Investigation of heavy exotic mesons with lattice QCD

"Physikalisches Kolloquium" – Universität Augsburg

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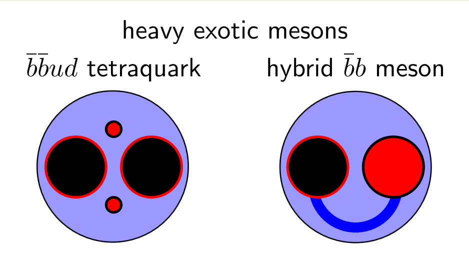
Outline

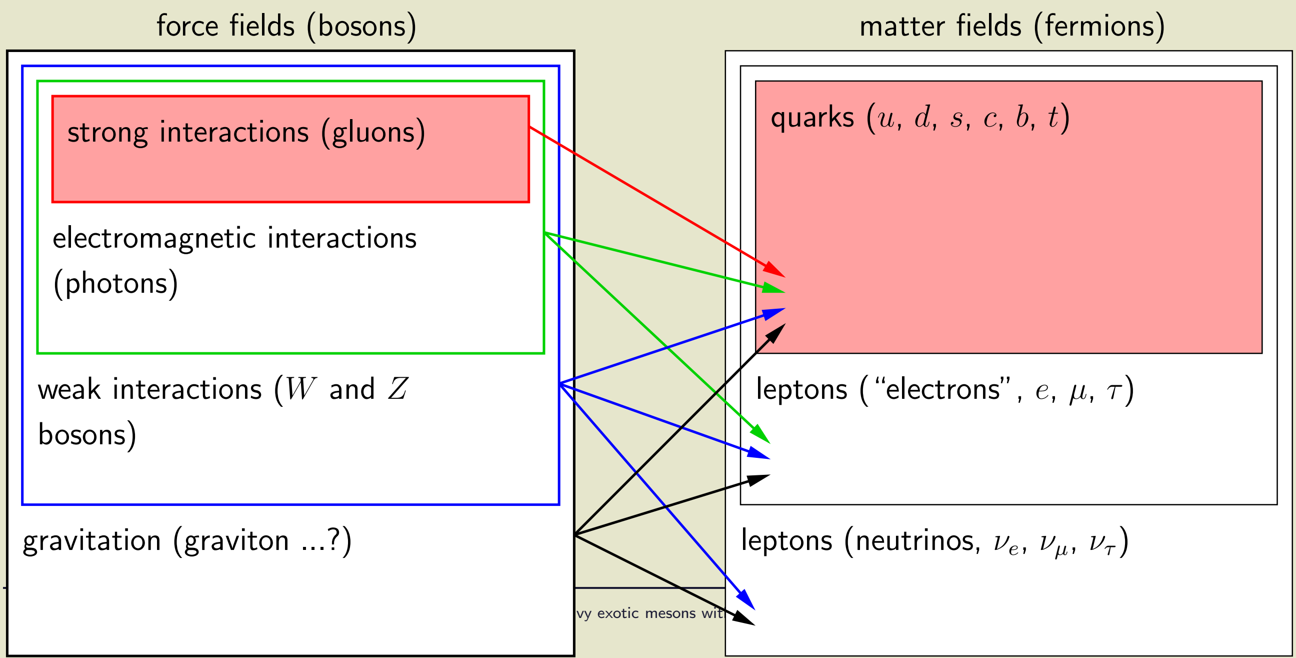
- **Part 1**:
  Basics of
  - **QCD** (= quantum chromodynamics),
  - **the computation of hadron masses in QCD**,
  - **lattice QCD** (= numerical QCD; rather technical, not ideally suited for a colloquium, very short).
- **Part 2**:
  Lattice QCD investigation of **heavy exotic mesons** (in the Born-Oppenheimer approximation).
  (a) **Tetraquarks**, composed of two heavy quarks $\bar{b}\bar{b}$ and two light quarks qq.
  (b) **Heavy hybrid mesons**, $\bar{b}b$ + gluons.



Part 1: Basics of QCD (quantum chromodynamics), the computation of hadron masses, lattice QCD

"Standard Model of particle physics"

- Four fundamental forces mediated by gauge bosons.
- Matter: Six types of quarks (= quark flavors), six types of leptons.
- **QCD**: the quantum field theory describing **quarks**, **gluons** and their **interactions** ... and consequently the structure, mass and decays of systems composed of **quarks** and **gluons** (so-called hadrons) ... e.g. of the proton, the neutron, but also of heavy exotic mesons.



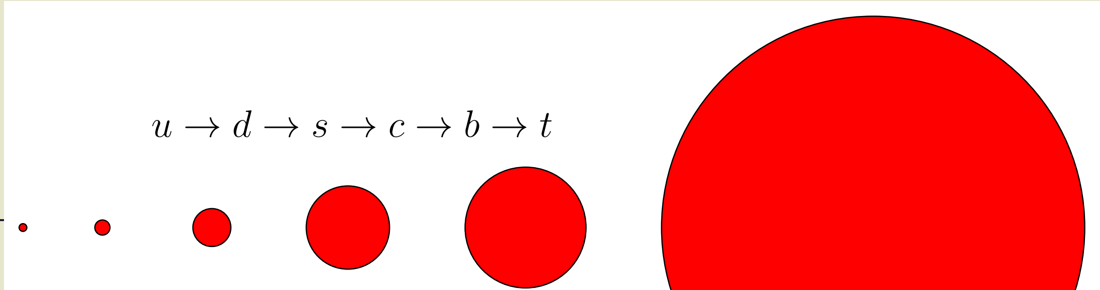
Quarks and gluons

- **Quarks and antiquarks** (spin $1/2$):
  - 6 flavors ... **up**, **down**, **strange**, **charm**, **bottom**, **top** (masses are quite different).
  - 3 colors ... **red**, **green**, **blue** ("three types of charges", similar to the electric charge).

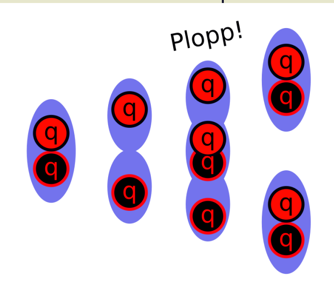
el. charge	$+2/3\,e$	$-1/3\,e$
	$m_{\text{up}} = 1.5 \ldots 3.3\,\text{MeV}/c^2$	$m_{\text{down}} = 3.5 \ldots 6.0\,\text{MeV}/c^2$
	$m_{\text{charm}} = 1160 \ldots 1340\,\text{MeV}/c^2$	$m_{\text{strange}} = 70 \ldots 130\,\text{MeV}/c^2$
	$m_{\text{top}} = 169100 \ldots 173300\,\text{MeV}/c^2$	$m_{\text{bottom}} = 4130 \ldots 4370\,\text{MeV}/c^2$

(e: elementary charge; $1\,\text{MeV}/c^2 = 1.79 \times 10^{-30}\,\text{kg}$)

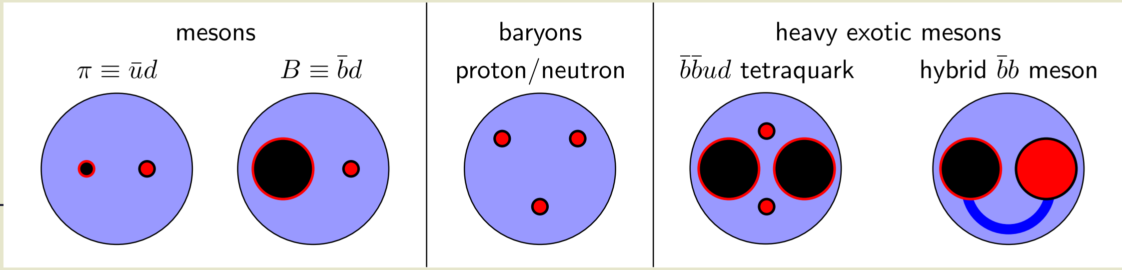
- **Gluons** (spin 1):
  - Massless particles, mediating the strong interaction between quarks.
  - Carry color charge (in contrast e.g. to photons, which do not carry electrical charge); this is the main reason for certain unexpected phenomena, in particular **confinement**.



