Dynamic structure factor of Luttinger liquids

Talk at the Laboratoire de Physique Theorique et de Hautes Energies
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P. Pirooznia and P. K., Eur. Phys. J. B 58, 201 (2007)
P. Pirooznia, F. Schütz and P.K., arXiv:0802.0970 [cond-mat.str-el] 7 Feb 2008

1. Introduction: Tomonaga-Luttinger model
2. RPA with quadratic energy dispersion
3. Previous work
4. Functional bosonization
5. Results for dynamic structure factor
6. Open problems and solution strategy: functional RG

1.Tomonaga-Luttinger model

Interacting fermions in 1D:

g-ology: marginal coupling constants for two-body scattering (including spin):

g_2: forward scattering, opposite Fermi points

g_4: forward scattering, same Fermi point

g_1: backward scattering

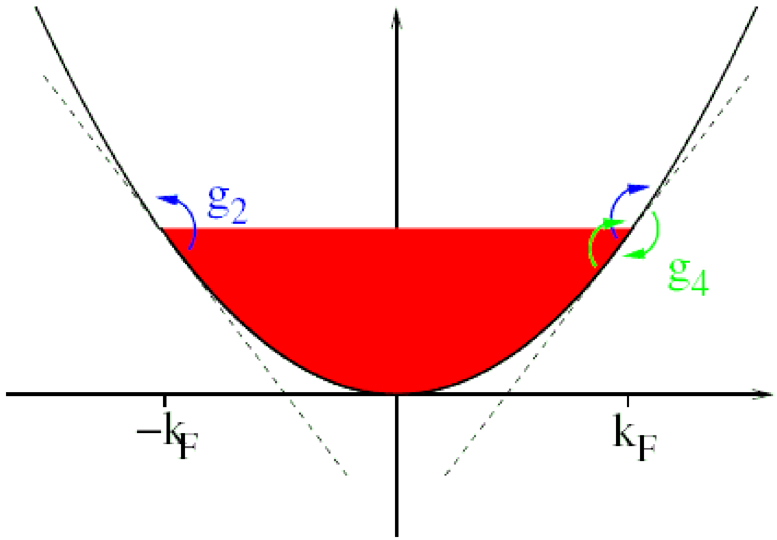
g_3: Umklapp scattering (commensurate fillings)

Tomonaga-Luttinger model:

- only forward scattering (small momentum transfers)
- linearized energy dispersion:

$$\xi_{k_F+k} = \epsilon_{k_F+k} - \epsilon_{k_F} \approx v_F k$$

- exactly solvable via bosonization



Bosonization of the Tomonaga-Luttinger model

Three versions of bosonization:

1.) Constructive bosonization: exact operator identify for fermionic field operator:
(Schönhammer 1997, von Delft+ Schöller 1998)

$$\hat{\psi}(x) = e^{ik_F x}\hat{\psi}_+(x) + e^{-ik_F x}\hat{\psi}_-(x) \qquad \hat{\psi}_\alpha(x) = \frac{1}{\sqrt{2\pi a}}\hat{F}_\alpha e^{2\pi i \frac{\hat{N}_\alpha x}{L}} e^{-i\hat{\phi}_\alpha(x)}$$

Klein factor

$$\hat{\phi}_\alpha(x) = \frac{i}{\sqrt{L}}\sum_{q>0}\sqrt{\frac{2\pi}{q}}\left[e^{iqx}b_{q,\alpha} - e^{-iqx}b^\dagger_{q,\alpha}\right]e^{-qa/2}$$

fermionic density: $$\hat{\rho}_\alpha(x) = \frac{\hat{N}_\alpha}{L} + \frac{\alpha}{2\pi}\frac{\partial\hat{\phi}_\alpha(x)}{\partial x} \qquad \hat{N}_\alpha =: \sum_q \hat{c}^\dagger_{q,\alpha}\hat{c}_{q,\alpha}:$$

2.) Field theoretical bosonization: sloppy treatment of Klein factors and cutoffs
(see for example Affleck, Tsvelik 1980s)

3.) Functional bosonization: Hubbard-Stratonovich Transformation
(Fogedby 1976, PK+Schönhammer 1995)

In all cases: Hamiltonian is quadratic in bosons
and can be solved exactly

TLM: dynamic structure factor

Want: spectral function for density fluctuations (=dynamic structure factor):

$$S(\omega, q) = \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dx e^{i(\omega t - qx)} \langle \delta\hat{\rho}(x,t)\delta\rho(0,0)\rangle$$

In TLM: density fluctuations (=bosons) do not interact; for spinless fermions:

$$\boxed{S_{\mathrm{TLM}}(\omega, q) = Z_q \delta(\omega - \omega_q)}$$

Dispersion of collective density mode (zero sound, ZS):

$$\omega_q = v_0 |q|$$

Velocity of ZS:

$$v_0 = A_0 v_F \quad , \quad A_0 = \sqrt{1 + g_0}$$

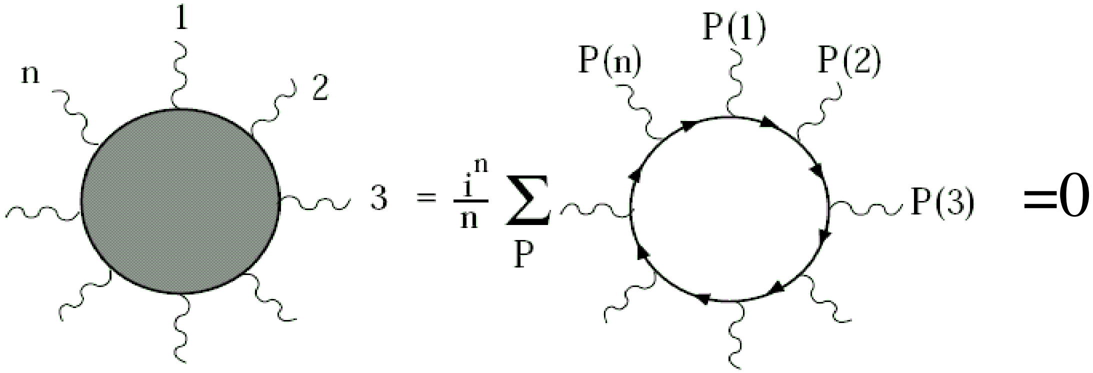
Weight of ZS mode:

$$Z_q = \frac{v_F q^2}{2\pi\omega_q} = \frac{|q|}{2\pi A_0}$$

TLM: Closed loop theorem:

Dzyaloshinskii, Larkin, 1973; T. Bohr, Nordita preprint, 1981

Symmetrized closed fermion loops with more than two external legs vanish



Random Phase Approximation (RPA) for density density correlation function is exact

$$S(\omega, q) = \pi^{-1}\mathrm{Im}\Pi(\omega + i0, q)$$

$$\Pi_{\mathrm{RPA}}(Q) = \frac{\Pi_0(Q)}{1 + f_q \Pi_0(Q)} \qquad \Pi_0(Q) = -\int_K G_0(K)G_0(K+Q)$$

