using lattice QCD.
    $\rightarrow$ full QCD results

    (2) Use standard techniques from quantum mechanics and $V(r)$ to study the dynamics of two heavy quarks (Schrödinger equation, scattering theory, etc.).
    $\rightarrow$ an approximation

    (+) Provides physical insights (e.g. forces between quarks, quark composition).

    (−) An approximation.
  - **Full lattice QCD computations of eigenvalues of the QCD Hamiltonian**:
    - Masses of stable hadrons correspond to energy eigenvalues at infinite volume.
    - Masses and decay widths of resonances can be calculated from the volume dependence of the energy eigenvalues (rather difficult).

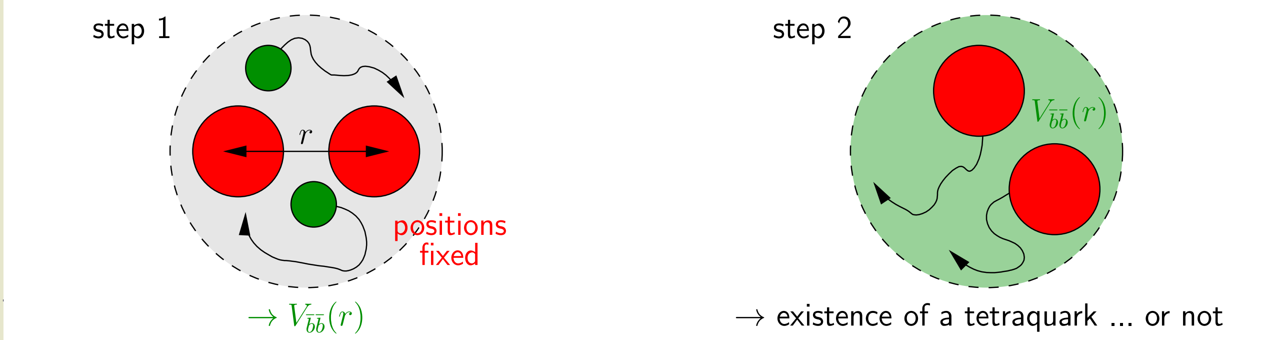
Basic idea: lattice QCD and BO

- Start with $\bar{b}\bar{b}qq$.
- $\bar{b}\bar{b}ud$ with $I(J^P) = 0(1^+)$ is the bottom counterpart of the experimentally observed T_{cc}.
  [R. Aaij *et al.* [LHCb], Nature Phys. **18**, 751-754 (2022) [arXiv:2109.01038]].
- Study such $\bar{b}\bar{b}qq$ tetraquarks in two steps:

  (1) **Compute potentials of the two static quarks $\bar{b}\bar{b}$ in the presence of two lighter quarks qq ($q \in \{u, d, s\}$) using lattice QCD.**

  (2) **Check, whether these potentials are sufficiently attractive to host bound states or resonances** ($\rightarrow$ tetraquarks) **by using techniques from quantum mechanics and scattering theory.**

(1) + (2) $\rightarrow$ Born-Oppenheimer approximation.



$\bar{b}\bar{b}qq$ / BB potentials (1)

- To determine $\bar{b}\bar{b}$ potentials $V_{qq,j_z,\mathcal{P},\mathcal{P}_x}(r)$, compute temporal correlation functions

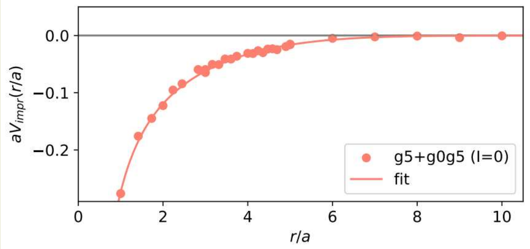
  $$\langle\Omega|\mathcal{O}^\dagger_{BB,\Gamma}(t)\mathcal{O}_{BB,\Gamma}(0)|\Omega\rangle \propto_{t\rightarrow\infty} e^{-V_{qq,j_z,\mathcal{P},\mathcal{P}_x}(r)t}$$

  of operators

  $$\mathcal{O}_{BB,\Gamma} \quad = \quad 2N_{BB}(\mathcal{C}\Gamma)_{AB}(\mathcal{C}\tilde{\Gamma})_{CD}\Big(\bar{Q}^a_C(-\mathbf{r}/2)q^a_A(-\mathbf{r}/2)\Big)\Big(\bar{Q}^b_D(+\mathbf{r}/2)q^b_B(+\mathbf{r}/2)\Big).$$

- **Many different channels** (isospin/light flavor, angular momentum, parity).
  $\rightarrow$ Attractive as well as repulsive potentials.
  $\rightarrow$ Potentials with different asymptotic values (two heavy-light mesons $\in \{B, B^*, B^*_0, B^*_1\}$).

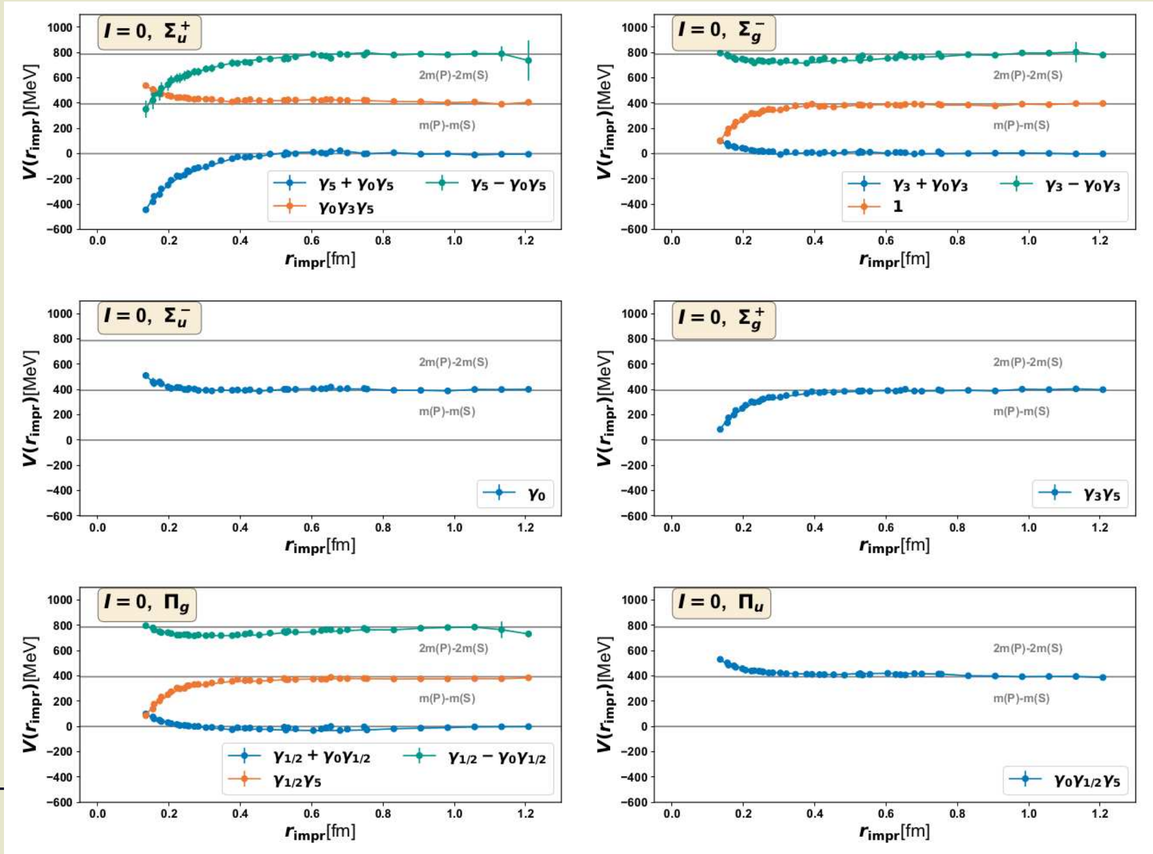
- The most attractive potential of a $B^{(*)}B^{(*)}$ meson pair has $(I, |j_z|, P, P_x) = (0, 0, +, -)$:
  - $\psi^{(f)}\psi^{(f')} = ud - du$, $\Gamma \in \{(1+\gamma_0)\gamma_5\, ,\, (1-\gamma_0)\gamma_5\}$.
  - $\bar{Q}\bar{Q} = \bar{b}\bar{b}$, $\tilde{\Gamma} \in \{(1+\gamma_0)\gamma_5\, ,\, (1+\gamma_0)\gamma_j\}$ (irrelevant).