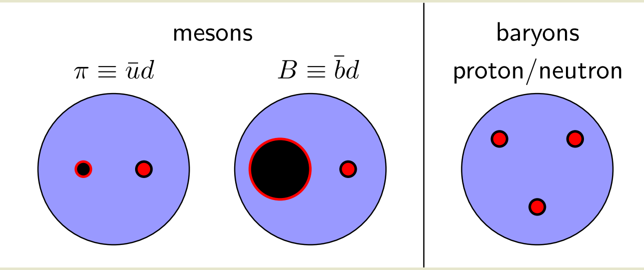
Confinement, hadrons



- One cannot observe/prepare isolated quarks ... they "always" appear in groups ... typically pairs or triplets, so-called **hadrons** ($\rightarrow$ **confinement**).
- **Hadrons**:
  - **Mesons**: Integer spin, typically quark-antiquark pairs.
    Examples: $\pi \equiv \bar{u}d$, $B \equiv \bar{b}d$, ...
    Exotic mesons in part 2 of this talk: $\bar{b}\bar{b}ud$ tetraquarks, $\bar{b}b$ hybrid mesons.
  - **Baryons**: Half-integer spin, typically triplets of quarks or antiquarks.
    Examples: proton $\equiv uud$, neutron $\equiv udd$, ...
  - Several hundred different types of mesons and baryons observed in experiments ("particle zoo"); they differ in
    * quark flavors (six possbilities for each quark/antiquark, u, d, s, c, b, t),
    * quantum numbers similar to that of the hydrogen atom (principal quantum number, total angular momentum J, parity P, ...).



Definition of QCD



- The definition of QCD is quite simple:

$$S = \int d^4x \Big(\sum_{f \in \{u,d,s,c,t,b\}} \bar{\psi}^{(f)} \Big(\gamma_\mu \Big(\partial_\mu - iA_\mu\Big) + m^{(f)}\Big)\psi^{(f)} + \frac{1}{2g^2}\mathrm{Tr}\Big(F_{\mu\nu}F_{\mu\nu}\Big)\Big)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu].$$

- QCD is a quantum field theory, particles are described by fields:
  - $\psi^{(f)}(\mathbf{r}, t)$, $\bar{\psi}^{(f)}(\mathbf{r}, t)$: **quark fields**.
  - $A_\mu(\mathbf{r}, t)$: **gluon field**.
  - A field excitation (an oscillation or a non-vanishing value of the field) at spatial point $\mathbf{r}$ and time t represents a particle at $(\mathbf{r}, t)$.
- No analytic solutions for e.g. meson or baryon masses, because
  - field equations are non-linear,
  - there is no small parameter (coupling constant), i.e. perturbation theory is typically not applicable.
- Numerical methods are mandatory $\rightarrow$ **lattice QCD**.

Computation of hadron masses (1)

- Lattice QCD computation of a hadron mass in three steps:
  (1) Define a suitable hadron creation operator $\mathcal{O}$.
  (2) Compute the temporal correlation function $C(t)$ of the hadron creation operator $\mathcal{O}$ numerically with lattice QCD.
  (3) Extract the hadron mass from the exponential decay of the correlation function $C(t)$.

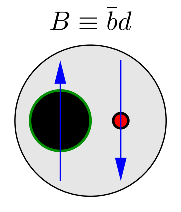
Computation of hadron masses (2)

Step (1): Define a suitable hadron creation operator $\mathcal{O}$

- A hadron creation operator is composed of quark field operators $\psi^{(f)}(\mathbf{r}) \equiv u(\mathbf{r}), d(\mathbf{r}), s(\mathbf{r}), c(\mathbf{r}), b(\mathbf{r}), t(\mathbf{r})$ and gluon field operators $A_\mu(\mathbf{r})$.
- $u(\mathbf{r})$ creates a u quark at $\mathbf{r}$, $d(\mathbf{r})$ creates a d quark at $\mathbf{r}$, etc.
- A **suitable hadron creation operator** O creates a state, which has the same quantum numbers as the hadron of interest and a similar quark and gluon structure:
  - Details are irrelevant.
  - **The resulting hadron mass is independent of these details.**
  - **Example:** B **meson**.
    * Essentially a quark antiquark pair $\bar{b}d$, has total angular momentum $J = 0$, parity $P = -$.
    * A suitable creation operator for a B meson at rest:

      $$\mathcal{O} \equiv \int d^3r \, \bar{b}(\mathbf{r})\gamma_5 d(\mathbf{r})$$

      (γ_5 leads to $J^P = 0^-$, $\int d^3r$ to momentum $\mathbf{p} = 0$).



Computation of hadron masses (3)

Step (2): Compute the temporal correlation function $C(t)$ of the hadron creation operator $\mathcal{O}$ numerically with lattice QCD

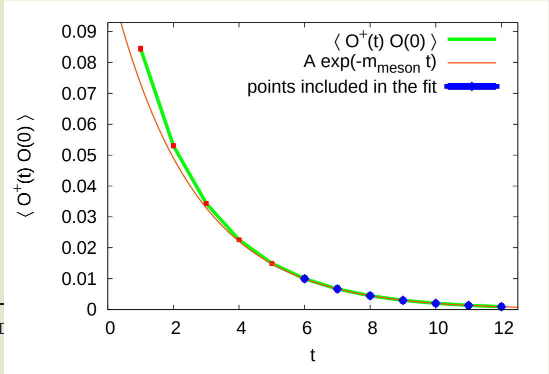
- **Correlation function**: $C(t) \equiv \langle\Omega|\mathcal{O}^\dagger(t)\mathcal{O}(0)|\Omega\rangle$ $\quad$ ($|\Omega\rangle$ = vacuum).
- Lattice QCD is very technical:
  - Advanced algorithms have to be implemented ...
  - ... the corresponding codes run on high performance computers for several weeks or even months ...
  - ... more on the following two slides.

Step (3): Extract the hadron mass from the exponential decay of the correlation function $C(t)$

- Elementary quantum mechanics leads to

$$C(t) \quad = \quad \langle\Omega|\mathcal{O}^\dagger(t)\mathcal{O}(0)|\Omega\rangle \quad \overset{t\rightarrow\infty}{\propto} \quad e^{-m_B t}.$$

- A fit of $Ae^{-m_B t}$ to the lattice QCD results for $C(t)$ provides the hadron mass of interest m_B.



Lattice QCD (1)

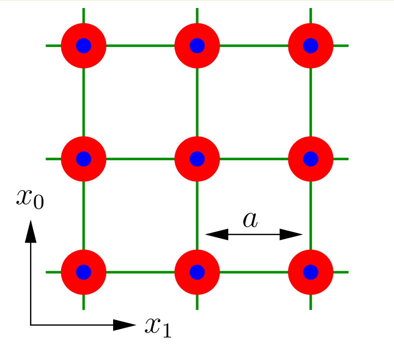
- **Goal**: Numerical computation of QCD observables, e.g. of a temporal correlation function, which allows to extract a hadron mass.