2.RPA with quadratic dispersion

P. Pirooznia and P. K., Eur. Phys. J. B 58, 201 (2007)

Forward scattering model (FSM): forward scattering interaction+ quadratic energy dispersion

$$S[\bar{c},c] = S_0[\bar{c},c] + \frac{1}{2}\int_Q f_q \rho_{-Q}\rho_Q \qquad \rho_Q = \int_K \bar{c}_K c_{K+Q}$$

$$S_0[\bar{c},c] = -\int_K (i\omega - \epsilon_k + \mu)\bar{c}_K c_K \qquad \epsilon_k = \frac{k^2}{2m}$$

Interaction is dominated by small momentum transfers $\boxed{q_c \ll k_F}$

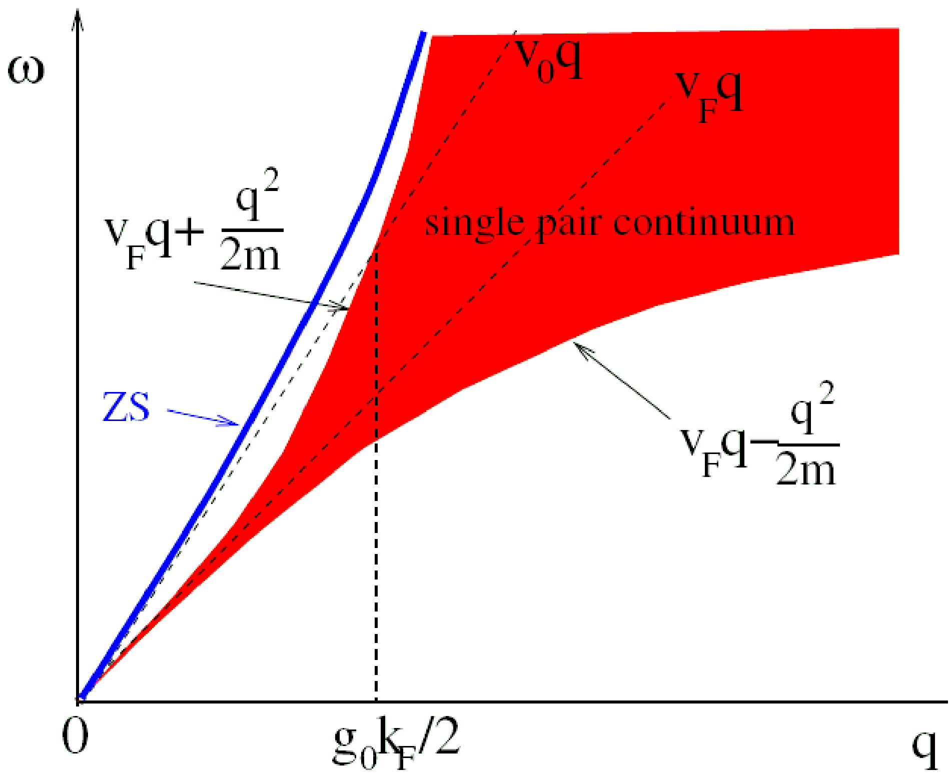
$$f_q = f_0\Theta(q_c - |q|) \quad \text{or} \quad f_q = f_0\frac{q_c^2}{q_c^2 + q^2}$$

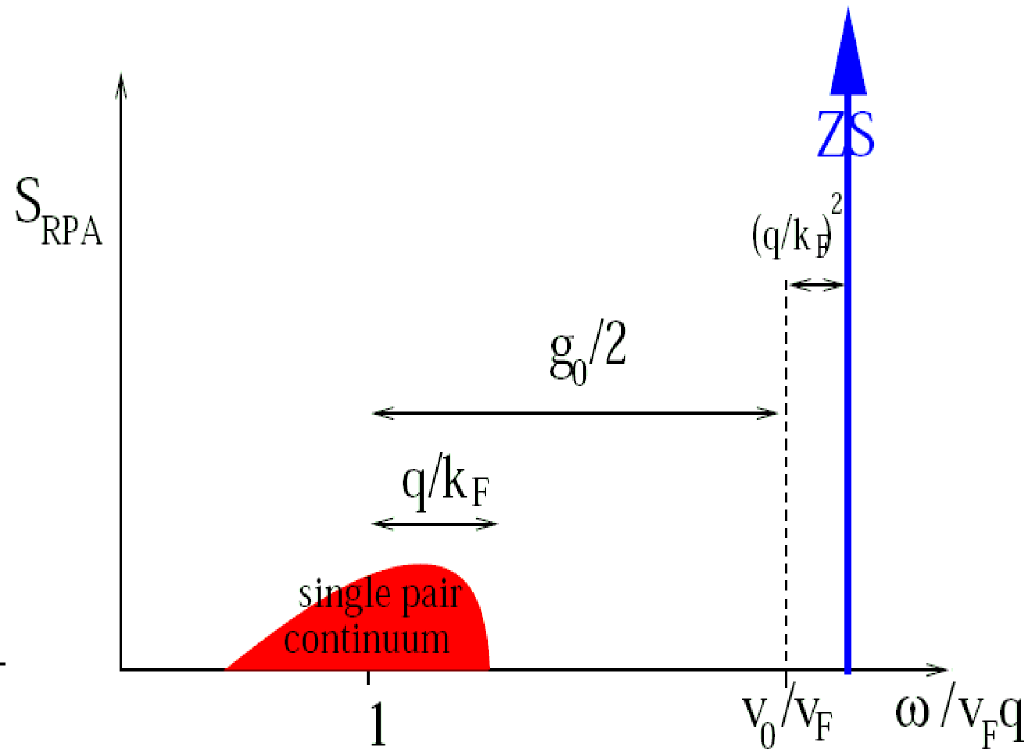
Free polarization can be calculated exactly at zero temperature:

$$\begin{aligned}\Pi_0(Q) &= -\frac{1}{L}\sum_k \frac{\Theta(-\xi_k) - \Theta(-\xi_{k+q})}{i\omega - \xi_{k+q} + \xi_k} \\ &= \frac{m}{\pi q}\ln\left|\frac{i\bar{\omega} + v_F q + \frac{q^2}{2m}}{i\bar{\omega} + v_F q - \frac{q^2}{2m}}\right| .\end{aligned}$$

RPA with quadratic dispersion: dynamic structure factor

$$g_0 = \nu_0 f_0 = \frac{f_0}{\pi v_F}$$ dimensionless interaction strength





- unphysical: single pair continuum at wrong energy $v_F|q|$ (and not at $v_0|q|$)
- but: single-pair continuum carries only small weight for $\frac{|q|}{k_F} \ll g_0$

3. Previous work

Two major strategies:

1.) Field theoretical bosonization:

curvature generates cubic interaction between bosonic fields

$$\mathcal{H}_{\rm int} = \frac{1}{12\pi m}[(\partial_x \hat{\phi}_-(x))^3 - (\partial_x \hat{\phi}_+(x))^3]$$