[P. Bicudo, K. Cichy, A. Peters, M.W., Phys. Rev. D **93**, 034501 (2016) [arXiv:1510.03441]]
[P. Bicudo, M. Marinkovic, L. Müller, M.W., unpublished ongoing work]

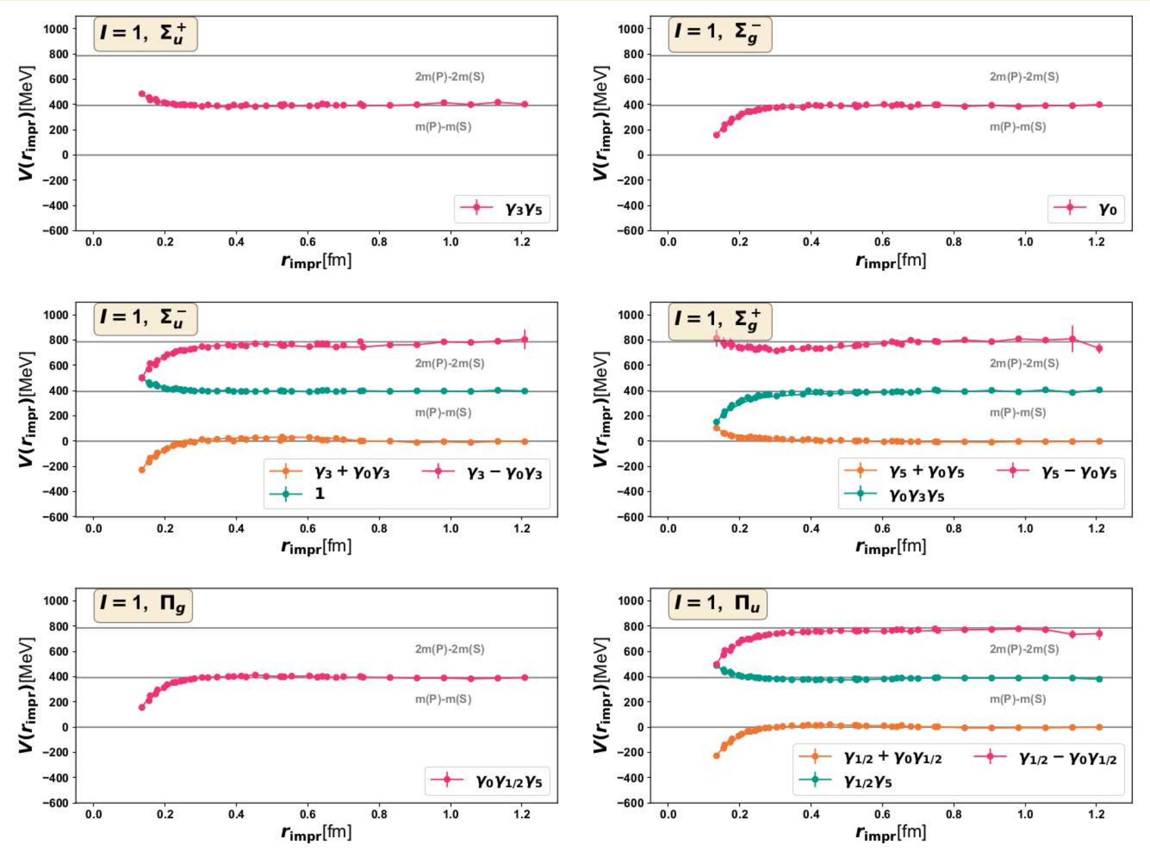
$\bar{b}\bar{b}qq$ / BB potentials (2)

[P. Bicudo, M. Marinkovic, L. Müller, M.W., unpublished ongoing work]



$\bar{b}\bar{b}qq$ / BB potentials (3)

[P. Bicudo, M. Marinkovic, L. Müller, M.W., unpublished ongoing work]



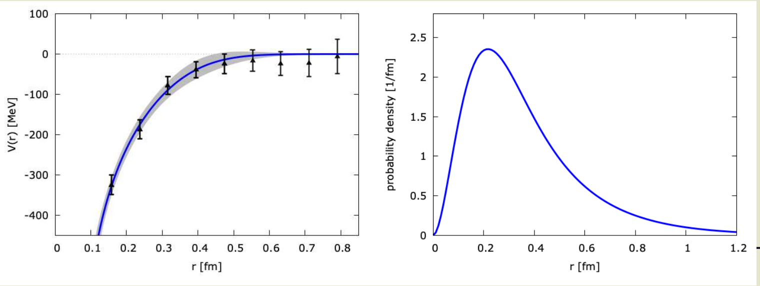
Stable $\bar{b}\bar{b}qq$ tetraquarks

- Solve the Schrödinger equation for the relative coordinate of the heavy quarks $\bar{b}\bar{b}$ using the previously computed $\bar{b}\bar{b}qq$ potentials,

$$\left(\frac{1}{m_b}\left(-\frac{d^2}{dr^2}+\frac{L(L+1)}{r^2}\right)+V_{qq,j_z,\mathcal{P},\mathcal{P}_x}(r)-2m_B\right)R(r) \quad = \quad ER(r).$$

- Possibly existing bound states, i.e. $E < 0$, indicate QCD-stable $\bar{b}\bar{b}qq$ tetraquarks.
- There is a bound state for orbital angular momentum $L = 0$ of $\bar{b}\bar{b}$:
  - Binding energy $E = -90^{+43}_{-36}$ MeV with respect to the BB^* threshold.
  - Quantum numbers: $I(J^P) = 0(1^+)$.

[P. Bicudo, M.W., Phys. Rev. D **87**, 114511 (2013) [arXiv:1209.6274]]



Further $\bar{b}\bar{b}qq$ results (1)

- **Are there further QCD-stable $\bar{b}\bar{b}qq$ tetraquarks with other $I(J^P)$ and light flavor quantum numbers?**

  $\rightarrow$ No, not for $qq = ud$ (both $I = 0, 1$), not for $qq = ss$.
  [P. Bicudo, K. Cichy, A. Peters, B. Wagenbach, M.W., Phys. Rev. D **92**, 014507 (2015) [arXiv:1505.00613]]

  $\rightarrow$ $\bar{b}\bar{b}us$ was not investigated.

  – Strong evidence from full QCD computations that a QCD-stable $\bar{b}\bar{b}us$ tetraquark exists.