- **Starting point**: **Path integral formulation of quantum field theory**,

$$C(t) \quad = \quad \langle\Omega|\mathcal{O}^\dagger(t)\mathcal{O}(0)|\Omega\rangle \quad = \quad \frac{1}{Z}\underbrace{\int\Big(\prod_f D\psi^{(f)}\,D\bar{\psi}^{(f)}\Big)DA_\mu}_{\text{path integral / functional integral}}\mathcal{O}^\dagger(t)\mathcal{O}(0)e^{-S}$$

  - $\int(\prod_f D\psi^{(f)}\,D\bar{\psi}^{(f)})DA_\mu$ is a so-called **path integral** ...
  - ... an integral over all mathematically possible quark and gluon field configurations $\psi^{(f)}(\mathbf{r},t)$ and $A_\mu(\mathbf{r},t)$ ...
  - ... i.e. an integral over a function space ...
  - ... at each of the infinitely many spacetime points $(\mathbf{r},t)$ one has to solve "ordinary 1-dimensional integrals" over the field values $\psi^{(f)}(\mathbf{r},t)$ and $A_\mu(\mathbf{r},t)$ ...
  - ... **i.e. a path integral is an infinitely-dimensional inegral.**

Lattice QCD (2)



- Numerical implementation:
  - Discretize spacetime with a hypercubic lattice with sufficiently small lattice spacing $a \approx 0.05\,\text{fm} \ldots 0.10\,\text{fm}$ $\rightarrow$ **continuum physics**.
  - Compactify spacetime with sufficiently large extent $L \approx 2.0\,\text{fm} \ldots 4.0\,\text{fm}$ (4-dimensional torus) $\rightarrow$ **no finite volume corrections**.
- The path integral is now an ordinary finite-dimensional integral,

$$\int \Big(\prod_f D\psi^{(f)}\, D\bar{\psi}^{(f)}\Big) DA_\mu \quad \rightarrow \quad \prod_{x_\nu \in \text{lattice}} \Big(\prod_f d\psi^{(f)}(x_\nu)\, d\bar{\psi}^{(f)}(x_\nu)\Big) dU_\mu(x_\nu).$$

- Typical dimension of a lattice QCD path integral:
  - x_ν: $32^4 \approx 10^6$ lattice sites.
  - $\psi = \psi_A^{a,(f)}$: 24 components ($\times 2$ particle/antiparticle, $\times 3$ color, $\times 4$ spin), 2 flavors.
  - $U_\mu = U_\mu^{ab}$: 32 components ($\times 8$ color, $\times 4$ spin).
  - In total: $32^4 \times (2 \times 24 + 32) \approx \mathbf{83 \times 10^6}$-dimensional integral.

$\rightarrow$ Specifically designed, technically advanced stochastic algorithms necessary ($\rightarrow$ error bars).
$\rightarrow$ High performance computers mandatory ($\rightarrow$ lattice QCD collaborations).

Goals of lattice QCD computations

- Typical goals of lattice QCD computations:
  - Verification or falsification of QCD by comparing lattice QCD results with experimental measurements (search for new physics).
  - Predictions of experimentally not yet observed mesons or baryons ($\rightarrow$ important input for experiments).
    "Does a $\bar{b}\bar{b}ud$ tetraquark exist? If yes, in which energy region?"
  - Investigations of the structure of mesons and baryons.
    "Has a $\bar{b}\bar{b}ud$ tetraquark a meson-meson or rather a diquark-antidiquark structure?"
    "How are the gluons arranged inside a hybrid meson?"
  - Resolving currently existing contradictions between experimental results and theoretical model calculations.
  - Computation of QCD observables that are experimentally difficult to measure (e.g. QCD at extreme temperatures).
  - ...

(+) No assumptions. No approximations. No model. First principles QCD results.

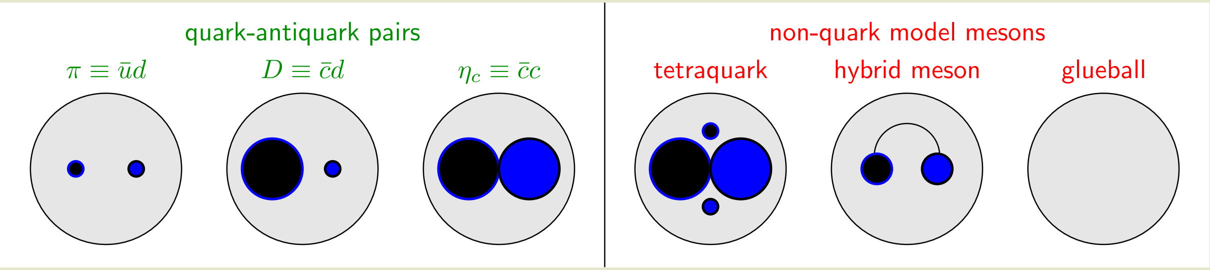
(−) Very time consuming ... lattice QCD projects typically span several years.

Part 2: Lattice QCD investigation of heavy exotic mesons (in the Born-Oppenheimer approximation)

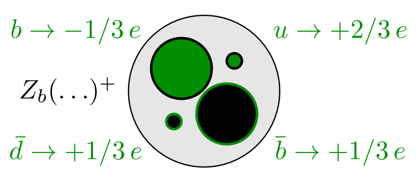
Exotic mesons (1)

- **Meson**: system of quarks and gluons with integer total angular momentum $J = 0, 1, 2, \ldots$
- Most mesons seem to be **quark-antiquark pairs** $\bar{q}q$, e.g. $\pi \equiv \bar{u}d$, $D \equiv \bar{c}d$, $\eta_s \equiv \bar{c}c$ (quark-antiquark model calculations reproduce the majority of experimental results).
- Certain mesons are poorly understood (e.g. significant discrepancies between experimental results and quark model calculations), could have a more complicated structure, e.g.
  - **2 quarks and 2 antiquarks (tetraquark)**,
  - **a quark-antiquark pair and gluons (hybrid meson)**,
  - **only gluons (glueball)**.

$\rightarrow$ Such mesons are referred to as **exotic mesons**.



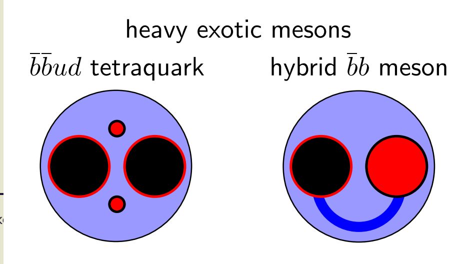
Exotic mesons (2)



- Experimental results on tetraquarks:
  - Electrically charged mesons $Z_b(10610)^+$ and $Z_b(10650)^+$: (2011)
    - The mass and decay channels indicate a $b\bar{b}$ pair.
    - $b\bar{b}$ is electrically neutral ... where does the charge come from?
    - In a tetraquark picture $Z_b(\ldots)^+ \equiv b\bar{b}u\bar{d}$ obvious ($u \to +2/3\,e$, $\bar{d} \to -1/3\,e$).
  - $T_{cc} = \bar{c}\bar{c}ud$ with isospin $I = 0$ und total angular momentum/parity $J^P = 1^+$: (2021)
    - Mass slightly below the lowest meson-meson threshold (DD^*).
    - Almost QCD-stable.

    [R. Aaij *et al.* [LHCb], Nature Phys. **18**, 751-754 (2022) [arXiv:2109.01038]].

- **In this talk exclusively heavy exotic mesons**:
  - Tetraquarks $\bar{b}\bar{b}qq$
    (light quarks $q \in \{u, d, s\}$; includes the "b quark counterpart" of the previously mentioned T_{cc}).
  - Hybrid mesons $\bar{b}b$ + gluons.



Two types of approaches

- Two types of approaches, when studying **heavy exotic mesons** with lattice QCD:
  - **Born-Oppenheimer approximation** (a 2-step procedure):
    - **The focus of the following slides.**

    (1) Compute the potential $V(r)$ of the two heavy quarks (approximated as static quarks) in the presence of two light quarks and/or gluons using lattice QCD.
    $\rightarrow$ full QCD results

    (2) Use standard techniques from quantum mechanics and $V(r)$ to study the dynamics of two heavy quarks (Schrödinger equation, scattering theory, etc.).
    $\rightarrow$ an approximation

    (+) Provides physical insights (e.g. forces between quarks, quark composition).

    (−) An approximation.
  - **Full lattice QCD computations of eigenvalues of the QCD Hamiltonian**:
    - **Not discussed in the following.**
    - Masses of stable hadrons correspond to energy eigenvalues at infinite volume.
    - Masses and decay widths of resonances can be calculated from the volume dependence of the energy eigenvalues (rather technical and difficult).