M. Schick, Phys. Rev. 1968
F. D. M. Haldane, J. Phys. 1981
K. Samokhin, J. Phys. 1998
S. Teber, EPJ B 2007, PRB 2007
G. Pereira,....,I. Affleck, PRL 2006, J. Stat. Phys. 2007, PRL 2008
D. Aristov, PRB 2007

Problems:

- without interactions, need to resum infinite orders in 1/m to recover known free fermion result
- mass shell singularity: finite order expansion in 1/m breaks down for $\omega = \pm v_0 q$

Origin of mass shell singularity:

Expansion of free polarization in powers of momentum

$$\Pi_0(i\omega, q) = \nu_0 \tilde{\Pi}_0(iy, p) \qquad iy = \frac{i\omega}{v_F q}\,,\; p = \frac{q}{2k_F}$$

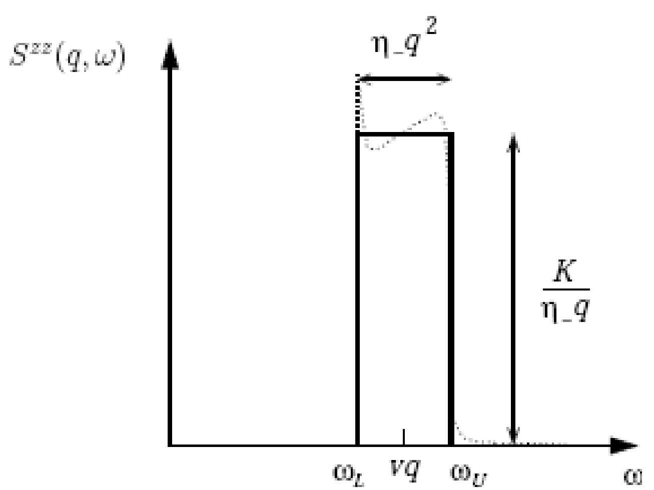
$$\tilde{\Pi}_0(iy,p) = \frac{1}{2p} \ln \left| \frac{iy+1+p}{iy+1-p} \right|$$

$$\tilde{\Pi}_0^{-1}(iy,p) = 1 + y^2 + p^2 \left[1 - \frac{4}{3(1+y^2)} \right] + O(p^4)$$

singular after analytic
continuation to real frequencies:

$$iy \to x + i0 = \frac{\omega}{v_F q} + i0$$

Pereira...Affleck: PRL 2006, J. Stat. Phys. 2007:
combine bosonization with Bethe-Ansatz for XXZ-chain:

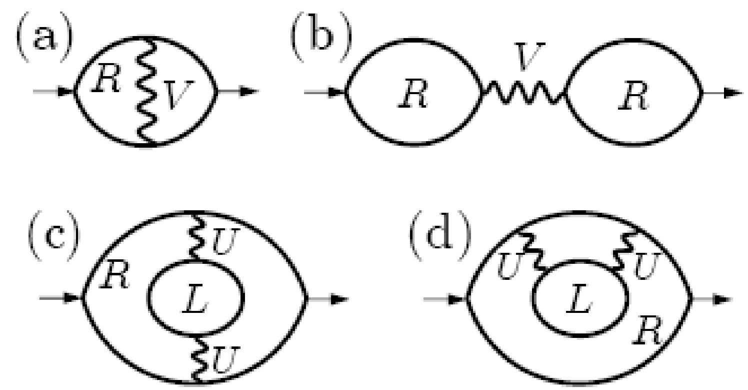


Problems:

- Jordan-Wigner trafo for XXZ-chain give interactions involving also large momentum transfers
- Form factor calculation includes only single-pair continuum

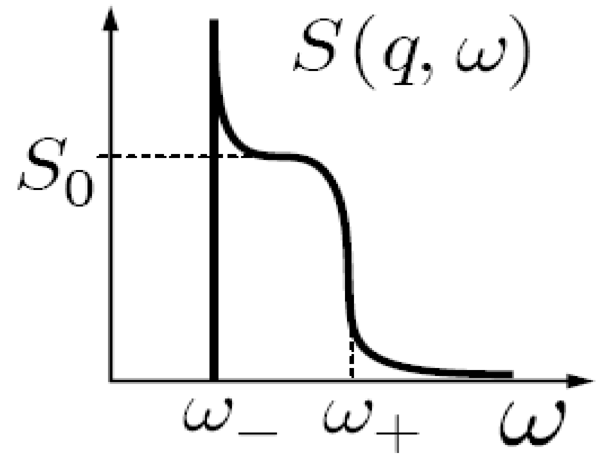
...previous work...

2.) Fermionic perturbation theory:



Pustilnik,...,Glazman: PRL 2006:

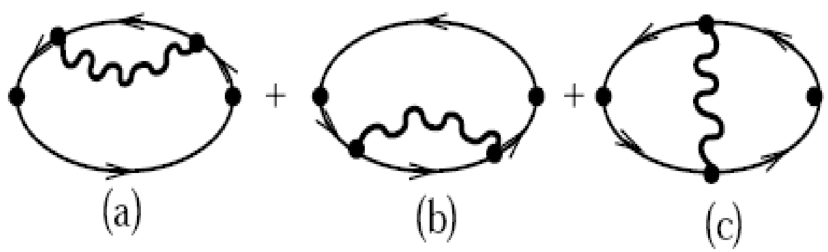
- expansion in bare interaction
- logarithmic singularities even to first order
- infinite resummation of selected diagrams
- non-universal powers law singularities
- do not worry about loop cancellation

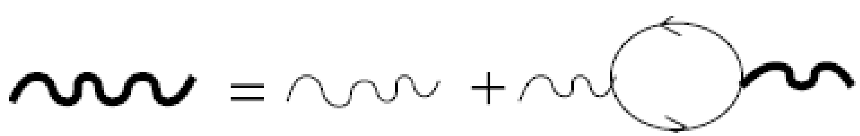


Pirooznia, PK, 2007:

- expansion to first order in RPA interaction
- no singularities apart from mass-shell
- ZS damping:

$$\gamma_q \approx \frac{\pi}{8}\frac{g_0^3}{A_0 A_+^4}\frac{|q|^3}{v_F m^2}$$





4. Functional bosonization

PHYSICAL REVIEW B VOLUME 52, NUMBER 15 15 OCTOBER 1995-I

Bosonization of interacting fermions in arbitrary dimension beyond the Gaussian approximation

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(Received 24 February 1995)

We use our recently developed functional bosonization approach to bosonize interacting fermions in arbitrary dimension d beyond the Gaussian approximation. Even in $d = 1$ the finite curvature of the energy dispersion at the Fermi surface gives rise to interactions between the bosons. In higher

See also PK, Springer Lecture Notes m48 (Springer, 1997)

Advantages:

- efficient way of setting up fermionic perturbation theory
- does not rely on expansion in powers of 1/m
- exact free polarization in non-interacting limit
- loop cancellation manifestly taken into account

