

Comparing meson-meson and diquark-antidiquark creation operators for a $\bar{b}\bar{b}ud$ tetraquark

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We compare two frequently discussed competing structures for a stable $\bar{b}\bar{b}ud$ tetraquark with quantum numbers $I(J^P) = 0(1^+)$ by considering a meson-meson as well as a diquark-antidiquark creation operator. We treat the heavy antiquarks as static with fixed positions and find diquark-antidiquark dominance for $\bar{b}\bar{b}$ separations $r \lesssim 0.2$ fm, while for $r \gtrsim 0.5$ fm the system essentially corresponds to a pair of B mesons. For the meson-meson to diquark-antidiquark ratio of the tetraquark we obtain around 58%/42%.

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1. Introduction

Anti-heavy-anti-heavy-light-light tetraquarks $\bar{Q}\bar{Q}qq$ are expected to be hadronically stable, if the antiquarks are sufficiently heavy (see e.g. Refs. [1–5]). This was confirmed numerically by lattice QCD computations using the Born-Oppenheimer approximation [6–10] as well as by full lattice QCD computations using four quarks of finite mass [11–15].

In this work (see also Ref. [16]) we continue our Born-Oppenheimer based lattice QCD studies and explore the structure of a theoretically predicted $\bar{b}\bar{b}ud$ tetraquark with quantum numbers $I(J^P) = 0(1^+)$. In particular we try to clarify, whether it resembles a meson-meson system or rather a diquark-antidiquark system. Experimentally, this tetraquark has not yet been observed, but its discovery potential is discussed in Refs. [17, 18].

2. Basic principle of our approach and summary of previous work

The Born-Oppenheimer approximation [19, 20] can be used to study $\bar{b}\bar{b}qq$ tetraquarks in a two step approach. In a first step, one treats the heavy \bar{b} quarks as static quarks \bar{Q} and computes $\bar{Q}\bar{Q}$ potentials in the presence of two lighter quarks qq ($q \in \{u, d, s\}$) using lattice QCD (see e.g. Refs. [7, 9, 21–24]). Then, in a second step, the resulting potentials are inserted into the Schrödinger equation to study the dynamics of the heavy \bar{b} quarks. Using standard techniques from quantum mechanics and scattering theory one can check, whether these potentials are sufficiently attractive to host bound states or resonances, which indicate the existence of $\bar{b}\bar{b}qq$ tetraquarks (see e.g. Refs. [6, 8, 10, 25]).

At large $\bar{Q}\bar{Q}$ separation r , the four quarks will form two static-light mesons $\bar{Q}q$ and $\bar{Q}q$ and the corresponding potential is equal to the sum of the two meson masses. A $\bar{Q}\bar{Q}$ potential in the presence of two lighter quarks qq depends on

- the light quark flavors (i.e. isospin and strangeness),
- the light quark spins (the static quark spins are irrelevant),
- parity, which can be related to the types of the mesons (negative parity B and B^* ground state mesons and positive parity B_0^* and B_1^* excitations).

Thus, there are quite a number of different channels, which were computed and are discussed in detail in Ref. [9]. Some of the corresponding potentials are attractive, others are repulsive, and they differ in their asymptotic values at large r .

To determine $\bar{Q}\bar{Q}$ potentials, one has to compute temporal correlation functions of suitably chosen creation operators. One possibility is to use operators of meson-meson type,

$$O_{BB} = 2(C\Gamma)_{AB}(C\tilde{\Gamma})_{CD} \left(\bar{Q}_C^a(-\mathbf{r}/2)\psi_A^{(f)a}(-\mathbf{r}/2) \right) \left(\bar{Q}_D^b(+\mathbf{r}/2)\psi_B^{(f')b}(+\mathbf{r}/2) \right), \quad (1)$$

where $C = \gamma_0\gamma_2$ is the charge conjugation matrix, A, B, C, D denote spin indices, a, b color indices and $\psi^{(f)}$ represent light quark field operators of flavor f . The most attractive potential corresponds to quantum numbers $(I, |j_z|, P, P_x) = (0, 0, +, -)$ (for a detailed discussion see Ref. [9]) and can be obtained by choosing $\psi^{(f)}\psi^{(f')} = ud - du$, $\Gamma = (1 + \gamma_0)\gamma_5$ and $\tilde{\Gamma} \in \{(1 + \gamma_0)\gamma_5, (1 + \gamma_0)\gamma_j\}$.