Part 2a: Tetraquarks, $\bar{b}\bar{b}qq$

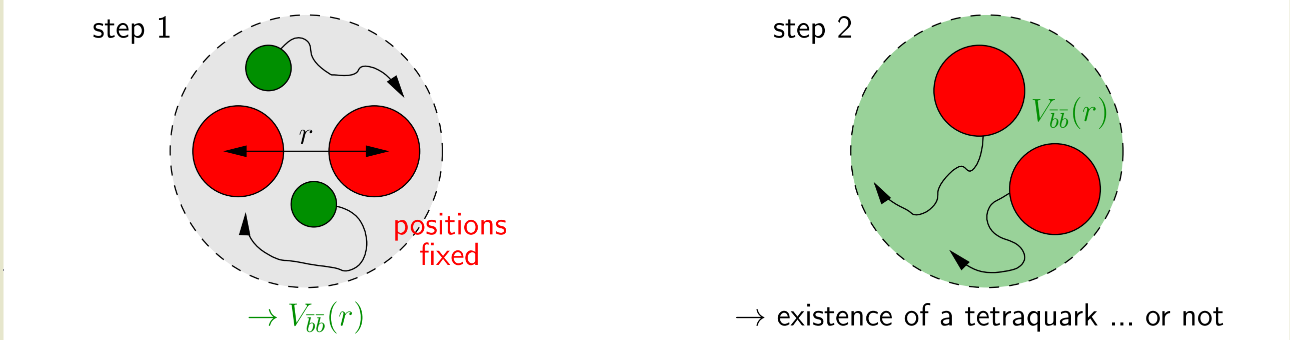
Basic idea: lattice QCD and BO

- **Basic idea**: Investigate the existence of $\bar{b}\bar{b}qq$ tetraquarks in two steps.

(BO1) **Compute potentials $V_{\bar{b}\bar{b}}(r)$ for the two static antiquarks ($\bar{b}\bar{b}$) in the presence of two lighter quarks (qq, $q \in \{u, d, s\}$) using lattice QCD.**

(BO2) **Check, whether these potentials are sufficiently attractive to host bound states or resonances** ($\rightarrow$ correspond to $\bar{b}\bar{b}qq$ tetraquarks) **by using techniques from quantum mechanics and scattering theory.**

- (1) + (2) $\rightarrow$ **Born-Oppenheimer-Approximation**:
  - Developed in the context of molecular and solid state physics.
    [M. Born, R. Oppenheimer, "Zur Quantentheorie der Molekeln," Annalen der Physik 389, Nr. 20, 1927]
  - Step (BO1) in the following not quantum mechanics, but (lattice) QCD.
  - Valid approximation for $m_q \ll m_b$ ($\bar{b}$ quarks almost a rest compared to light quarks).



BO1: $\bar{b}\bar{b}qq/BB$ potentials (1)

- To determine $\bar{b}\bar{b}$ potentials $V_{qq,j_z,\mathcal{P},\mathcal{P}_x}(r)$, compute temporal correlation functions

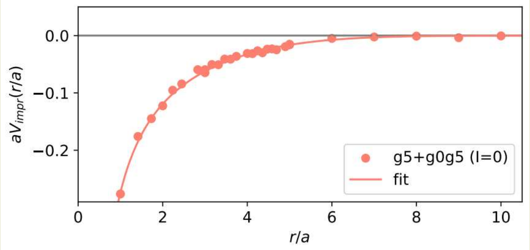
  $$\langle\Omega|\mathcal{O}^\dagger_{BB,\Gamma}(t)\mathcal{O}_{BB,\Gamma}(0)|\Omega\rangle \propto_{t\to\infty} e^{-V_{qq,j_z,\mathcal{P},\mathcal{P}_x}(r)t}$$

  of operators

  $$\mathcal{O}_{BB,\Gamma} \quad = \quad 2N_{BB}(\mathcal{C}\Gamma)_{AB}(\mathcal{C}\tilde{\Gamma})_{CD}\Big(\bar{Q}^a_C(-\mathbf{r}/2)q^a_A(-\mathbf{r}/2)\Big)\Big(\bar{Q}^b_D(+\mathbf{r}/2)q^b_B(+\mathbf{r}/2)\Big).$$

- **Many different channels** (isospin/light flavor, angular momentum, parity).
  $\rightarrow$ Attractive as well as repulsive potentials.
  $\rightarrow$ Potentials with different asymptotic values (two heavy-light mesons $\in \{B, B^*, B_0^*, B_1^*\}$).

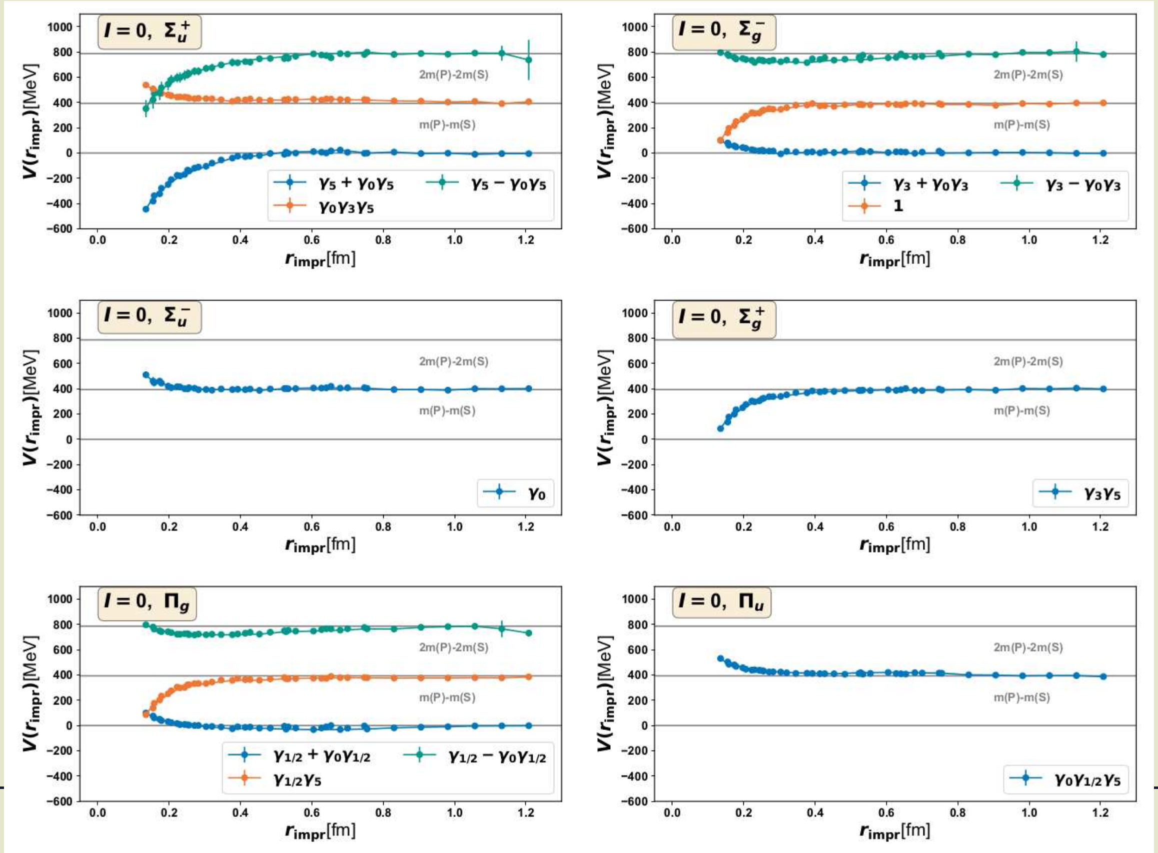
- The most attractive potential of a $B^{(*)}B^{(*)}$ meson pair has $(I, |j_z|, P, P_x) = (0, 0, +, -)$:
  - $\psi^{(f)}\psi^{(f')} = ud - du$, $\Gamma \in \{(1+\gamma_0)\gamma_5\,,\,(1-\gamma_0)\gamma_5\}$.
  - $\bar{Q}\bar{Q} = \bar{b}\bar{b}$, $\tilde{\Gamma} \in \{(1+\gamma_0)\gamma_5\,,\,(1+\gamma_0)\gamma_j\}$ (irrelevant).