Functional bosonization (here with vaccum expectation values)

1.) Decouple interaction via bosonic Hubbard-Stratonovich field

$$\frac{\mathcal{Z}}{\mathcal{Z}_0} = \frac{\int \mathcal{D}[\bar{c}, c, \phi] e^{-S_0[\bar{c}, c] - S_0[\phi] - S_1[\bar{c}, c, \phi]}}{\int \mathcal{D}[\bar{c}, c, \phi] e^{-S_0[\bar{c}, c] - S_0[\phi]}}$$

$$S_0[\phi] = \int_Q f_q^{-1} \phi_{-Q} \phi_Q \qquad S_1[\bar{c}, c, \phi] = i \int_Q \int_K \bar{c}_{K+Q} c_K \phi_Q$$

2.) Take vacuum expectation value into account:

$$\phi_Q = -i \delta_{Q,0} \bar{\phi} + \delta \phi_Q \qquad \bar{\phi} = f_0 \rho_0 \qquad \rho_0 = \int_K G_0(K) = \frac{1}{V} \sum_k \Theta(\mu - \epsilon_k - f_0 \rho_0)$$

$$G_0(K) = \frac{1}{i\omega - \epsilon_k - f_0 \rho_0 + \mu} \qquad \frac{k_F^2}{2m} = \mu - f_0 \rho_0$$

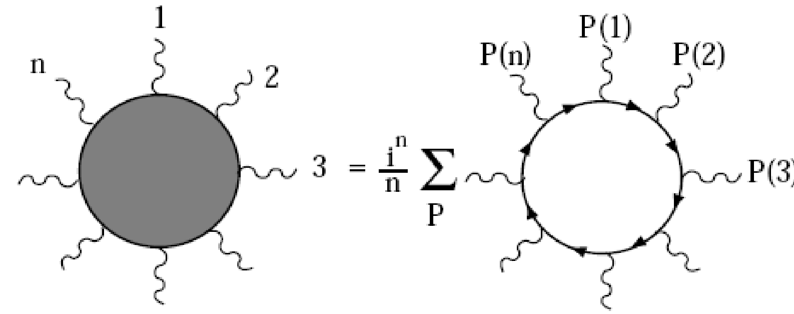
2.) Integrate over fermions:

$$\langle \delta\phi_Q \delta\phi_{Q'} \rangle = \frac{\int \mathcal{D}[\delta\phi] e^{-S_{\text{eff}}[\delta\phi]} \delta\phi_Q \delta\phi_{Q'}}{\int \mathcal{D}[\delta\phi] e^{-S_{\text{eff}}[\delta\phi]}} = \delta_{Q+Q',0} \frac{1}{f_q^{-1} + \Pi_*(Q)}$$

$$S_{\text{eff}}[\delta\phi] = S_2[\delta\phi] + S_{\text{int}}[\delta\phi] \qquad S_2[\delta\phi] = \frac{1}{2} \int_Q [f_q^{-1} + \Pi_0(Q)] \delta\phi_{-Q} \delta\phi_Q$$

Bosonic interactions within functional bosonization:

$$S_{\rm int}[\delta\phi] = \sum_{n=3}^{\infty} \frac{1}{n!} \int_{Q_1} \cdots \int_{Q_n} \delta_{Q_1+\ldots+Q_n,0} \times \Gamma_0^{(n)}(Q_1,\ldots,Q_n)\delta\phi_{Q_1}\ldots\delta\phi_{Q_n}$$



Interaction vertices are proportional to symmetrized closed fermion loops:

$$\Gamma_0^{(n)}(Q_1,\ldots,Q_n) = i^n (n-1)! L_S^{(n)}(-Q_1,\ldots,-Q_n)$$

$$L_S^{(n)}(Q_1,\ldots,Q_n) = \frac{1}{n!} \sum_{P(1,\ldots,n)} \int_K G_0(K) G_0(K-Q_{P(1)})\, G_0(K-Q_{P(1)}-Q_{P(2)}) \cdots G_0(K - \sum_{j=1}^{n-1} Q_{P(j)})$$

Can be calculated exactly for 1D fermions with quadratic dispersion:

(Neumayer, Metzner 1999; Pirooznia, Schütz, P.K. arXiv:0802.0970, Feb. 2008)

Non-symmetrized loop:

$$L^{(n)}(Q_1,\ldots,Q_n) = -\sum_{\substack{i,j=1\\ i<j}}^{n} \left[\prod_{\substack{l=1\\ l\neq i,j}}^{n} H_{ijl} \right]^{-1} \Pi_0(Q_{ij})$$

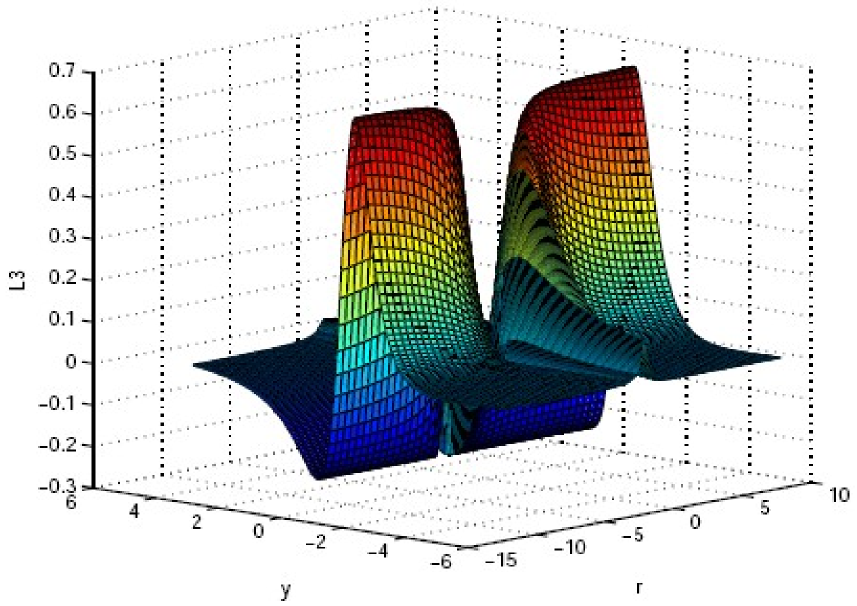
$$H_{ijl} = \frac{1}{q_{ij}} \left[i(\omega_{il} q_{lj} - q_{il}\omega_{lj}) - \frac{q_{li} q_{lj} q_{ij}}{2m} \right]$$

$$q_{ij} = \bar{q}_i - \bar{q}_j \qquad \omega_{ij} = \bar{\omega}_i - \bar{\omega}_j$$