Lattice data points computed on 2-flavor ETMC gauge link configurations (see Table 1 and Refs. [26–28]) are shown in Figure 1 (left plot). These results are consistently parameterized by

$$V(r) = -\frac{\alpha}{r} \exp\left(-\left(\frac{r}{d}\right)^p\right) + V_0 \quad (2)$$

with $\alpha = 0.293$, $d = 0.356$ fm and $p = 2.74$ (the constant V_0 contains the self energy of the static quarks and is physically irrelevant; within statistical errors $V_0 = 2m_{sl}$, where m_{sl} is the mass of the lightest static-light meson).

ensemble	β	a in fm	$(L/a)^3 \times T/a$	κ	μ	m_{PS} in MeV
B40.24	3.90	0.079(3)	$24^3 \times 48$	0.160856	0.004	340(13)
C30.32	4.05	0.063(2)	$32^3 \times 64$	0.157010	0.003	325(10)

Table 1: ETMC gauge link ensembles used in this work (for details see Refs. [26–28]).

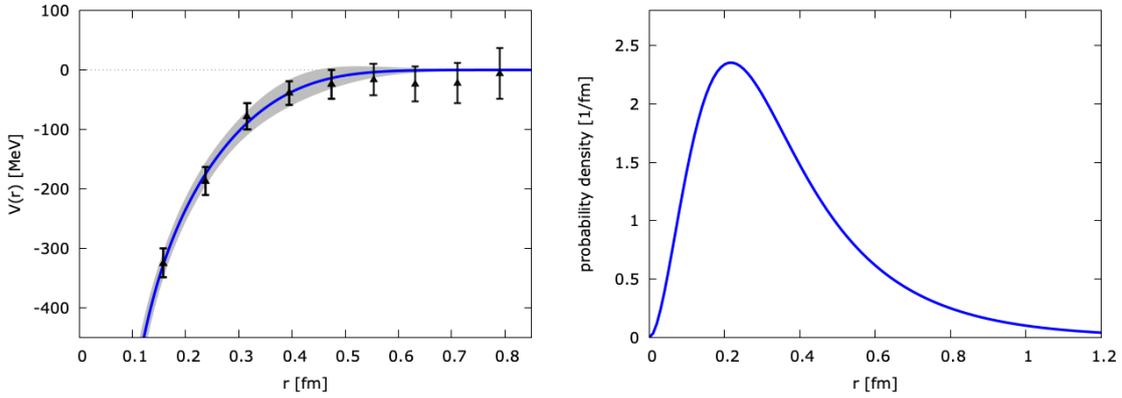


Figure 1: (left) Lattice QCD results for the most attractive $\bar{Q}\bar{Q}$ potential with quantum numbers $(I, |j_z|, P, P_x) = (0, 0, +, -)$ together with the parameterization (2). (right) Radial probability density of the $\bar{b}\bar{b}$ separation $p_r(r) = 4\pi|R(r)|^2$. (The results shown in the two plots correspond to ensemble B40.24 and are taken from Ref. [6].)