[P. Bicudo, K. Cichy, A. Peters, M.W., Phys. Rev. D **93**, 034501 (2016) [arXiv:1510.03441]]
[P. Bicudo, M. Marinkovic, L. Müller, M.W., PoS **LATTICE2024**, 124 (2024) [arXiv:2409.10786]]

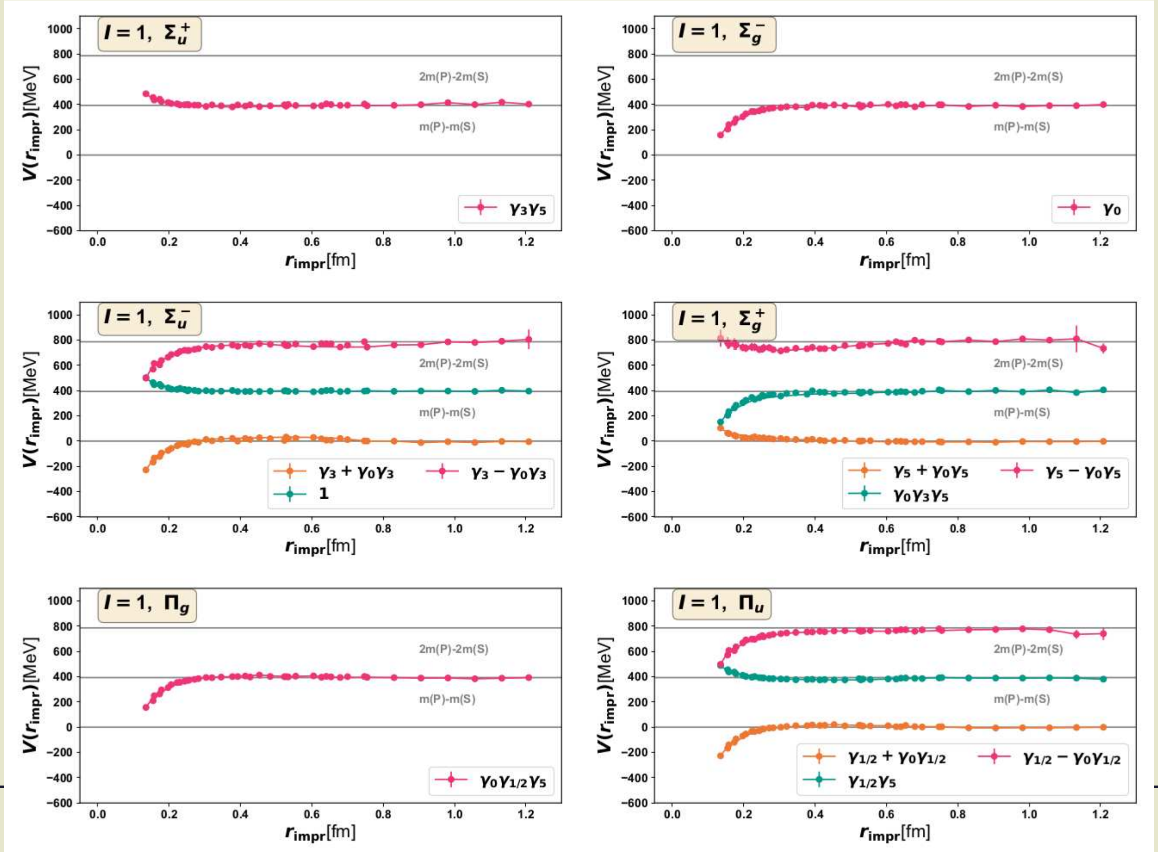
BO1: $\bar{b}\bar{b}qq/BB$ potentials (2)

[P. Bicudo, M. Marinkovic, L. Müller, M.W., PoS **LATTICE2024**, 124 (2024) [arXiv:2409.10786]]



BO1: $\bar{b}\bar{b}qq/BB$ potentials (3)

[P. Bicudo, M. Marinkovic, L. Müller, M.W., PoS **LATTICE2024**, 124 (2024) [arXiv:2409.10786]]



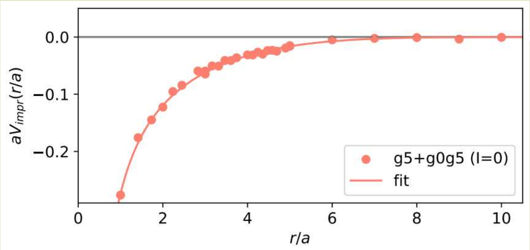
BO2: Stable $\bar{b}\bar{b}qq$ tetraquarks

- Solve the Schrödinger equation for the relative coordinate of the heavy quarks $\bar{b}\bar{b}$ using the previously computed $\bar{b}\bar{b}qq$ potentials,

$$\left(\frac{1}{m_b}\left(-\frac{d^2}{dr^2}+\frac{L(L+1)}{r^2}\right)+V_{qq,j_z,\mathcal{P},\mathcal{P}_x}(r)-2m_B\right)R(r) \quad = \quad ER(r).$$

- Possibly existing bound states, i.e. $E < 0$, indicate QCD-stable $\bar{b}\bar{b}qq$ tetraquarks.
- There is a bound state for orbital angular momentum $L = 0$ of $\bar{b}\bar{b}$:
  - Binding energy $E = -90^{+43}_{-36}$ MeV with respect to the BB^* threshold.
  - Quantum numbers: $I(J^P) = 0(1^+)$.

[P. Bicudo, M.W., Phys. Rev. D **87**, 114511 (2013) [arXiv:1209.6274]]



BO2: Further $\bar{b}\bar{b}qq$ results (1)

- **Are there further QCD-stable $\bar{b}\bar{b}qq$ tetraquarks with other $I(J^P)$ and light flavor quantum numbers?**

  $\rightarrow$ No, not for $qq = ud$ (both $I = 0, 1$), not for $qq = ss$.
  [P. Bicudo, K. Cichy, A. Peters, B. Wagenbach, M.W., Phys. Rev. D **92**, 014507 (2015) [arXiv:1505.00613]]

  $\rightarrow$ $\bar{b}\bar{b}us$ was not investigated.

  – Strong evidence from full QCD computations that a QCD-stable $\bar{b}\bar{b}us$ tetraquark exists.

- **Effect of heavy quark spins:**

  – Expected to be $\mathcal{O}(m_{B^*} - m_B) = \mathcal{O}(45\,\text{MeV})$.

  – Previously ignored (potentials of static quarks are independent of the heavy spins).

  – In [P. Bicudo, J. Scheunert, M.W., Phys. Rev. D **95**, 034502 (2017) [arXiv:1612.02758]] included in a crude phenomenological way via a BB^* and a B^*B^* coupled channel Schrödinger equation with the experimental mass difference $m_{B^*} - m_B$ as input.

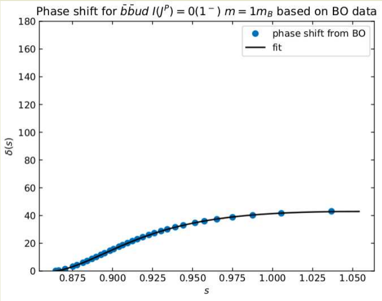
  $\rightarrow$ Binding energy reduced from around $90\,\text{MeV}$ to $59\,\text{MeV}$.

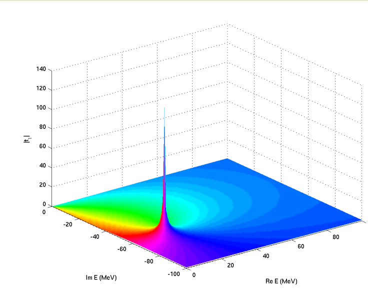
  $\rightarrow$ Physical reason: the previously discussed attractive potential does not only correspond to a lighter BB^* pair, but has also a heavier B^*B^* contribution.

BO2: Further $\bar{b}\bar{b}qq$ results (2)

- **Are there $\bar{b}\bar{b}qq$ tetraquark resonances?**

  – In
  [P. Bicudo, M. Cardoso, A. Peters, M. Pflaumer, M.W., Phys. Rev. D **96**, 054510 (2017) [arXiv:1704.02383]]
  resonances studied via standard scattering theory from quantum mechanics textbooks.





  → Heavy quark spins ignored.