Symmetrized 3-loop:

$$2L_S^{(3)}(i\omega_1, q_1; i\omega_2, q_2; -i\omega_1 - i\omega_2, -q_1 - q_2) = \frac{\nu_0}{mv_F^2}\tilde{L}_S^{(3)}(iy_1, p_1; iy_2, p_2)$$

$$\tilde{L}_S^{(3)}(iy_1, p_1; iy_2, p_2) = \frac{1}{(y_1 - y_2)^2 + (p_1 + p_2)^2}\left[\left(\frac{1}{s_1} + \frac{1}{s_2}\right)\tilde{\Pi}_0(iy_1 s_1 + iy_2 s_2, p_1 + p_2) - \frac{1}{s_2}\tilde{\Pi}_0(iy_1, p_1) - \frac{1}{s_1}\tilde{\Pi}_0(iy_2, p_2)\right]$$



$$iy = \frac{i\omega}{v_F q}, \; p = \frac{q}{2k_F}$$

$$s_1 = \frac{p_1}{p_1 + p_2} = \frac{r}{r+1}$$

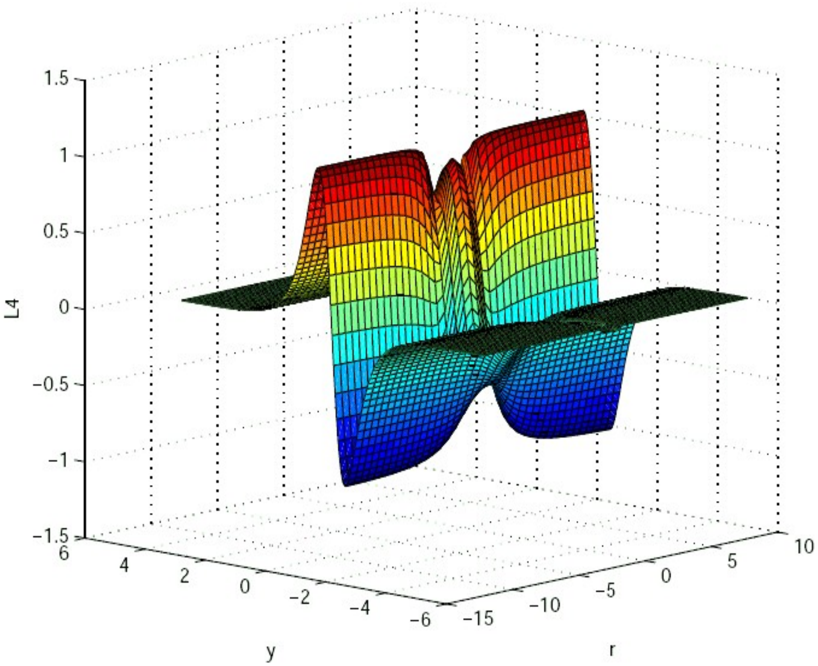
$$s_2 = \frac{p_2}{p_1 + p_2} = \frac{1}{r+1}$$

$$r = \frac{p_1}{p_2}$$

Symmetrized 4-loop:

$$6L_S^{(4)}(i\omega_1, q_1; -i\omega_1, -q_1; i\omega_2, q_2; -i\omega_2, -q_2) = \frac{\nu_0}{(mv_F^2)^2}\tilde{L}_S^{(4)}(iy_1, p_1; iy_2, p_2)$$

$$\begin{aligned}
\tilde{L}_S^{(4)}(iy_1, p_1; iy_2, p_2) &= 2\frac{(p_1^2+3p_2^2)y_-^4 + 2(p_1^2-p_2^2)^2 y_-^2 + (p_1^2-p_2^2)^3}{p_2^2[y_-^2+p_+^2]^2[y_-^2+p_-^2]^2}\tilde{\Pi}_0(iy_1, p_1) \\
&+ 2\frac{(p_2^2+3p_1^2)y_-^4 + 2(p_2^2-p_1^2)^2 y_-^2 + (p_2^2-p_1^2)^3}{p_1^2[y_-^2+p_+^2]^2[y_-^2+p_-^2]^2}\tilde{\Pi}_0(iy_2, p_2) \\
&- \frac{\tilde{\Pi}_0(iy_1 s_1 + iy_2 s_2, p_+)}{s_1^2 s_2^2[y_-^2+p_+^2]^2} - \frac{\tilde{\Pi}_0(iy_1 r_1 + iy_2 r_2, p_-)}{r_1^2 r_2^2[y_-^2+p_-^2]^2} \\
&+ \frac{2y_-\mathrm{Im}[W(iy_1,p_1) - W(iy_2,p_2)]}{[y_-^2+p_+^2][y_-^2+p_-^2]} - \mathrm{Re}[W(iy_1,p_1)W(iy_2,p_2)]
\end{aligned}$$



$$\begin{aligned}
y_\pm &= y_1 \pm y_2 \\
p_\pm &= p_1 \pm p_2
\end{aligned}$$

$$W(iy, p) = \frac{1}{2p}\left[\frac{1}{iy+1+p} - \frac{1}{iy+1-p}\right]$$

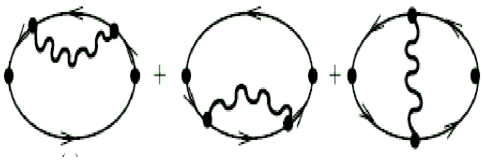
$$C_\pm(iy_-, p_1, p_2) = \frac{1}{p_1 p_2[iy_- - p_\pm]}$$

Functional bosonization: expansion to one bosonic loop:

small parameter $p_0 \equiv \dfrac{q_0}{2k_F} \ll 1$

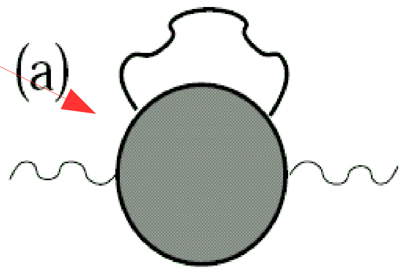
momentum transfer cutoff q_0

equivalent to

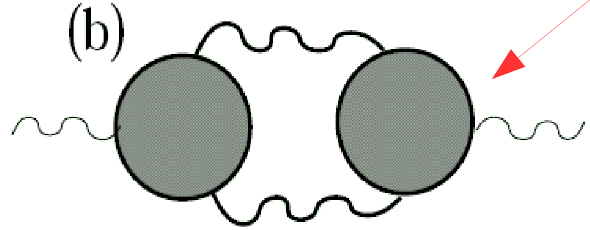


Aslamasov-Larkin diagram

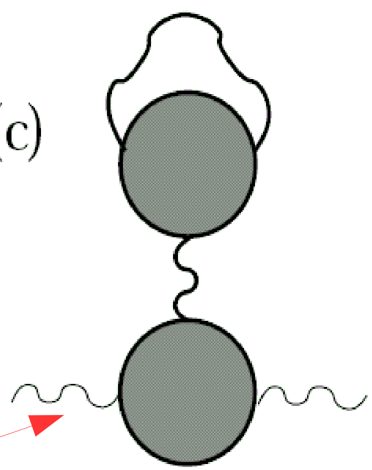
(a)



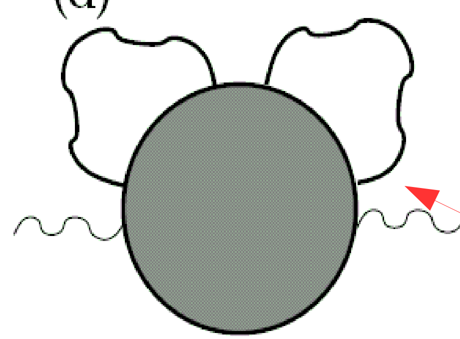
(b)



(c)



(d)



two bosonic loops
neglect because
proportional to 1/m^4

renormalizes ZS velocity

5. Results for spectral line-shape

Further approximation:

- Retain only leading 1/m-dependence of higher order loops (approximation A)

$$(n-1)! L_S^{(n)}(Q_1, \ldots, Q_n) = \frac{\nu_0}{(m v_F^2)^{n-2}} \tilde{L}_S^{(n)}(Q_1, \ldots, Q_n)$$

take here limit 1/m -> 0

- Interaction with sharp cutoff in momentum space $f_q = f_0 \Theta(q_0 - |q|)$

➡ all integrations can be done exactly!