- **Effect of heavy quark spins:**

  – Expected to be $\mathcal{O}(m_{B^*} - m_B) = \mathcal{O}(45\,\text{MeV})$.

  – Previously ignored (potentials of static quarks are independent of the heavy spins).

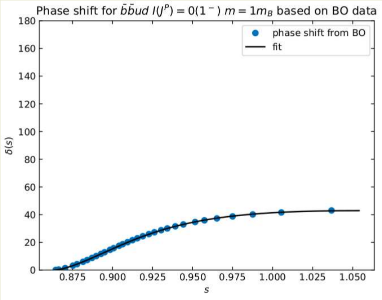
  – In [P. Bicudo, J. Scheunert, M.W., Phys. Rev. D **95**, 034502 (2017) [arXiv:1612.02758]] included in a crude phenomenological way via a BB^* and a B^*B^* coupled channel Schrödinger equation with the experimental mass difference $m_{B^*} - m_B$ as input.

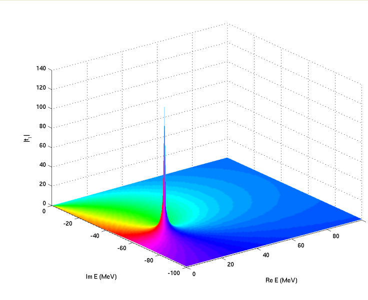
  $\rightarrow$ Binding energy reduced from around $90\,\text{MeV}$ to $59\,\text{MeV}$.

  $\rightarrow$ Physical reason: the previously discussed attractive potential does not only correspond to a lighter BB^* pair, but has also a heavier B^*B^* contribution.

Further $\bar{b}\bar{b}qq$ results (2)

- **Are there $\bar{b}\bar{b}qq$ tetraquark resonances?**
  - In
    [P. Bicudo, M. Cardoso, A. Peters, M. Pflaumer, M.W., Phys. Rev. D **96**, 054510 (2017) [arXiv:1704.02383]]
    resonances studied via standard scattering theory from quantum mechanics textbooks.





→ Heavy quark spins ignored.

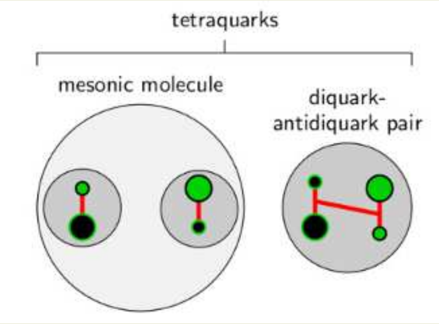
→ Indication for $\bar{b}\bar{b}ud$ tetraquark resonance with $I(J^P) = 0(1^-)$ found, $E = 17^{+4}_{-4}$ MeV above the BB threshold, decay width $\Gamma = 112^{+90}_{-103}$ MeV.

  - In
    [J. Hoffmann, A. Zimermmane-Santos, M.W., PoS **LATTICE2022**, 262 (2023) [arXiv:2211.15765]]
    [J. Hoffmann, M.W., unpublished ongoing work]
    heavy quark spins included.

→ $\bar{b}\bar{b}ud$ resonance shifted upwards, slightly above the B^*B^* threshold.

→ Physical reason: the relevant attractive potential does not only correspond to a lighter BB pair, but has also a heavier B^*B^* contribution.

Further $\bar{b}\bar{b}qq$ results (3)

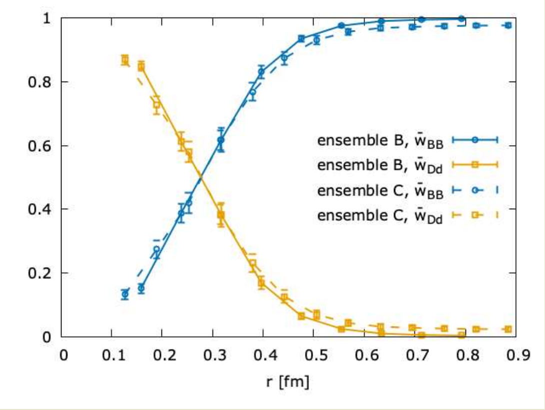


- **Structure of the QCD-stable $\bar{b}\bar{b}ud$ tetraquark: meson-meson (BB) versus diquark-antidiquark (Dd).**
  - Use not just one but two operators,

$$\mathcal{O}_{BB,\Gamma} = 2N_{BB}(\mathcal{C}\Gamma)_{AB}(\mathcal{C}\tilde{\Gamma})_{CD}\Big(\bar{Q}^a_C(-\mathbf{r}/2)\psi^{(f)a}_A(-\mathbf{r}/2)\Big)\Big(\bar{Q}^b_D(+\mathbf{r}/2)\psi^{(f')b}_B(+\mathbf{r}/2)\Big)$$

$$\mathcal{O}_{Dd,\Gamma} = -N_{Dd}\epsilon^{abc}\Big(\psi^{(f)b}_A(\mathbf{z})(\mathcal{C}\Gamma)_{AB}\psi^{(f')c}_B(\mathbf{z})\Big)$$
$$\epsilon^{ade}\Big(\bar{Q}^f_C(-\mathbf{r}/2)U^{fd}(-\mathbf{r}/2;\mathbf{z})(\mathcal{C}\tilde{\Gamma})_{CD}\bar{Q}^g_D(+\mathbf{r}/2)U^{ge}(+\mathbf{r}/2;\mathbf{z})\Big),$$

compare the contribution of each operator to the $\bar{b}\bar{b}$ potential $V_{qq,j_z,\mathcal{P},\mathcal{P}_x}(r)$.
[P. Bicudo, A. Peters, S. Velten, M.W., Phys. Rev. D **103**, 114506 (2021) [arXiv:2101.00723]]



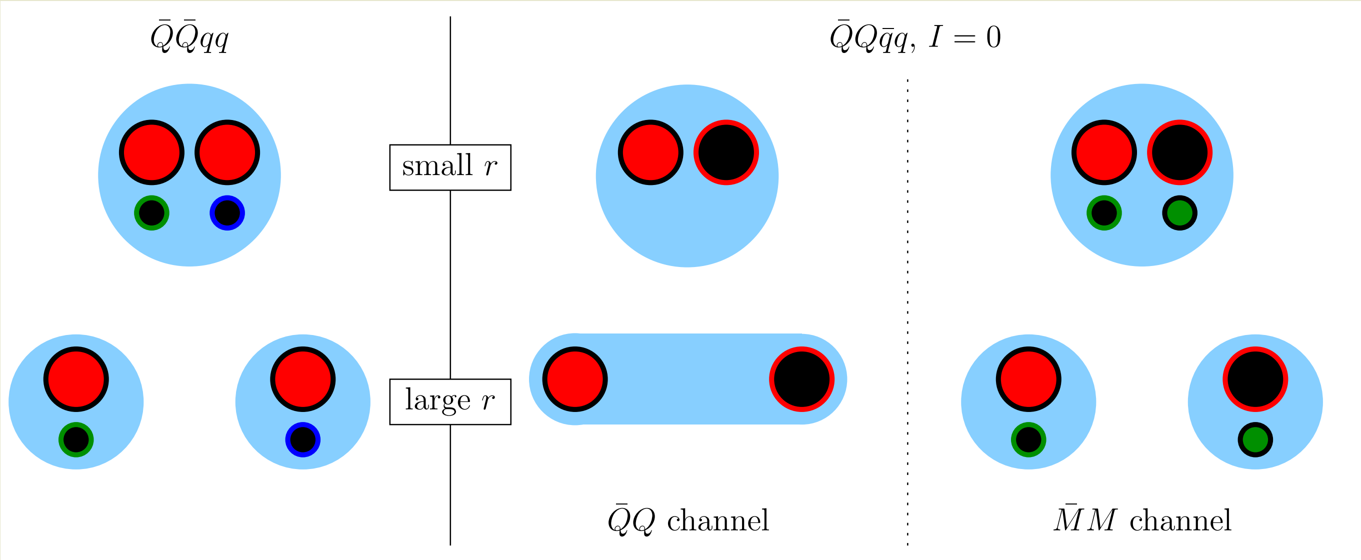
→ $r \lesssim 0.2\,\mathrm{fm}$: Clear diquark-antidiquark dominance.

