

Renormalization of Conserving Selfconsistent Dyson equations

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Motivation

- Thermodynamics of strongly interacting systems
- Conservation laws, detailed balance, thermodynamical consistency
- Finite width effects (resonance, damping, ⋯)

Concepts

- Real time quantum field theory
- The Φ -derivable scheme (example $O(N)$)
- Renormalization
- Restoration of symmetries
- Gauge Symmetries and Vector Mesons

Schwinger-Keldysh Formalism

#2

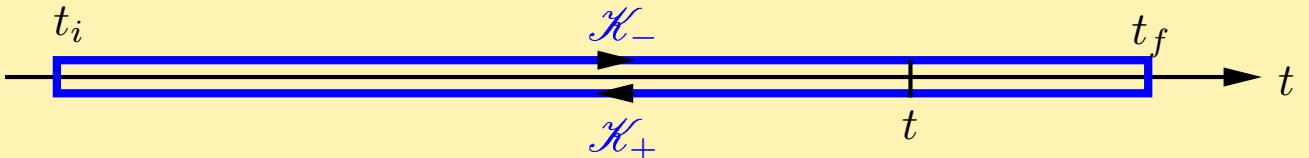
- Initial statistical operator ρ_i at $t = t_i$
- Time evolution

$$\langle O(t) \rangle = \text{Tr} \left[\underbrace{\rho(t_i) \mathcal{T}_a \left\{ \exp \left[+i \int_{t_i}^t dt' \mathbf{H}_I(t') \right] \right\}}_{\text{anti time-ordered}} \right]$$

$$= \mathbf{O}_I(t)$$

$$\underbrace{\mathcal{T}_c \left\{ \exp \left[-i \int_{t_i}^t dt' \mathbf{H}_I(t') \right] \right\}}_{\text{time-ordered}}.$$

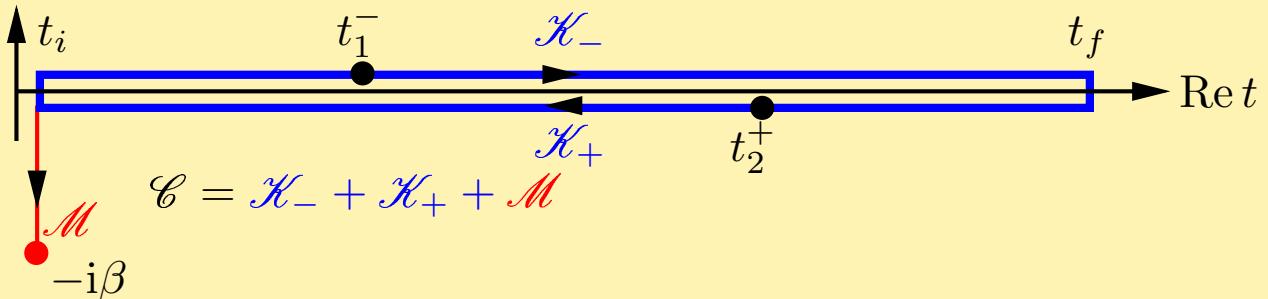
- Difference to vacuum: Contour-ordered Green's functions



$$\mathcal{C} = \mathcal{K}_- + \mathcal{K}_+$$

- In equilibrium: $\rho = \exp(-\beta \mathbf{H})/Z$ with $Z = \text{Tr} \exp(-\beta \mathbf{H})$
- Imaginary part of the time contour

Im t



- Correlation functions with real times: $iG_{\mathcal{C}}(x_1^-, x_2^+)$
- Fields periodic (bosons) or anti-periodic (fermions)
- Feynman rules \Rightarrow time integrals \rightarrow contour integrals

The Φ -Functional

#3

- Introduce **local** and **bilocal** auxiliary sources
- Generating functional

$$Z[J, K] = N \int D\phi \exp \left[iS[\phi] + i \{ J_1 \phi_1 \}_1 + \left\{ \frac{i}{2} K_{12} \phi_1 \phi_2 \right\}_{12} \right]$$

- Generating functional for **connected diagrams**

$$Z[J, K] = \exp(iW[J, K])$$

- The **mean field** and the **connected Green's function**

$$\underbrace{\varphi_1 = \frac{\delta W}{\delta J_1}, \quad G_{12} = -\frac{\delta^2 W}{\delta J_1 \delta J_2}}_{\text{standard quantum field theory}} \Rightarrow \frac{\delta W}{\delta K_{12}} = \frac{1}{2} [\varphi_1 \varphi_2 + iG_{12}]$$