When solving the radial Schrödinger equation for that potential,

$$\left(\frac{1}{m_b} \left(-\frac{d^2}{dr^2} + \frac{L(L+1)}{r^2} \right) + V(r) - 2m_{sl} \right) R(r) = ER(r) \quad (3)$$

(m_b denotes the b quark mass, which can be estimated e.g. by the mass of the B meson), one finds for orbital angular momentum $L = 0$ a bound state with binding energy $-E = 38(18)$ MeV [6]. The radial probability density of that state, $p_r(r) = 4\pi|R(r)|^2$, is shown in Figure 1 (right plot) indicating that the $\bar{b}\bar{b}$ separation is typically between 0.1 fm and 0.5 fm. Using the Pauli principle for the \bar{b} quarks one can conclude that the quantum numbers of the corresponding $\bar{b}\bar{b}ud$ tetraquark are $I(J^P) = 0(1^+)$.

3. Structure of the $\bar{b}\bar{b}ud$ tetraquark

To investigate the structure of the $\bar{b}\bar{b}ud$ tetraquark, we consider two significantly different creation operators, which both probe the $(I, |j_z|, P, P_x) = (0, 0, +, -)$ sector: a meson-meson (or BB) operator as given in Eq. (1) and a diquark-antidiquark (or Dd) operator

$$\begin{aligned} \mathcal{O}_{Dd} = & -\epsilon^{abc} \left(\psi_A^{(f)b}(\mathbf{0})(C\Gamma)_{AB}\psi_B^{(f')c}(\mathbf{0}) \right) \\ & \epsilon^{ade} \left(\bar{Q}_C^f(-\mathbf{r}/2)U^{fd}(-\mathbf{r}/2; \mathbf{0})(C\tilde{\Gamma})_{CD}\bar{Q}_D^g(+\mathbf{r}/2)U^{ge}(+\mathbf{r}/2; \mathbf{0}) \right) \end{aligned} \quad (4)$$

with U denoting straight parallel transporters. We choose the same $\psi^{(f)}\psi^{(f')}$, Γ and $\tilde{\Gamma}$ as in Eq. (1). With these two operators we computed the 2×2 correlation matrix

$$C_{jk}(t) = \left\langle \mathcal{O}_j^\dagger(t_2)\mathcal{O}_k(t_1) \right\rangle = \langle \Omega | \mathcal{O}_j^\dagger(t_2)\mathcal{O}_k(t_1) | \Omega \rangle = \langle \Phi_j(t_2) | \Phi_k(t_1) \rangle, \quad (5)$$

where $|\Omega\rangle$ denotes the vacuum and $|\Phi_j\rangle = \mathcal{O}_j|\Omega\rangle$ are meson-meson ($j = BB$) and diquark-antidiquark ($j = Dd$) trial states.

3.1 BB and Dd percentages as functions of the $\bar{Q}\bar{Q}$ separation for the anti-static-anti-static-light-light system

In this subsection we focus on the $\bar{Q}\bar{Q}ud$ system with static antiquarks with fixed positions. We defined the trial state

$$|\Phi_{b,d}\rangle = b|\Phi_{BB,(1+\gamma_0)\gamma_5}\rangle + d|\Phi_{Dd,(1+\gamma_0)\gamma_5}\rangle \quad (6)$$

and determined the coefficients b and d such that the trial state is as similar to the ground state as possible. This amounts to minimizing effective energies

$$V_{b,d}^{\text{eff}}(r, t) = -\frac{1}{a} \log \left(\frac{C_{[b,d][b,d]}(t)}{C_{[b,d][b,d]}(t-a)} \right), \quad C_{[b,d][b,d]}(t) = \begin{pmatrix} b \\ d \end{pmatrix}_j^\dagger C_{jk}(t) \begin{pmatrix} b \\ d \end{pmatrix}_k \quad (7)$$

with respect to b and d . Since the optimization is independent of the normalization and the relative phase of b and d , we consider the weights or percentages of BB and Dd defined via

$$w_{BB} = \frac{|b|^2}{|b|^2 + |d|^2}, \quad w_{Dd} = \frac{|d|^2}{|b|^2 + |d|^2} = 1 - w_{BB}. \quad (8)$$

For fixed $\bar{Q}\bar{Q}$ separation r the percentages w_{BB} and w_{Dd} depend only weakly on t as shown in Figure 2 for selected separations. To fully eliminate the t dependence, we fit constants $\bar{w}_{BB}(r)$ and $\bar{w}_{Dd}(r)$ to the lattice data points $w_{BB}(r, t)$ and $w_{Dd}(r, t)$ for fixed r , but several t .