→ $0.5\,\mathrm{fm} \lesssim r$: Essentially a meson-meson system.

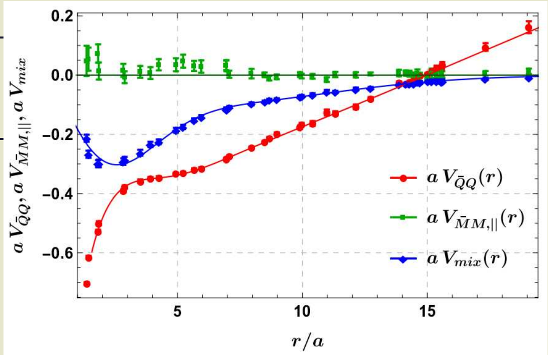
→ Integrate over t to estimate the composition of the tetraquark: $\%BB \approx 60\%$, $\%Dd \approx 40\%$.

Quarkonium, $I = 0$: difference to $\bar{Q}\bar{Q}qq$ (1)

- Now quarkonium with $I = 0$, i.e. $\bar{Q}Q$ and/or $\bar{Q}Q\bar{q}q$ (with $\bar{q}q = (\bar{u}u + \bar{d}d)/\sqrt{2}, \bar{s}s$).
- Technically more complicated than $\bar{Q}\bar{Q}qq$, because there are two channels:
  - Quarkonium channel, $\bar{Q}Q$ (with $Q \equiv b$).
  - Heavy-light meson-meson channel, $\bar{M}M$ (with $M = \bar{Q}q$), "string breaking".



Quarkonium, $I = 0$: ...



- Lattice computation of potentials for both channels ($\bar{Q}Q$ and $\bar{M}M$) needed, additionally also a mixing potential:
  - Pioneering work:
    [G. S. Bali *et al.* [SESAM Collaboration], Phys. Rev. D **71**, 114513 (2005) [hep-lat/0505012]]
    Rather heavy u/d quark masses ($m_\pi \approx 650\,\text{MeV}$), only 2 flavors, not $2+1$.
  - More recent work:
    [J. Bulava, B. Hörz, F. Knechtli, V. Koch, G. Moir, C. Morningstar and M. Peardon, Phys. Lett. B **793**, 493-498 (2019) [arXiv:1902.04006]]
    Unfortunately, mixing potential not computed.
  - Several assumptions needed to adapt the "Bali results" to $2+1$ flavors and physical quark masses.

$\rightarrow$ Potential for a coupled channel Schrödiger equation (see next slide):

$$V(\mathbf{r}) = \begin{pmatrix} V_{\bar{Q}Q}(r) & V_\text{mix}(r)(1 \otimes \mathbf{e}_r) & (1/\sqrt{2})V_\text{mix}(r)(1 \otimes \mathbf{e}_r) \\ V_\text{mix}(r)(1 \otimes \mathbf{e}_r) & V_{\bar{M}M}(r) & 0 \\ (1/\sqrt{2})V_\text{mix}(r)(1 \otimes \mathbf{e}_r) & 0 & V_{\bar{M}M}(r) \end{pmatrix}.$$

Quarkonium, $I = 0$: SE

- Schrödinger equation non-trivial:
  - 3 coupled channels, $\bar{Q}Q$, MM (3 components), $M_s M_s$ (3 components).
  - Static potentials used as input have other symmetries than quarkonium.

$$\left(-\frac{1}{2}\mu^{-1}\left(\partial_r^2 + \frac{2}{r}\partial_r - \frac{\mathbf{L}^2}{r^2}\right) + V(\mathbf{r}) + \begin{pmatrix} E_{\text{threshold}} & 0 & 0 \\ 0 & 2m_M & 0 \\ 0 & 0 & 2m_{M_s} \end{pmatrix} - E\right)\psi(\mathbf{r}) \quad = \quad 0.$$

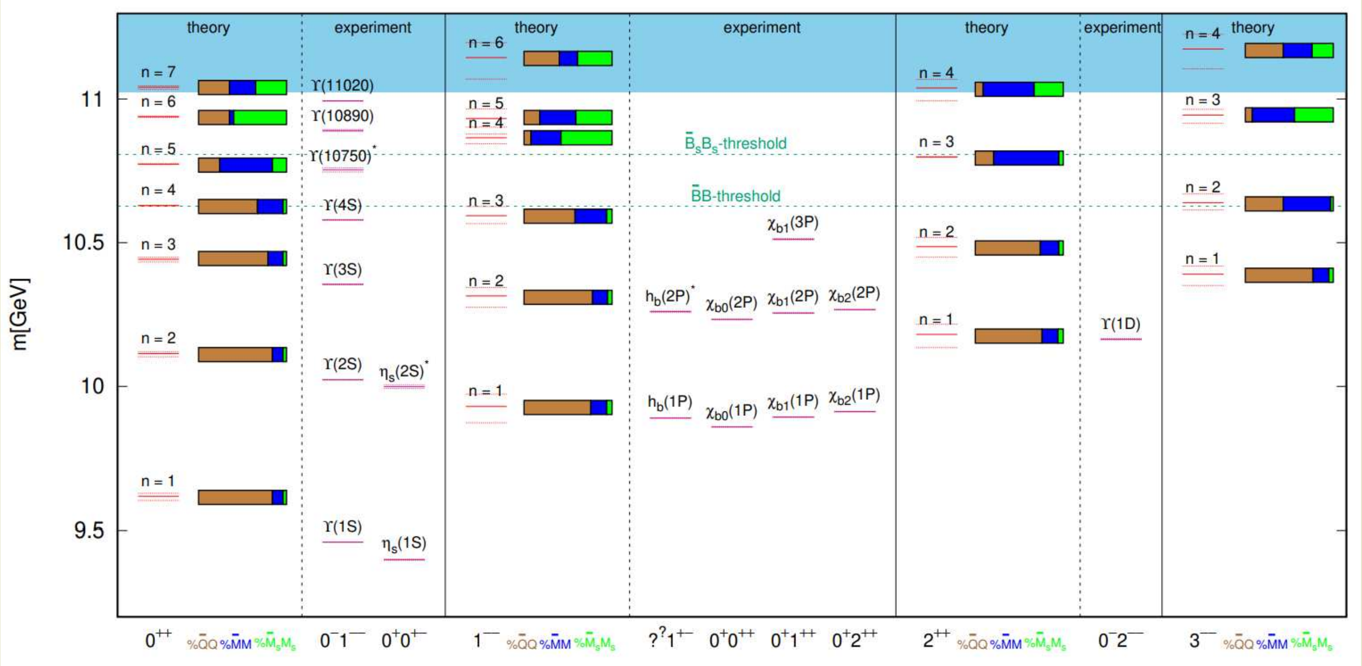
  - Project to definite total angular momentum $\tilde{J}$ (excluding the heavy quark spins),
    - 7 coupled PDEs $\rightarrow$ 3 coupled ODEs for $\tilde{J} = 0$,
    - 7 coupled PDEs $\rightarrow$ 5 coupled ODEs for $\tilde{J} \geq 1$
  - Add scattering boundary conditions.
- Determine scattering amplitudes and T matrices from the Schrödinger equation, find poles of $T_{\tilde{J}}$ in the complex energy plane to identify bound states and resonances.
- The components of the resulting wave functions provide the compositions of the states, i.e. the quarkonium and meson-meson (= tetraquark) percentages $\%\bar{Q}Q$ and $\%\bar{M}M$.