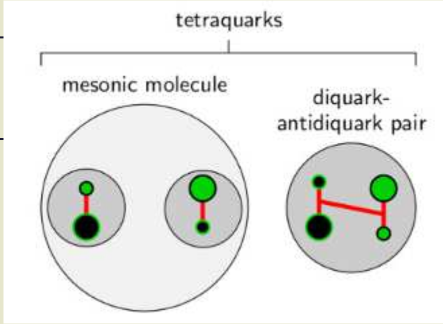
  → Indication for $\bar{b}\bar{b}ud$ tetraquark resonance with $I(J^P) = 0(1^-)$ found, $E = 17^{+4}_{-4}$ MeV above the BB threshold, decay width $\Gamma = 112^{+90}_{-103}$ MeV.

  – In
  [J. Hoffmann, A. Zimermmane-Santos, M.W., PoS **LATTICE2022**, 262 (2023) [arXiv:2211.15765]]
  [J. Hoffmann, M.W., unpublished ongoing work]
  heavy quark spins included.

  → $\bar{b}\bar{b}ud$ resonance shifted upwards, slightly above the B^*B^* threshold.

  → Physical reason: the relevant attractive potential does not only correspond to a lighter BB pair, but has also a heavier dominating B^*B^* contribution.

BO2: Further $\bar{b}\bar{b}qq$ results (3)

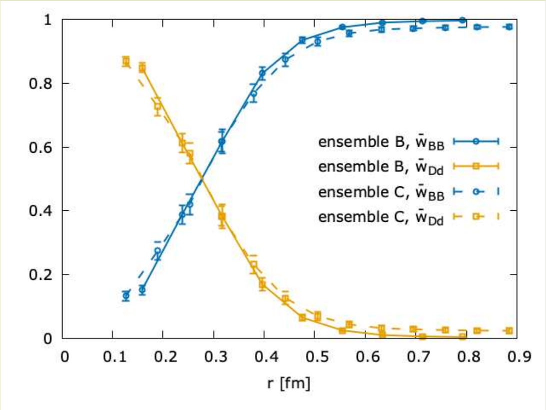


- **Structure of the QCD-stable $\bar{b}\bar{b}ud$ tetraquark: meson-meson (BB) versus diquark-antidiquark (Dd).**
  - Use not just one but two operators,

$$\mathcal{O}_{BB,\Gamma} = 2N_{BB}(\mathcal{C}\Gamma)_{AB}(\mathcal{C}\tilde{\Gamma})_{CD}\Big(\bar{Q}^a_C(-\mathbf{r}/2)\psi^{(f)a}_A(-\mathbf{r}/2)\Big)\Big(\bar{Q}^b_D(+\mathbf{r}/2)\psi^{(f')b}_B(+\mathbf{r}/2)\Big)$$

$$\mathcal{O}_{Dd,\Gamma} = -N_{Dd}\epsilon^{abc}\Big(\psi^{(f)b}_A(\mathbf{z})(\mathcal{C}\Gamma)_{AB}\psi^{(f')c}_B(\mathbf{z})\Big)$$
$$\epsilon^{ade}\Big(\bar{Q}^f_C(-\mathbf{r}/2)U^{fd}(-\mathbf{r}/2;\mathbf{z})(\mathcal{C}\tilde{\Gamma})_{CD}\bar{Q}^g_D(+\mathbf{r}/2)U^{ge}(+\mathbf{r}/2;\mathbf{z})\Big),$$

    compare the contribution of each operator to the $\bar{b}\bar{b}$ potential $V_{qq,j_z,\mathcal{P},\mathcal{P}_x}(r)$.
    [P. Bicudo, A. Peters, S. Velten, M.W., Phys. Rev. D **103**, 114506 (2021) [arXiv:2101.00723]]



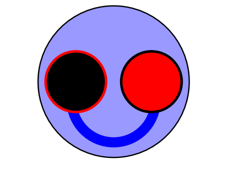
→ $r \lesssim 0.2\,\text{fm}$: Clear diquark-antidiquark dominance.

→ $0.5\,\text{fm} \lesssim r$: Essentially a meson-meson system.

→ Integrate over t to estimate the composition of the tetraquark: $\%BB \approx 60\%$, $\%Dd \approx 40\%$.

Part 2b: hybrid mesons, $\bar{b}b + $ gluons

BO1: hybrid $\bar{b}b$ potentials (1)

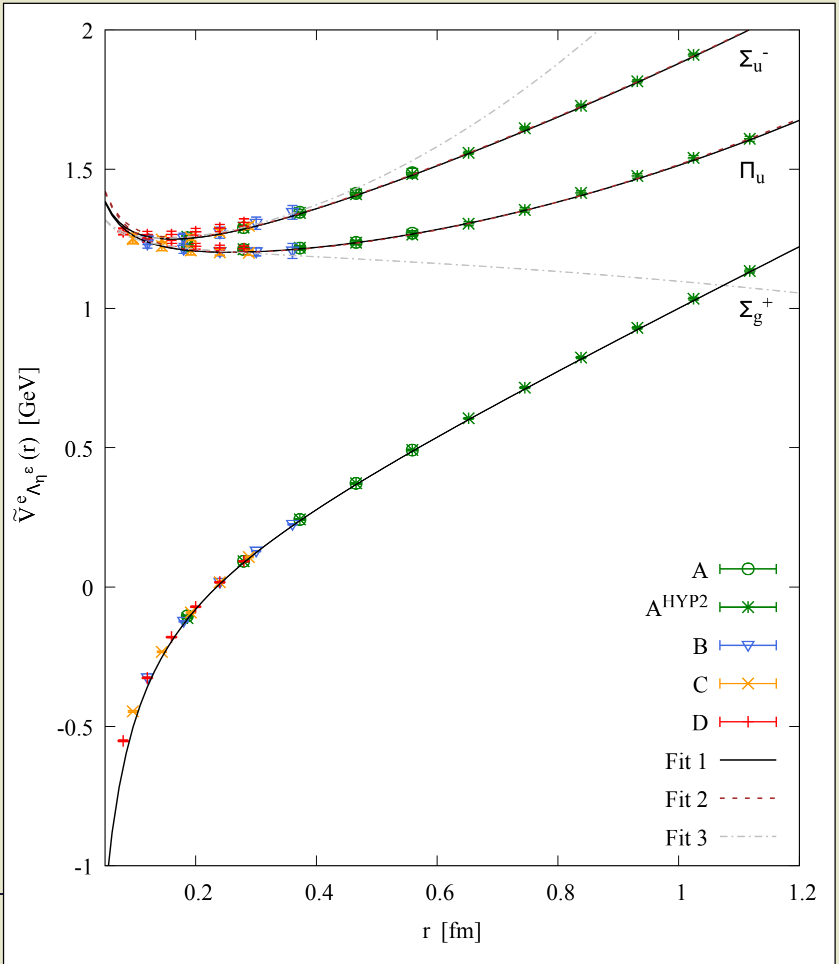


- Now heavy hybrid mesons, i.e. $\bar{b}b +$ gluons, also in the Born-Oppenheimer approximation.
- Non-trivial gluon distributions, i.e. gluons contribute to the quantum numbers of hybrid $\bar{b}b$ potentials:
  - Absolute total angular momentum with respect to the $\bar{Q}Q$ separation axis (z axis): $\Lambda = 0, 1, 2, \ldots \equiv \Sigma, \Pi, \Delta, \ldots$
  - Parity combined with charge conjugation: $\eta = +, - = g, u$.
  - Relection along an axis perpendicular to the $\bar{Q}Q$ separation axis (x axis): $\epsilon = +, -$.
- The ordinary static potential has quantum numbers $\Lambda_\eta^\epsilon = \Sigma_g^+$.
- Particularly interesting: the two lowest hybrid static potentials with $\Lambda_\eta^\epsilon = \Pi_u, \Sigma_u^-$.
- References:

[K. J. Juge, J. Kuti, C. J. Morningstar, Nucl. Phys. Proc. Suppl. **63**, 326 (1998) [hep-lat/9709131]
[C. Michael, Nucl. Phys. A **655**, 12 (1999) [hep-ph/9810415]
...
[P. Bicudo, N. Cardoso, M. Cardoso, Phys. Rev. D **98**, 114507 (2018) [arXiv:1808.08815 [hep-lat]]]
[S. Capitani, O. Philipsen, C. Reisinger, C. Riehl. M.W., Phys. Rev. D **99**, 034502 (2019) [arXiv:1811.11046 [hep-lat]]]