In Figure 3 we show the percentages \bar{w}_{BB} and \bar{w}_{Dd} as functions of the $\bar{Q}\bar{Q}$ separation r for the two ensembles B40.24 and C30.32. For $r \lesssim 0.2$ fm there is clear diquark-antidiquark dominance. For $0.2 \text{ fm} \lesssim r \lesssim 0.3$ fm diquark-antidiquark dominance turns into meson-meson dominance. For $0.5 \text{ fm} \lesssim r$ the system is essentially a pair of static-light mesons. There is no significant difference between the two ensembles and our results for \bar{w}_{BB} and \bar{w}_{Dd} seem to be essentially independent of the lattice spacing a .

As an alternative to \bar{w}_{BB} and \bar{w}_{Dd} one can also study eigenvector components obtained from a standard generalized eigenvalue problem. Results on the BB and Dd percentages are very similar. For details see Ref. [16].

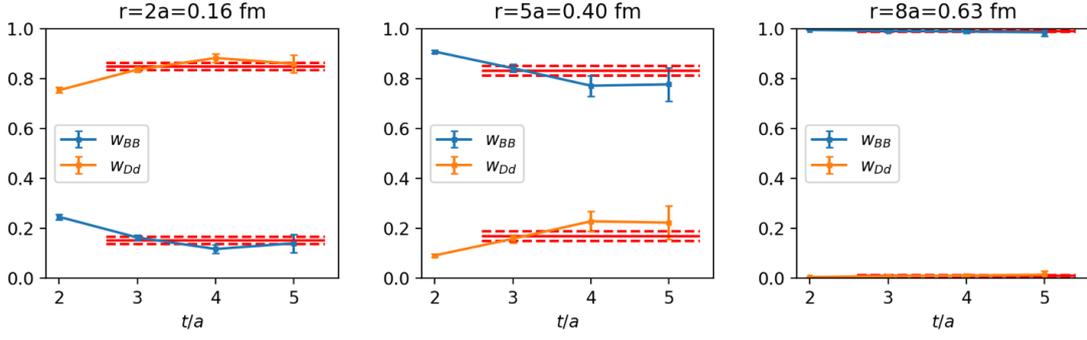


Figure 2: w_{BB} and $w_{Dd} = 1 - w_{BB}$, the normalized absolute squares of the coefficients of the optimized trial state for several fixed r as functions of t for ensemble B40.24. The horizontal red lines indicate fits to constants \bar{w}_{BB} and \bar{w}_{Dd} .

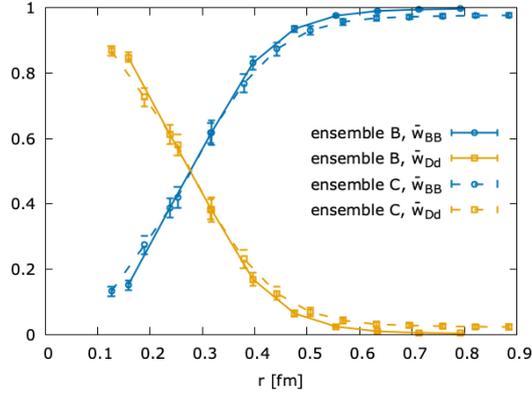


Figure 3: \bar{w}_{BB} and $\bar{w}_{Dd} = 1 - \bar{w}_{BB}$, the normalized absolute squares of the coefficients of the optimized trial state, as functions of r for both ensembles.