[P. Bicudo, M. Cardoso, N. Cardoso, M.W., Phys. Rev. D **101**, 034503 (2020) [arXiv:1910.04827]]
[P. Bicudo, N. Cardoso, L. Müller, M.W., Phys. Rev. D **103**, 074507 (2021) [arXiv:2008.05605]]
[P. Bicudo, N. Cardoso, L. Müller, M.W., Phys. Rev. D **107**, 094515 (2023) [arXiv:2205.11475]]

	theory			experiment			
$\tilde{J}^{PC}$	n	m[GeV]	Γ[MeV]	name	m[GeV]	Γ[MeV]	$I^G(J^{PC})$
0^{++}	1	9.618^{+10}_{-15}	-	$\eta_b(1S)$	9.399(2)	10(5)	$0^+(0^{+-})$
				$\Upsilon_b(1S)$	9.460(0)	≈ 0	$0^-(1^{--})$
	2	10.114^{+7}_{-11}	-	$\eta_b(2S)_{\text{BELLE}}$	9.999(6)	-	$0^+(0^{+-})$
				$\Upsilon(2S)$	10.023(0)	≈ 0	$0^-(1^{--})$
	3	10.442^{+7}_{-9}	-	$\Upsilon(3S)$	10.355(1)	≈ 0	$0^-(1^{--})$
	4	10.629^{+1}_{-1}	$49.3^{+5.4}_{-3.9}$	$\Upsilon(4S)$	10.579(1)	21(3)	$0^-(1^{--})$
	5	10.773^{+1}_{-2}	$15.9^{+2.9}_{-4.4}$	$\Upsilon(10750)_{\text{BELLE II}}$	10.753(7)	36(22)	$0^-(1^{--})$
	6	10.938^{+2}_{-2}	$61.8^{+7.6}_{-8.0}$	$\Upsilon(10860)$	10.890(3)	51(7)	$0^-(1^{--})$
	7	11.041^{+5}_{-7}	$45.5^{+13.5}_{-8.2}$	$\Upsilon(11020)$	10.993(1)	49(15)	$0^-(1^{--})$
1^{--}	1	9.930^{+43}_{-52}	-	$\chi_{b0}(1P)$	9.859(1)	-	$0^+(0^{++})$
				$h_b(1P)$	9.890(1)	-	$?^?(1^{+-})$
				$\chi_{b1}(1P)$	9.893(1)	-	$0^+(1^{++})$
				$\chi_{b2}(1P)$	9.912(1)	-	$0^+(2^{++})$
	2	10.315^{+29}_{-40}	-	$\chi_{b0}(2P)$	10.233(1)	-	$0^+(0^{++})$
				$\chi_{b1}(2P)$	10.255(1)	-	$0^+(1^{++})$
				$h_b(2P)_{\text{BELLE}}$	10.260(2)	-	$?^?(1^{+-})$
				$\chi_{b2}(2P)$	10.267(1)	-	$0^+(2^{++})$
	3	10.594^{+32}_{-28}	-	$\chi_{b1}(3P)$	10.512(2)	-	$0^+(0^{++})$
	4	10.865^{+37}_{-21}	$67.5^{+5.1}_{-4.9}$				
	5	10.932^{+33}_{-54}	$101.8^{+7.3}_{-5.1}$				
	6	11.144^{+52}_{-75}	$25.0^{+1.1}_{-1.3}$				
2^{++}	1	10.181^{+35}_{-46}	-	$\Upsilon(1D)$	10.164(2)	-	$0^-(2^{--})$
	2	10.486^{+32}_{-36}	-				
	3	10.799^{+2}_{-2}	$13.0^{+2.1}_{-2.0}$				
	4	11.038^{+30}_{-44}	$40.8^{+2.0}_{-2.8}$				
3^{--}	1	10.390^{+28}_{-39}	-				
	2	10.639^{+31}_{-25}	$2.4^{+1.5}_{-0.9}$				
	3	10.944^{+20}_{-29}	$46.8^{-4.6}_{+6.2}$				
	4	11.174^{+51}_{-69}	$1.9^{+2.1}_{-1.4}$				

Bottomonium, $I = 0$: results (2)

- Quarkonium component,
  $\bar{B}B$ component,
  $\bar{B}_s B_s$ component.



Quarkonium, $I = 0$: $1/m_Q$ corrections (1)

- Potentials of static quarks are independent of the heavy spins.
  $\rightarrow$ Systematic errors are possibly large,
  $\mathcal{O}(m_{B^*} - m_B) = \mathcal{O}(45\,\text{MeV})$ (b quarks), $\mathcal{O}(m_{D^*} - m_D) = \mathcal{O}(140\,\text{MeV})$ (c quarks).

- Such spin effects and further corrections due to the finite heavy quark mass can be expressed order by order in $1/m_Q$ in terms of Wilson loops with field strength insertions.
  [E. Eichten and F. Feinberg, Phys. Rev. D **23**, 2724 (1981)]
  [N. Brambilla, A. Pineda, J. Soto and A. Vairo, Phys. Rev. D **63**, 014023 (2001) [arXiv:hep-ph/0002250]]

- Existing crude computations up to order $1/m_Q^2$.
  [Y. Koma and M. Koma, Nucl. Phys. B **769**, 79-107 (2007) [arXiv:hep-lat/0609078]]

- **Compute these $1/m_Q$ and $1/m_Q^2$ corrections more precisely using gradient flow.**
  [M. Eichberg, M.W., PoS **LATTICE2023**, 068 (2024) [arXiv:2311.06560 [hep-lat]]
  [M. Eichberg, invited talk at QWG 2024]

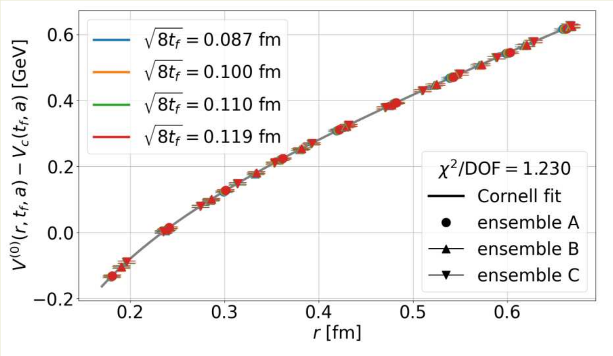
Quarkonium, $I = 0$: $1/m_Q$ corrections (2)

- $\bar{Q}Q$ potential:

$$V(r) \; = \; V^{(0)}(r) + \frac{1}{m_Q} V^{(1)}(r) + \frac{1}{m_Q^2} \Big(V_{\text{SD}}^{(2)}(r) + V_{\text{SI}}^{(2)}(r) \Big) + \mathcal{O}(1/m_Q^3).$$