- Smooth interaction $f_q = f_0 + \frac{1}{2} f_0'' q^2 + O(q^4), \quad \text{with } f_0'' \neq 0$

➡ new momentum scale $q_c = \frac{1}{m|f_0''|}$ $\qquad q_c \ll q_0 \ll k_F$

Expansion strategy for dynamic structure factor :

$$S(\omega, q) = \frac{\nu_0}{\pi} \mathrm{Im} \left[\frac{1}{g_p + \tilde{\Pi}_*^{-1}(x + i0, p)} \right]$$

$$x = \omega/(v_F q)$$
$$p = q/(2k_F)$$

inverse irreducible polarization

$$\tilde{\Pi}_*^{-1}(iy, p) = 1 + y^2 - \frac{p^2}{3} \frac{1 - 3y^2}{1 + y^2} - (1 + y^2)^2 \tilde{\Pi}_1(iy, p) - (1 + y^2)^2 \tilde{\Pi}_2(iy, p) + O(p_0^3)$$

from free polarization — first order in effective interaction — second order in effective interaction

$$\tilde{\Pi}_1(iy, p) = -\int_{-\infty}^{\infty} dp' |p'| \int_{-\infty}^{\infty} \frac{dy'}{2\pi} \tilde{f}_g(iy', p') \tilde{L}_S^{(4)}(iy, p, iy', p')$$

$$\tilde{\Pi}_2^{\mathrm{AL}}(iy, p) = -\int_{-\infty}^{\infty} dp' |p'| \int_{-\infty}^{\infty} \frac{dy'}{2\pi} \tilde{f}_g(iy', p') \tilde{f}_g \left(\frac{iyp + iy'p'}{p + p'}, p + p' \right) [\tilde{L}_S^{(3)}(iy, p, iy', p')]^2$$

(Aslamasov-Larkin diagram)

Effective interaction: bosonic propagator

$$\tilde{f}_*(iy,p) \approx \tilde{f}_g(iy,p) = \frac{\tilde{g}_p}{1+\tilde{g}_p\tilde{\Pi}_0(iy,p)} \qquad \tilde{g}_p = g + \frac{1}{2}g_0'' p^2 + O(p^4).$$

renormalized coupling constant depends on counter-terms:

$$g = \frac{g_0 + Z_1}{Z_2} - 1 \qquad Z_i = 1 + p_0^2 g_i, \quad i = 1,2.$$

such that effective bosonic propagator depends on true ZS velocity:

$$\frac{v}{v_F} = \sqrt{\frac{Z_1 + g_0}{Z_2}} \equiv x_0 \equiv \sqrt{1+g}.$$

...after frequency integration (theorem of residues):

$$\tilde{\Pi}_*^{-1}(iy,p) \;=\; 1+y^2+p^2+I_H(1-y^2)+\tilde{I}(iy,p)\; +O(p_0^3).$$

$$I_H \;=\; -\frac{2g}{1+g}\int_0^\infty dpp\left[\frac{1+\frac{\tilde{g}_p}{2}}{\sqrt{1+\tilde{g}_p}}-1\right]$$

$$\tilde{I}(iy,p)=\frac{1}{2}\int_0^\infty dp'p'\left[\tilde{J}(iy,p,p')+\tilde{J}(-iy,p,p')\right]$$

$$\begin{aligned}
\tilde{J}(iy,p,p') \;=\; & \frac{p'^4\tilde{F}_1(iy,p')+p'^2p^2\tilde{F}_2(iy,p')+p^4F_3(iy,p')}{x_{p'}[a_{p'}^2p'^2-(1+iy)^2p^2][b_{p'}^2p'^2-(1-iy)^2p^2]}+\frac{8p^2(1-iy)}{b_{p'}^2p'^2-(1-iy)^2p^2} \\
&+\frac{\tilde{g}_{p'+p}(p'+p)^2\left[(p'+p)(1+2iyx_{p'}+x_{p'}^2)-p(y^2+x_{p'}^2)\right]^2}{2x_{p'}\left[(p'+p)^2-(x_{p'}p'+iyp)^2\right]\left[x_{p'+p}^2(p'+p)^2-(x_{p'}p'+iyp)^2\right]} \\
&+\frac{\tilde{g}_{p'}|p'+p|p'\left[p'(1+2iyx_{p'+p}+x_{p'+p}^2)+p(y^2+x_{p'+p}^2)\right]^2}{2x_{p'+p}\left[p'^2-\left(x_{p'+p}(p'+p)-iyp\right)^2\right]\left[x_{p'}^2p'^2-\left(x_{p+p'}(p+p')-iyp\right)^2\right]} \\
&+\frac{(p'+p)\left[8(p'+p)-4(1-iy)p+\tilde{g}_{p'+p}(p'+p)\left[\frac{p'+p}{p}(3+iy)+\frac{1}{2}(1+iy)^2\right]\right]}{x_{p'+p}^2(p'+p)^2-[p'+p-(1-iy)p]^2} \\
&+\frac{|p'+p|\left[-8p'-4(1-iy)p+\tilde{g}_{p'}p'\left[\frac{p'}{p}(3+iy)-\frac{1}{2}(1+iy)^2\right]\right]}{x_{p'}^2p'^2-[p'+(1-iy)p]^2}+(p\to -p).
\end{aligned}$$

for smooth cutoff:
no mass shell
singularities !