3.2 BB and Dd percentages for the $\bar{b}\bar{b}ud$ tetraquark

The total meson-meson and diquark-antidiquark percentages of the $\bar{b}\bar{b}ud$ tetraquark can be obtained by numerically solving the integrals

$$\%_{BB} = \int dr p_r(r) \bar{w}_{BB}(r) \quad , \quad \%_{Dd} = \int dr p_r(r) \bar{w}_{Dd}(r) = 1 - \%_{BB}, \quad (9)$$

where $p_r(r) = 4\pi|R(r)|^2$ is the radial probability density discussed in section 2 and shown in Figure 1 (right plot). We find $\%_{BB} = 0.58$ and $\%_{Dd} = 0.42$. These results indicate that the $\bar{b}\bar{b}ud$ tetraquark with quantum numbers $I(J^P) = 0(1^+)$ is a linear superposition of a meson-meson system and a diquark-antidiquark system with slight meson-meson dominance. This is supported by a recent full lattice QCD study of the same system using four quarks of finite mass [14, 29].

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References

- [1] J. P. Ader, J. M. Richard and P. Taxil, “Do narrow heavy multi-quark states exist?,” *Phys. Rev. D* **25**, 2370 (1982).
- [2] J. I. Ballot and J. M. Richard, “Four quark states in additive potentials,” *Phys. Lett. B* **123**, 449-451 (1983).
- [3] H. J. Lipkin, “A model independent approach to multi-quark bound states,” *Phys. Lett. B* **172**, 242-247 (1986).
- [4] L. Heller and J. A. Tjon, “On the existence of stable dimesons,” *Phys. Rev. D* **35**, 969 (1987).
- [5] J. Carlson, L. Heller and J. A. Tjon, “Stability of dimesons,” *Phys. Rev. D* **37**, 744 (1988).
- [6] P. Bicudo and M. Wagner, “Lattice QCD signal for a bottom-bottom tetraquark,” *Phys. Rev. D* **87**, no. 11, 114511 (2013) [arXiv:1209.6274 [hep-ph]].
- [7] Z. S. Brown and K. Orginos, “Tetraquark bound states in the heavy-light heavy-light system,” *Phys. Rev. D* **86**, 114506 (2012) [arXiv:1210.1953 [hep-lat]].
- [8] P. Bicudo, K. Cichy, A. Peters, B. Wagenbach and M. Wagner, “Evidence for the existence of $ud\bar{b}\bar{b}$ and the non-existence of $ss\bar{b}\bar{b}$ and $cc\bar{b}\bar{b}$ tetraquarks from lattice QCD,” *Phys. Rev. D* **92**, no. 1, 014507 (2015) [arXiv:1505.00613 [hep-lat]].
- [9] P. Bicudo, K. Cichy, A. Peters and M. Wagner, “ BB interactions with static bottom quarks from Lattice QCD,” *Phys. Rev. D* **93**, no. 3, 034501 (2016) [arXiv:1510.03441 [hep-lat]].
- [10] P. Bicudo, J. Scheunert and M. Wagner, “Including heavy spin effects in the prediction of a $\bar{b}\bar{b}ud$ tetraquark with lattice QCD potentials,” *Phys. Rev. D* **95**, no. 3, 034502 (2017) [arXiv:1612.02758 [hep-lat]].
- [11] A. Francis, R. J. Hudspith, R. Lewis and K. Maltman, “Lattice prediction for deeply bound doubly heavy tetraquarks,” *Phys. Rev. Lett.* **118**, no. 14, 142001 (2017) [arXiv:1607.05214 [hep-lat]].