- $V^{(0)}(r)$: the ordinary static potential.
  - Can be extracted from Wilson loops.
  - **Technically rather simple.**

[M. Eichberg, invited talk at QWG 2024]



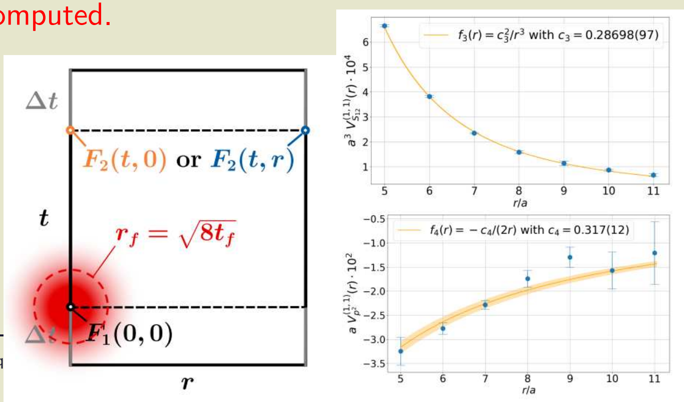
Quarkonium, $I = 0$: $1/m_Q$ corrections (3)

- $\bar{Q}Q$ potential:

$$V(r) \quad = \quad V^{(0)}(r) + \frac{1}{m_Q} V^{(1)}(r) + \frac{1}{m_Q^2}\Big(V_{\text{SD}}^{(2)}(r) + V_{\text{SI}}^{(2)}(r)\Big) + \mathcal{O}(1/m_Q^3).$$

- $V^{(1)}(r)$, $V_{\text{SD}}^{(2)}(r)$, $V_{\text{SI}}^{(2)}(r)$: corrections to the ordinary static potential, $\propto 1/m_Q$ and $\propto 1/m_Q^2$:
  - Can be extracted from Wilson loops with chromo field insertions.
    $\rightarrow$ Loops with much larger temporal extent needed.
  - Chromo field insertions exhibit rather large discretization errors:
    $\rightarrow$ "Renormalization" via gradient flow (a controlled smoothing of the gluon field).
    $\rightarrow$ Not only continuum limit, but also zero flow time limit needed.
  - Matching coefficients have to be computed.
  - **Technically very difficult.**

[M. Eichberg, invited talk at QWG 2024]



Heavy hybrid mesons: potentials (1)

- Now heavy hybrid mesons, i.e. $\bar{Q}Q +$ gluons.
- (Hybrid) static potentials can be characterized by the following quantum numbers:
  - Absolute total angular momentum with respect to the $\bar{Q}Q$ separation axis (z axis): $\Lambda = 0, 1, 2, \ldots \equiv \Sigma, \Pi, \Delta, \ldots$
  - Parity combined with charge conjugation: $\eta = +, - = g, u$.
  - Relection along an axis perpendicular to the $\bar{Q}Q$ separation axis (x axis): $\epsilon = +, -$.
- The ordinary static potential has quantum numbers $\Lambda_\eta^\epsilon = \Sigma_g^+$.
- Particularly interesting: the two lowest hybrid static potentials with $\Lambda_\eta^\epsilon = \Pi_u, \Sigma_u^-$.
- References:

[K. J. Juge, J. Kuti, C. J. Morningstar, Nucl. Phys. Proc. Suppl. **63**, 326 (1998) [hep-lat/9709131]
[C. Michael, Nucl. Phys. A **655**, 12 (1999) [hep-ph/9810415]
[G. S. Bali *et al.* [SESAM and TχL Collaborations], Phys. Rev. D **62**, 054503 (2000) [hep-lat/0003012]
[K. J. Juge, J. Kuti, C. Morningstar, Phys. Rev. Lett. **90**, 161601 (2003) [hep-lat/0207004]
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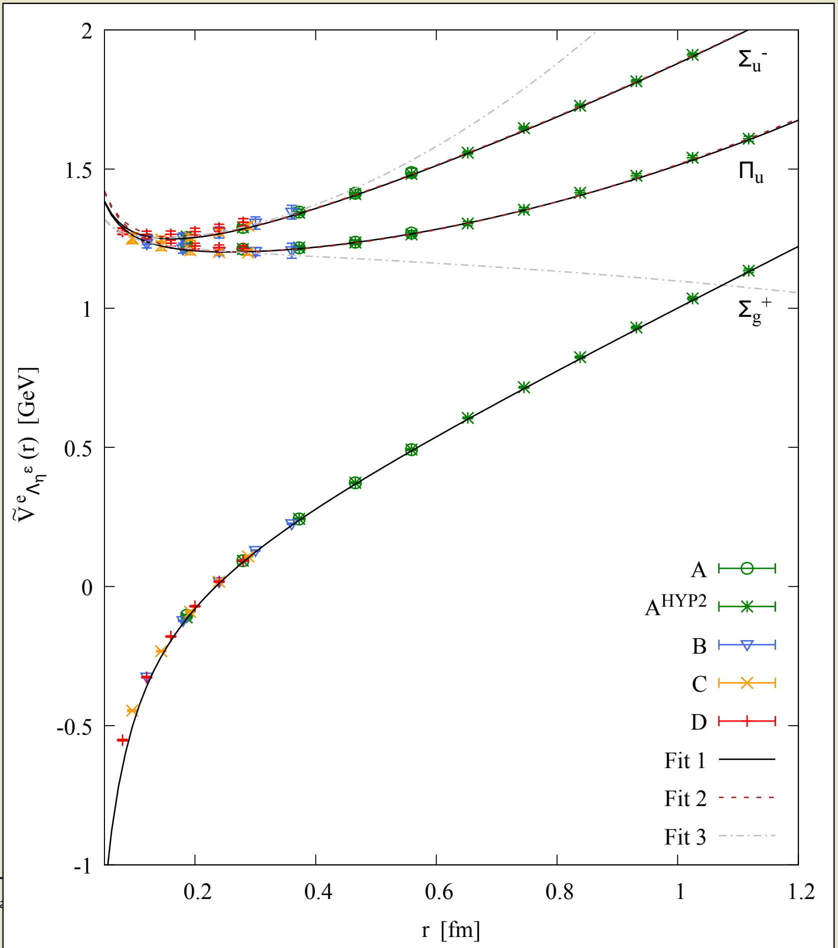
[G. S. Bali, A. Pineda, Phys. Rev. D **69**, 094001 (2004) [hep-ph/0310130]
[P. Bicudo, N. Cardoso, M. Cardoso, Phys. Rev. D **98**, 114507 (2018) [arXiv:1808.08815 [hep-lat]]]
[S. Capitani, O. Philipsen, C. Reisinger, C. Riehl. M.W., Phys. Rev. D **99**, 034502 (2019) [arXiv:1811.11046 [hep-lat]]]

Heavy hybrid mesons: potentials (2)

- [C. Schlosser, M.W., Phys. Rev. D **105**, 054503 (2022) [arXiv:2111.00741]]
- Computation of $1/m_Q$ and $1/m_Q^2$ corrections using gradient flow in progress.
  [C. Schlosser, M.W., unpublished ongoing work]



Heavy hybrid mesons: SE

- Solve Schrödinger equations for the relative coordinate of $\bar{Q}Q$ using hybrid static potentials,

$$\Big(-\frac{1}{2\mu}\frac{d^2}{dr^2} + \frac{L(L+1) - 2\Lambda^2 + J_{\Lambda_\eta^\epsilon}(J_{\Lambda_\eta^\epsilon}+1)}{2\mu r^2} + V_{\Lambda_\eta^\epsilon}(r)\Big) u_{\Lambda_\eta^\epsilon;L,n}(r) \quad = \quad E_{\Lambda_\eta^\epsilon;L,n} u_{\Lambda_\eta^\epsilon;L,n}(r).$$

  Energy eigenvalues $E_{\Lambda_\eta^\epsilon;L,n}$ correspond to masses of $\bar{Q}Q$ hybrid mesons.