Spectral line-shape for smooth cutoff:

- calculation controlled for

$$q \gtrsim \quad q_* = \frac{8\pi Z_{\mathrm{w}} x_0}{3\sqrt{3}} q_c$$

$$q_c = 1/(m|f_0''|)$$

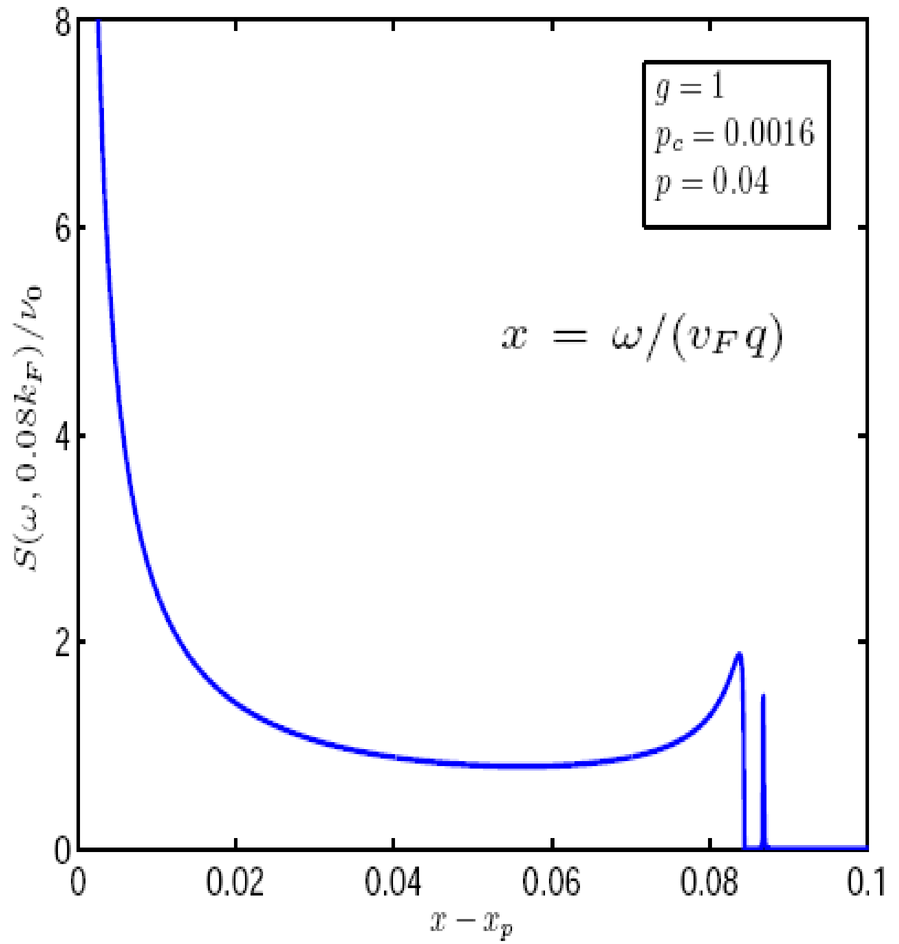
- width of ZS resonance

$$\gamma_q = v_F q \tilde{\gamma}_p = \frac{3}{8\pi x_0} \frac{q^3}{2mq_c}.$$

- logarithmic threshold singularity at

$$\omega_q^- \equiv v_F q x_p = vq + \frac{\mathrm{sign} f_0''}{2\pi x_0} \frac{q^3}{2mq_c}$$

$$S(\omega, q) \sim \frac{\nu_0}{2x_0 Z_2 |\eta_p|} \frac{1}{(x - x_p) \ln^2 \left[\frac{4\tilde{\gamma}_p}{x - x_p}\right]}$$



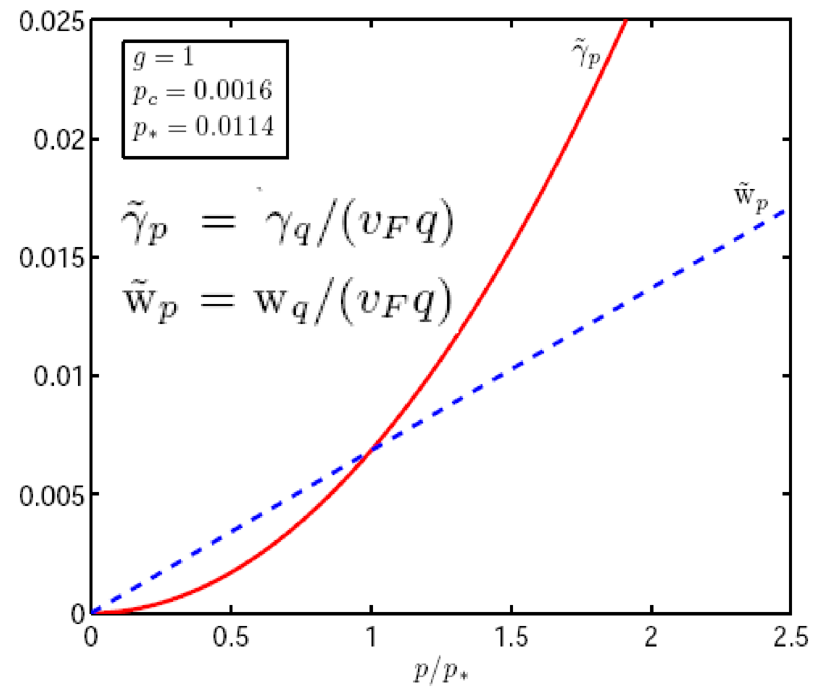
Width of the zero-sound resonance:

Two different long wavelength regimes in FSM:

- intermediate: $q_c \ll q \ll k_F$

$$\gamma_q \propto q^3/(mq_c)$$

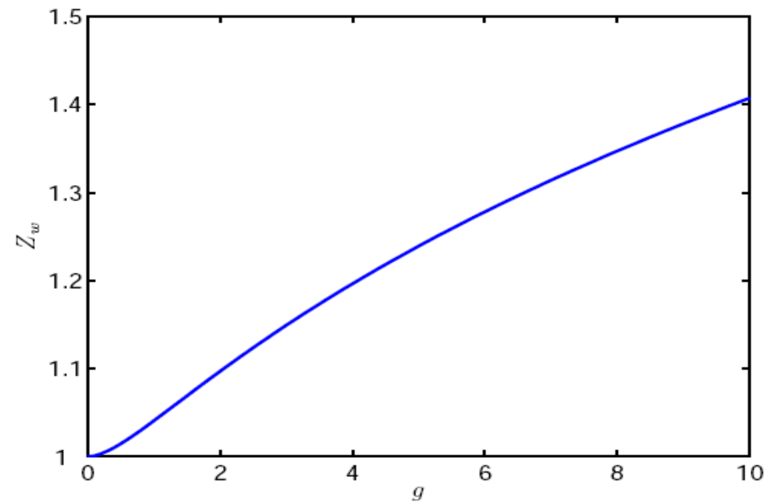
$$q_c = 1/(m|f_0''|)$$



- asymptotic: $q \ll q_c$

estimate ZS damping from width single-pair particle-hole continuum:

$$\mathrm{w}_q = Z_{\mathrm{w}} \frac{q^2}{2\sqrt{3}m}$$



Threshold exponent:

Hypothesis: logarithmic singularity can be exponentiated:

$$2x_0(x-x_p)\left\{1+\eta_p \ln\left[\frac{4\tilde{\gamma}_p}{x-x_p}\right]\right\} \rightarrow 2x_0(x-x_p)\left[\frac{4\tilde{\gamma}_p}{x-x_p}\right]^{\eta_p}$$