- [12] A. Francis, R. J. Hudspith, R. Lewis and K. Maltman, “Evidence for charm-bottom tetraquarks and the mass dependence of heavy-light tetraquark states from lattice QCD,” *Phys. Rev. D* **99**, no. 5, 054505 (2019) [arXiv:1810.10550 [hep-lat]].
- [13] P. Junnarkar, N. Mathur and M. Padmanath, “Study of doubly heavy tetraquarks in Lattice QCD,” *Phys. Rev. D* **99**, no. 3, 034507 (2019) [arXiv:1810.12285 [hep-lat]].
- [14] L. Leskovec, S. Meinel, M. Pflaumer and M. Wagner, “Lattice QCD investigation of a doubly-bottom $\bar{b}\bar{b}ud$ tetraquark with quantum numbers $I(J^P) = 0(1^+)$,” *Phys. Rev. D* **100**, no. 1, 014503 (2019) [arXiv:1904.04197 [hep-lat]].
- [15] R. J. Hudspith, B. Colquhoun, A. Francis, R. Lewis and K. Maltman, “A lattice investigation of exotic tetraquark channels,” *Phys. Rev. D* **102**, 114506 (2020) [arXiv:2006.14294 [hep-lat]].
- [16] P. Bicudo, A. Peters, S. Velten and M. Wagner, “Importance of meson-meson and of diquark-antidiquark creation operators for a $\bar{b}\bar{b}ud$ tetraquark,” *Phys. Rev. D* **103**, no. 11, 114506 (2021) [arXiv:2101.00723 [hep-lat]].
- [17] A. Ali, A. Y. Parkhomenko, Q. Qin and W. Wang, “Prospects of discovering stable double-heavy tetraquarks at a Tera-Z factory,” *Phys. Lett. B* **782**, 412-420 (2018) [arXiv:1805.02535 [hep-ph]].
- [18] A. Ali, Q. Qin and W. Wang, “Discovery potential of stable and near-threshold doubly heavy tetraquarks at the LHC,” *Phys. Lett. B* **785**, 605-609 (2018) [arXiv:1806.09288 [hep-ph]].
- [19] M. Born and R. Oppenheimer, “Zur Quantentheorie der Molekeln,” *Annalen der Physik* **389**, 457 (1927).
- [20] E. Braaten, C. Langmack and D. H. Smith, “Born-Oppenheimer approximation for the XYZ mesons,” *Phys. Rev. D* **90**, no. 1, 014044 (2014) [arXiv:1402.0438 [hep-ph]].
- [21] W. Detmold, K. Orginos and M. J. Savage, “ BB potentials in quenched lattice QCD,” *Phys. Rev. D* **76**, 114503 (2007) [arXiv:hep-lat/0703009 [hep-lat]].
- [22] M. Wagner [ETM], “Forces between static-light mesons,” *PoS LATTICE2010*, 162 (2010) [arXiv:1008.1538 [hep-lat]].
- [23] G. Bali *et al.* [QCDSF], “Static-light meson-meson potentials,” *PoS LATTICE2010*, 142 (2010) [arXiv:1011.0571 [hep-lat]].
- [24] M. Wagner [ETM], “Static-static-light-light tetraquarks in lattice QCD,” *Acta Phys. Polon. Supp.* **4**, 747-752 (2011) [arXiv:1103.5147 [hep-lat]].
- [25] P. Bicudo, M. Cardoso, A. Peters, M. Pflaumer and M. Wagner, “ $ud\bar{b}\bar{b}$ tetraquark resonances with lattice QCD potentials and the Born-Oppenheimer approximation,” *Phys. Rev. D* **96**, no. 5, 054510 (2017) [arXiv:1704.02383 [hep-lat]].
- [26] P. Boucaud *et al.* [ETM], “Dynamical twisted mass fermions with light quarks,” *Phys. Lett. B* **650**, 304-311 (2007) [arXiv:hep-lat/0701012 [hep-lat]].

- [27] P. Boucaud *et al.* [ETM], “Dynamical twisted mass fermions with light quarks: simulation and analysis details,” *Comput. Phys. Commun.* **179**, 695-715 (2008) [arXiv:0803.0224 [hep-lat]].
- [28] R. Baron *et al.* [ETM], “Light meson physics from maximally twisted mass lattice QCD,” *JHEP* **08**, 097 (2010) [arXiv:0911.5061 [hep-lat]].
- [29] M. Pflaumer, L. Leskovec, S. Meinel and M. Wagner, “Existence and non-existence of doubly heavy tetraquark bound states,” [arXiv:2108.10704 [hep-lat]].