  [E. Braaten, C. Langmack, D. H. Smith, Phys. Rev. D **90**, 014044 (2014) [arXiv:1402.0438]]
  [M. Berwein, N. Brambilla, J. Tarrus Castella, A. Vairo, Phys. Rev. D **92**, 114019 (2015) [arXiv:1510.04299]]
  [R. Oncala, J. Soto, Phys. Rev. D **96**, 014004 (2017) [arXiv:1702.03900]]

- **Recent work to include heavy spin and $1/m_Q$ corrections.**

  [N. Brambilla, G. Krein, J. Tarrus Castella, A. Vairo, Phys. Rev. D **97**, 016016 (2018) [arXiv:1707.09647]]
  [N. Brambilla, W. K. Lai, J. Segovia, J. Tarrus Castella, A. Vairo, Phys. Rev. D **99**, 014017 (2019) [arXiv:1805.07713]]
  [C. Schlosser, M.W., unpublished ongoing work]

Hybrid flux tubes (1)

- We are interested in

$$\Delta F^2_{\mu\nu,\Lambda_\eta^\epsilon}(r;\mathbf{x}) \quad = \quad \langle 0_{\Lambda_\eta^\epsilon}(r)|F^2_{\mu\nu}(\mathbf{x})|0_{\Lambda_\eta^\epsilon}(r)\rangle - \langle\Omega|F^2_{\mu\nu}|\Omega\rangle.$$

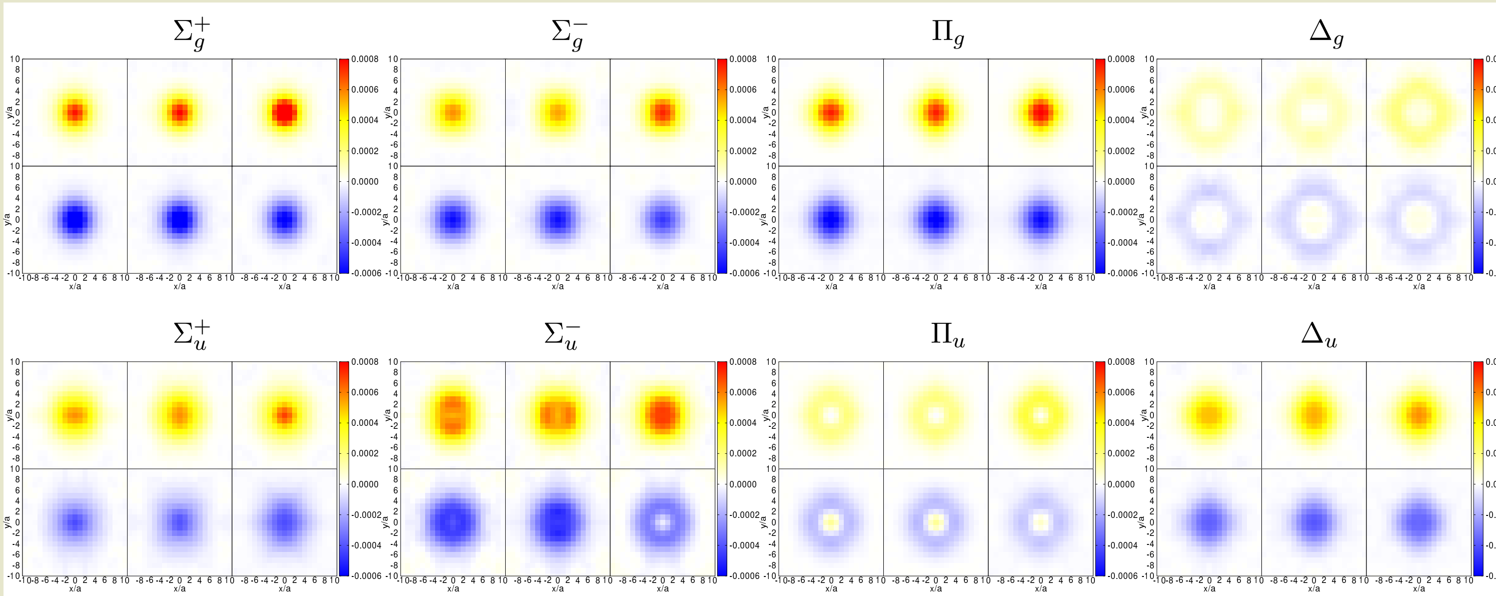
  - $F^2_{\mu\nu}(\mathbf{x})$, $F^2_{\mu\nu}$: squared chromoelectric/chromomagnetic field strength.
  - $|0_{\Lambda_\eta^\epsilon}(r)\rangle$: "hybrid static potential (ground) state" (r denotes the $\bar{Q}Q$ separation).
  - $|\Omega\rangle$: vacuum state.

- The sum over the six independent $\Delta F^2_{\mu\nu,\Lambda_\eta^\epsilon}(r;\mathbf{x})$ is proportional to the chromoelectric and -magnetic energy density of hybrid flux tubes.

Hybrid flux tubes (2)

- $\Delta F^2_{\mu\nu,\Lambda_\eta^\epsilon}(r;\mathbf{x})$, SU(2), mediator plane (x-y plane with Q, $\bar{Q}$ at $(0,0,\pm r/2)$), $r \approx 0.8\,\text{fm}$.
  [L. Müller, O. Philipsen, C. Reisinger, M.W., Phys. Rev. D **100**, 054503 (2019) [arXiv:1907.014820]]]
- For results for $\Lambda_\eta^\epsilon = \Sigma_g^+, \Sigma_u^+, \Pi_u$ see also
  [P. Bicudo, N. Cardoso and M. Cardoso, Phys. Rev. D **98**, 114507 (2018) [arXiv:1808.08815]]

ΔE_x^2	ΔE_y^2	ΔE_z^2
ΔB_x^2	ΔB_y^2	ΔB_z^2



Hybrid flux tubes ($r \approx 0.48\,$fm)

- $\Delta F^2_{\mu\nu,\Lambda_\eta^\epsilon}(r;\mathbf{x})$, SU(2), separation plane (x-z plane with Q, $\bar{Q}$ at $(0,0,\pm r/2)$), $r \approx 0.8\,$fm.
  [L. Müller, O. Philipsen, C. Reisinger, M.W., Phys. Rev. D **100**, 054503 (2019) [arXiv:1907.014820]]]

- For results for $\Lambda_\eta^\epsilon = \Sigma_g^+, \Sigma_u^+, \Pi_u$ see also
  [P. Bicudo, N. Cardoso and M. Cardoso, Phys. Rev. D **98**, 114507 (2018) [arXiv:1808.08815]]