BO1: hybrid $\bar{b}b$ potentials (2)

- [C. Schlosser, M.W., Phys. Rev. D **105**, 054503 (2022) [arXiv:2111.00741]]



BO2: hybrid $\bar{b}b$ mesons

- Solve Schrödinger equations for the relative coordinate of the $\bar{b}b$ pair using hybrid static potentials, e.g.

$$\bigg(-\frac{1}{2\mu}\frac{d^2}{dr^2}+\frac{L(L+1)-2\Lambda^2+J_{\Lambda_\eta^\epsilon}(J_{\Lambda_\eta^\epsilon}+1)}{2\mu r^2}+V_{\Lambda_\eta^\epsilon}(r)\bigg)u_{\Lambda_\eta^\epsilon;L,n}(r) \quad = \quad E_{\Lambda_\eta^\epsilon;L,n}u_{\Lambda_\eta^\epsilon;L,n}(r).$$

  Energy eigenvalues $E_{\Lambda_\eta^\epsilon;L,n}$ correspond to masses of $\bar{b}b$ hybrid mesons.
  [E. Braaten, C. Langmack, D. H. Smith, Phys. Rev. D **90**, 014044 (2014) [arXiv:1402.0438]]
  [M. Berwein, N. Brambilla, J. Tarrus Castella, A. Vairo, Phys. Rev. D **92**, 114019 (2015) [arXiv:1510.04299]]
  [R. Oncala, J. Soto, Phys. Rev. D **96**, 014004 (2017) [arXiv:1702.03900]]

- Interpretation and implications of resulting spectra less clear than e.g. for $\bar{b}\bar{b}ud$ tetraquarks:
  - No obvious hybrid candidates in experimentally measured meson spectra.
  - Multi-hadron states (e.g. $\bar{b}b$ + pion(s)) ignored, heavy quark spins neglected.

- Recent ongoing work to include heavy spin and $1/m_Q$ corrections.
  [N. Brambilla, G. Krein, J. Tarrus Castella, A. Vairo, Phys. Rev. D **97**, 016016 (2018) [arXiv:1707.09647]]
  [N. Brambilla, W. K. Lai, J. Segovia, J. Tarrus Castella, A. Vairo, Phys. Rev. D **99**, 014017 (2019) [arXiv:1805.07713]]
  [C. Schlosser, M.W., arXiv:2501.08844]

Hybrid flux tubes (1)

- Compute with lattice QCD the space-dependent chromoelectric and chromomagnetic energy densities of the gluons for states, which correspond to hybrid $\bar{b}b$ potentials.

$$\Delta F^2_{\mu\nu,\Lambda^\epsilon_\eta}(r;\mathbf{x}) \quad = \quad \langle 0_{\Lambda^\epsilon_\eta}(r)|F^2_{\mu\nu}(\mathbf{x})|0_{\Lambda^\epsilon_\eta}(r)\rangle - \langle\Omega|F^2_{\mu\nu}|\Omega\rangle .$$

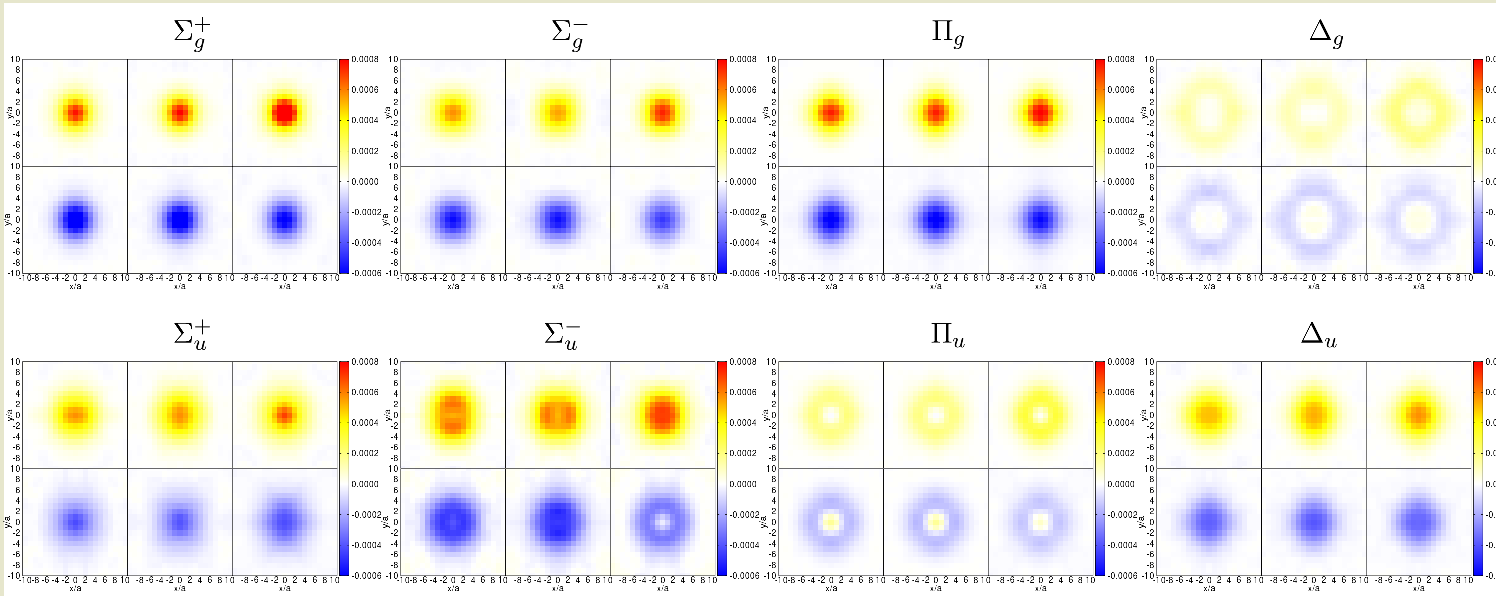
  - $F^2_{\mu\nu}(\mathbf{x})$, $F^2_{\mu\nu}$: Squared chromoelectric/chromomagnetic field strength.
  - $|0_{\Lambda^\epsilon_\eta}(r)\rangle$: "Hybrid static potential (ground) state" (r denotes the $\bar{Q}Q$ separation).
  - $|\Omega\rangle$: Vacuum state.

$\rightarrow$ **Visualize hybrid flux tubes between a quark b and an antiquark $\bar{b}$.**

Hybrid flux tubes (2)

- $\Delta F^2_{\mu\nu,\Lambda_\eta^\epsilon}(r;\mathbf{x})$, SU(2), mediator plane (x-y plane with b, $\bar{b}$ at $(0,0,\pm r/2)$), $r \approx 0.80\,\mathrm{fm}$.
  [L. Müller, O. Philipsen, C. Reisinger, M.W., Phys. Rev. D **100**, 054503 (2019) [arXiv:1907.014820]]]
- For results for $\Lambda_\eta^\epsilon = \Sigma_g^+, \Sigma_u^+, \Pi_u$ see also
  [P. Bicudo, N. Cardoso and M. Cardoso, Phys. Rev. D **98**, 114507 (2018) [arXiv:1808.08815]]