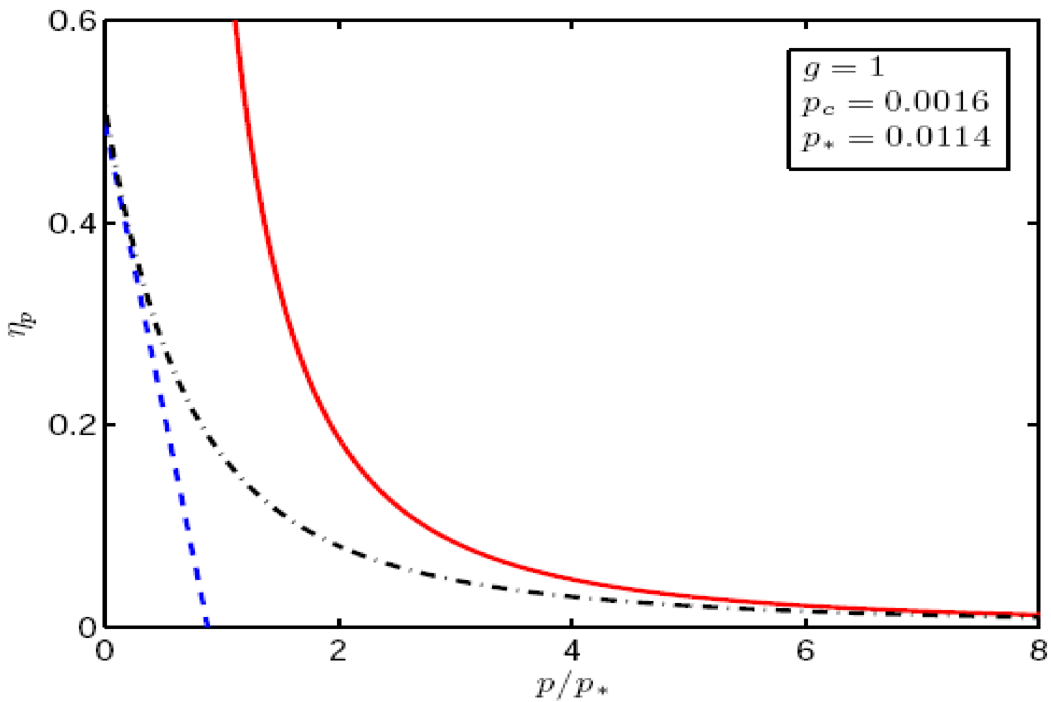
algebraic threshold singularity:

$$S(\omega,q)\sim\frac{\nu_0}{2x_0Z_2}\frac{|\eta_p|}{(4\tilde{\gamma}_p)^{2\eta_p}}\frac{1}{[x-x_p]^{\mu_p}}$$

Threshold exponent:

$$\mu_p = 1-2\eta_p = 1+\operatorname{sign} f_0''\frac{3p_*^2}{2p^2}$$

Compare with estimate of Pustilnik et al (2006): $\mu_p \approx \dfrac{p}{2\pi p_c}$



6. Open problems + solution strategy

1.) Evaluation of functional-bosonization results
for irreducible polarisation with exact 3- and 4-loops

- Avoid approximation A (simplified loops)
- Power counting does not work due to loop cancellation!

2.) Existence of threshold singularities in non-integrable models?

3.) What happens if interaction involves also large momentum transfers?

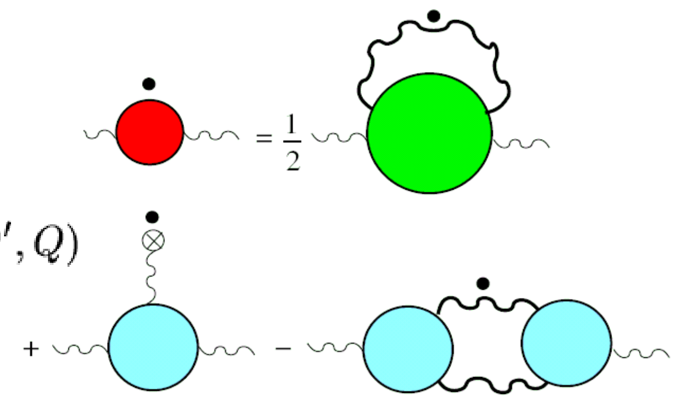
- Reconcile present results with calculations in exactly solvable models

4.) Need non-perturbative approach which regularizes all singularities and
explicitly includes loop cancellation:
FRG with momentum transfer cutoff (F. Schütz, L. Bartosch, PK, PRB 2005)

Functional RG with momentum transfer cutoff:

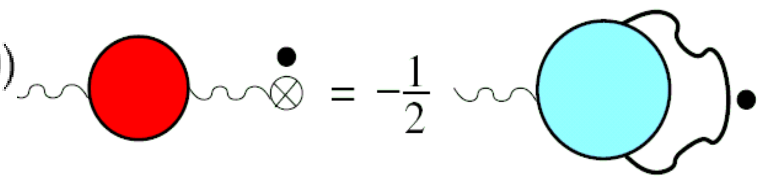
FRG flow equation for irreducible polarization:

$$\begin{aligned}\partial_\Lambda \Pi_\Lambda(Q) \; &= \; \frac{1}{2}\int_{Q'} \dot{F}_\Lambda(Q')\Gamma^{(4)}_\Lambda(Q',-Q',Q,-Q) + \Gamma^{(3)}_\Lambda(Q,-Q,0)\partial_\Lambda\bar{\phi}_\Lambda \\ &- \int_{Q'} \dot{F}_\Lambda(Q')F_\Lambda(Q+Q')\Gamma^{(3)}_\Lambda(-Q,Q+Q',-Q')\Gamma^{(3)}_\Lambda(Q',-Q-Q',Q)\end{aligned}$$



And for vaccum expectation value of HS field

$$[f_0^{-1} + \Pi_\Lambda(0)]\partial_\Lambda\bar{\phi}_\Lambda = -\frac{1}{2}\int_{Q'} \dot{F}_\Lambda(Q')\Gamma^{(3)}_\Lambda(Q',-Q',0)$$



Truncation: approximate higher order vertices by their initial condition
=symmetrized fermion loops!

Technical problems:
- Solve 2D integro-differential equation
- Numerical analytic continuation

Summary, Conclusions

- Dynamic structure factor of forward scattering model in 1d:
  - naïve expansion in powers of 1/m ill defined
  - best method: functional bosonization
  - for q->0: zero sound damping scales as q^2/m
  - Intermediate regime: hump with threshold singularity width $\gamma_q \propto q^3/(mq_c)$
    $q_c \ll q \ll k_F$
- Lots of work to be done:
  - Full evaluation of functional bosonization for finite 1/m
  - Analysis within functional renormalization group
- General class of problems:
  - physical effect is essentially determined by RG irrelevant operators
  - continuum field theory does not work
  - promising method: functional renormalization group