ΔE_x^2	ΔE_y^2	ΔE_z^2
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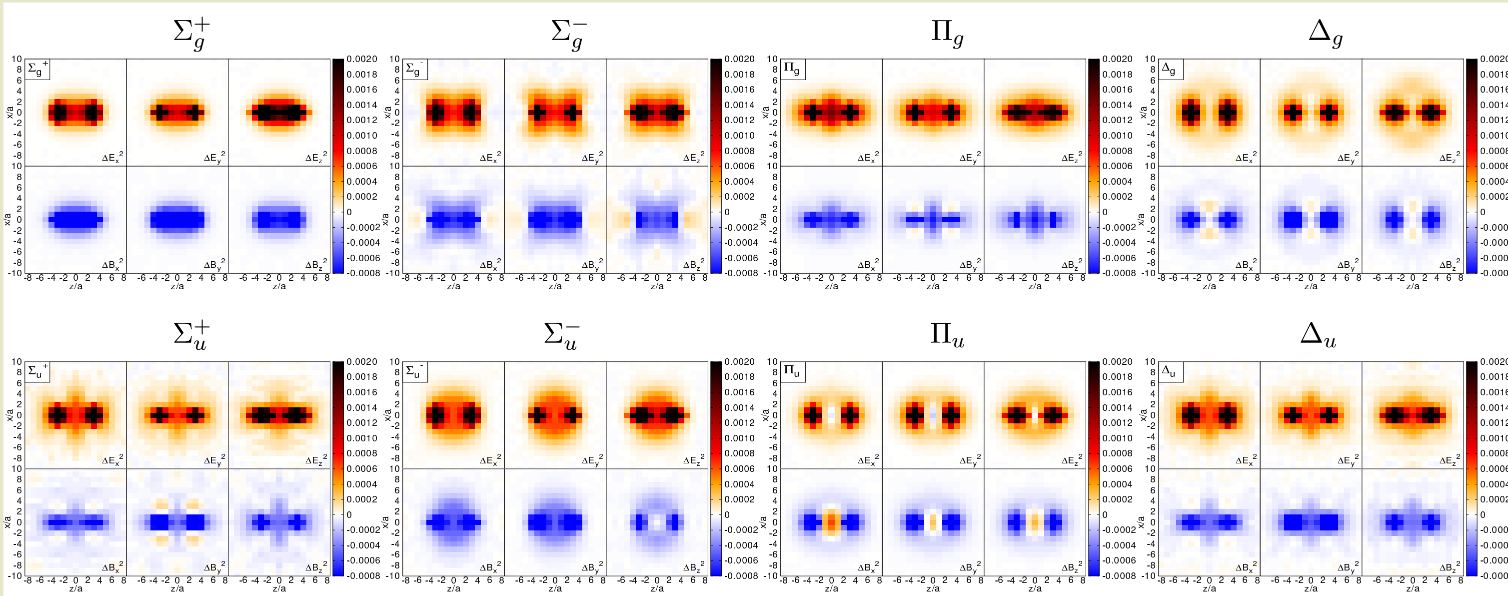
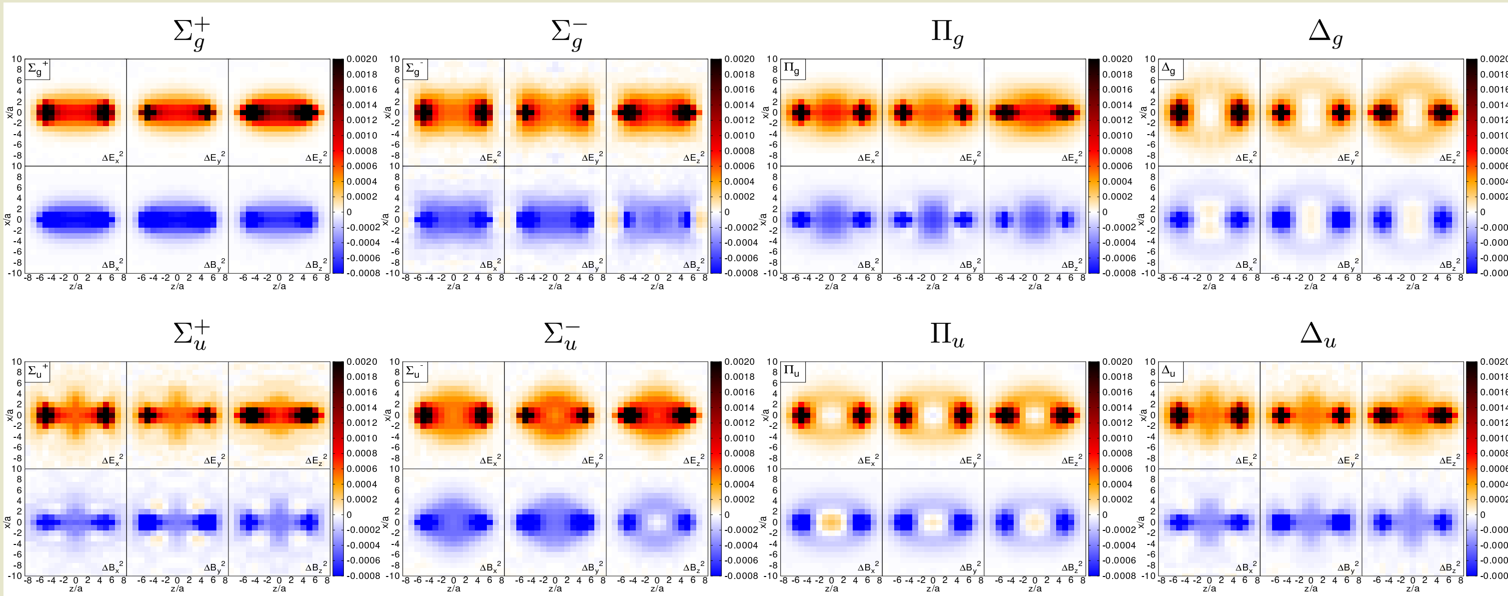


Hybrid flux tubes ($r \approx 0.80\,\textbf{fm}$)

- $\Delta F^2_{\mu\nu,\Lambda^\epsilon_\eta}(r;\mathbf{x})$, SU(2), separation plane (x-z plane with Q, $\bar{Q}$ at $(0,0,\pm r/2)$), $r \approx 0.8\,\text{fm}$.
[L. Müller, O. Philipsen, C. Reisinger, M.W., Phys. Rev. D **100**, 054503 (2019) [arXiv:1907.014820]]]

- For results for $\Lambda^\epsilon_\eta = \Sigma^+_g, \Sigma^+_u, \Pi_u$ see also
[P. Bicudo, N. Cardoso and M. Cardoso, Phys. Rev. D **98**, 114507 (2018) [arXiv:1808.08815]]

ΔE_x^2	ΔE_y^2	ΔE_z^2
ΔB_x^2	ΔB_y^2	ΔB_z^2



Summary

- **Investigations of heavy exotic mesons within the Born-Oppenheimer approximation using static potentials computed with lattice QCD**:
  - mainly open flavor four-quark states,
    tetraquarks $\bar{Q}\bar{Q}qq$, (heavy quarks $Q \in \{b, c\}$, light quarks $q \in \{u, d, s\}$)
  - but also
    tetraquarks $\bar{Q}Q\bar{q}q$ and **hybrid mesons** $\bar{Q}Q$ + gluons (very brief).
- Why b quarks? Why not only c quarks, which are more relevant in the context of PANDA?
  - b quarks are technically simpler, e.g. Heavy Quark Effective Theory or Non Relativistic QCD applicable/more accurate.
  - Systems with $Q = b$ are physically simpler, e.g. certain tetraquarks are QCD-stable.
  - **Long-term goal**: accurate predictions for exotic mesons with c quarks.
  - **Important intermediate steps**: computations with b quarks (or even static quarks).