ΔE_x^2	ΔE_y^2	ΔE_z^2
ΔB_x^2	ΔB_y^2	ΔB_z^2

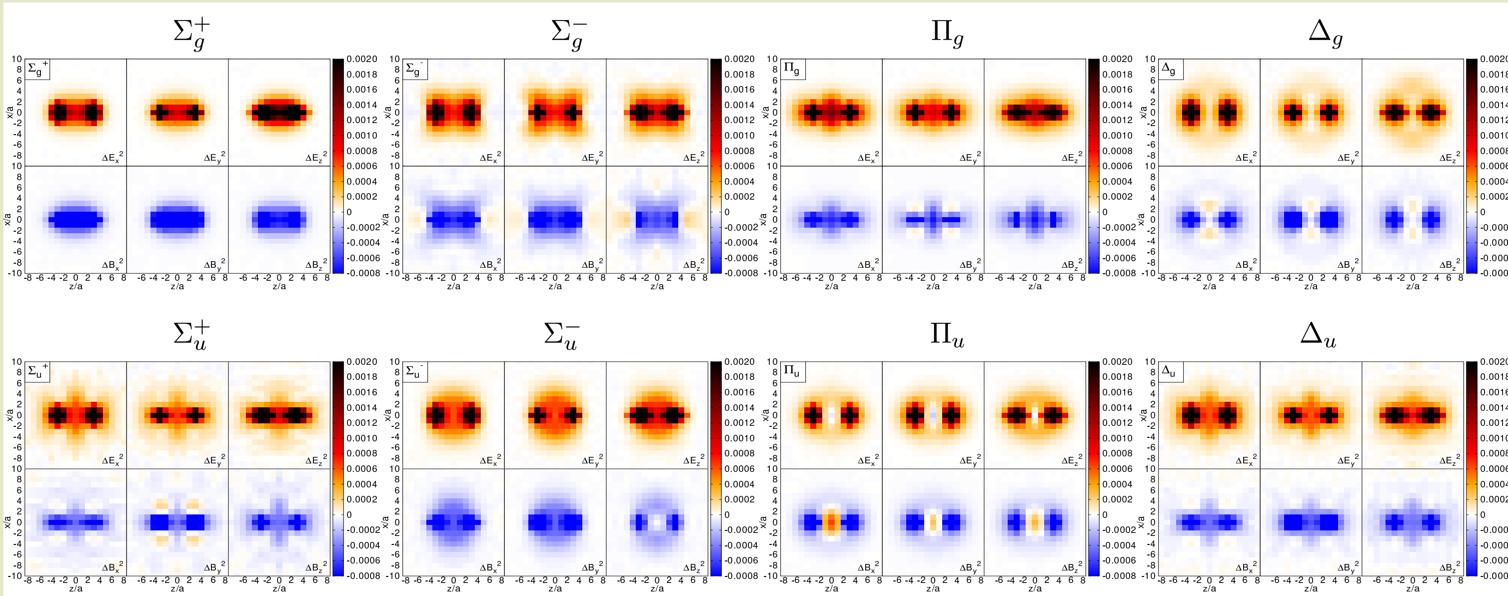


Hybrid flux tubes, $r \approx 0.48\,\text{fm}$

- $\Delta F^2_{\mu\nu,\Lambda^\epsilon_\eta}(r;\mathbf{x})$, SU(2), separation plane (x-z plane with b, $\bar{b}$ at $(0,0,\pm r/2)$), $r \approx 0.8\,\text{fm}$.
  [L. Müller, O. Philipsen, C. Reisinger, M.W., Phys. Rev. D **100**, 054503 (2019) [arXiv:1907.014820]]]

- For results for $\Lambda^\epsilon_\eta = \Sigma^+_g, \Sigma^+_u, \Pi_u$ see also
  [P. Bicudo, N. Cardoso and M. Cardoso, Phys. Rev. D **98**, 114507 (2018) [arXiv:1808.08815]]

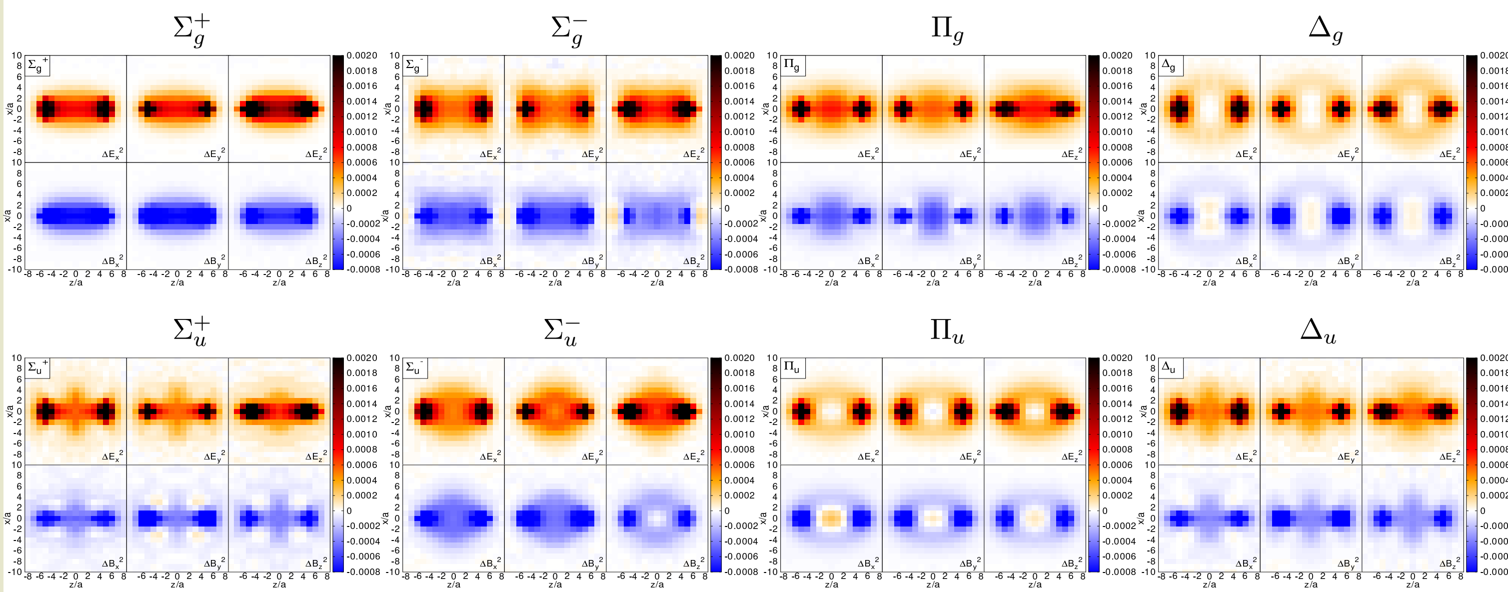
ΔE_x^2	ΔE_y^2	ΔE_z^2
ΔB_x^2	ΔB_y^2	ΔB_z^2



Hybrid flux tubes, $r \approx 0.80\,$fm

- $\Delta F^2_{\mu\nu,\Lambda_\eta^\epsilon}(r;\mathbf{x})$, SU(2), separation plane (x-z plane with b, $\bar{b}$ at $(0,0,\pm r/2)$), $r \approx 0.8\,$fm.
  [L. Müller, O. Philipsen, C. Reisinger, M.W., Phys. Rev. D **100**, 054503 (2019) [arXiv:1907.014820]]]
- For results for $\Lambda_\eta^\epsilon = \Sigma_g^+, \Sigma_u^+, \Pi_u$ see also
  [P. Bicudo, N. Cardoso and M. Cardoso, Phys. Rev. D **98**, 114507 (2018) [arXiv:1808.08815]]

ΔE_x^2	ΔE_y^2	ΔE_z^2
ΔB_x^2	ΔB_y^2	ΔB_z^2



Summary

- **Goal: Start with the fundamental theory of QCD, carry out lattice QCD computations, understand existence/masses/composition of heavy exotic mesons.**
- Heavy exotic mesons: Born-Oppenheimer approximation applicable.
  (1) Lattice QCD computation of $\bar{b}b$ potentials.
  (treatment of light degrees of freedom in QCD)
  (2) Solve a Schrödinger equation using potentials from step (1).
  (treatment of heavy degrees of freedom in quantum mechanics)
- Selected results:
  - Lattice QCD computation of $\bar{b}\bar{b}qq/BB$ potentials and hybrid $\bar{b}b$ potentials.
  - Prediction of a stable $\bar{b}\bar{b}ud$ tetraquark with quantum numbers $I(J^P) = 0(1^+)$.
  - Investigation of the structure of this tetraquark: $\%BB \approx 60\%$, $\%Dd \approx 40\%$.
  - Lattice QCD computation of gluonic energy densities for hybrid $\bar{b}b$ mesons (visualization of flux tubes).