- Legendre transformation for φ and G :

$$\Gamma[\varphi, G] = W[J, K] - \{\varphi_1 J_1\}_1 - \frac{1}{2} \{(\varphi_1 \varphi_2 + iG_{12}) K_{12}\}_{12}$$

- Exact closed form:

$$\begin{aligned} \Gamma[\varphi, G] = & S_0[\varphi] + \frac{i}{2} \text{Tr} \ln(-iG^{-1}) + \frac{i}{2} \{D_{12}^{-1}(G_{12} - D_{12})\}_{12} \\ & + \Phi[\varphi, G] \Leftarrow \text{all closed 2PI interaction diagrams} \end{aligned}$$

$$D_{12} = (-\square - m^2)^{-1}$$

Equations of Motion

#4

- Physical solution defined by vanishing **auxiliary sources**:

$$\frac{\delta \Gamma}{\delta \varphi_1} = -\mathbf{J}_1 - \{K_{12}\varphi_2\}_2 \stackrel{!}{=} 0$$

$$\frac{\delta \Gamma}{\delta G_{12}} = -\frac{i}{2} K_{12} \stackrel{!}{=} 0$$

- Equation of motion for the **mean field** φ

$$-\square \varphi - m^2 \varphi := j = -\frac{\delta \Phi}{\delta \varphi}$$

- for the “full” propagator $G \Rightarrow$ Dyson’s equation:

$$-i(D_{12}^{-1} - G_{12}^{-1}) := -i\Sigma = 2 \frac{\delta \Phi}{\delta G_{21}}$$

- Integral form of Dyson’s equation:

$$G_{12} = D_{12} + \{D_{11'}\Sigma_{1'2'}G_{2'2}\}_{1'2'}$$

- **Closed set** of equations of for φ and G

“Diagrammar”

#5

- $O(N)$ -theory

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \vec{\phi})(\partial^\mu \vec{\phi}) - \frac{m^2}{2}\vec{\phi}^2 - \frac{\lambda}{4!}(\vec{\phi}^2)^2$$

- 2PI Generating Functional

$$i\Phi = \underbrace{\text{[diagram with four blue crosses at vertices]}}_{\text{mean field part}} + \underbrace{\text{[diagram with one green loop]}}_{\text{Correlations}} + \underbrace{\text{[diagram with two green loops connected by a horizontal line]}}_{\text{Correlations}} + \underbrace{\text{[diagram with three green loops connected by a horizontal line]}}_{\text{Correlations}} + \dots$$

- Mean field equation of motion

$$i(\square + m^2)\varphi = \text{[diagram with a black dot and four blue crosses]} + \text{[diagram with a black dot and one green loop]} + \text{[diagram with a black dot, a green loop, and a point labeled 'x' on the line]} + \dots$$

- Self-energy

$$-i\Sigma_{12} = \underbrace{\text{[diagram with a black dot and two blue crosses on a horizontal line]}}_{\text{mass terms}} + \underbrace{\text{[diagram with a black dot and one green loop on a horizontal line]}}_{\text{damping width}} + \underbrace{\text{[diagram with a black dot, two green loops, and two blue crosses on a horizontal line]}}_{\text{(momentum dependent)}} + \dots$$

Properties of the Φ -derivable Approximations

#6

Why using the Φ -functional?

- Truncation of the Series of diagrams for Φ
- ☞ Expectation values for currents are conserved
⇒ “Conserving Approximations”
- In equilibrium $i\Gamma[\varphi, G] = \ln Z(\beta)$
(thermodynamical potential)
- consistent treatment of **Dynamical quantities** (real time formalism) and **thermodynamical bulk properties** (imaginary time formalism) like **energy, pressure, entropy**
- Real- and Imaginary-Time quantities “glued” together by **Analytic properties** from (anti-)periodicity conditions of the fields (**KMS-condition**)
- Self-consistent set of equations for self-energies and mean fields

Problem of Renormalization

#7

Why renormalization?

- ☞ Diagrams UV-divergent
- ☞ Control the physical parameters in vacuum
- ☞ Temperature dependence from theory alone

How to renormalize self-consistent diagrams?

- ☞ In terms of perturbation theory: Resummation of all self-energy insertions in propagators
- ☞ Self-consistent diagrams with explicit nested and overlapping sub-divergences
- ☞ “Hidden” sub-divergences from self-consistency

How to manage it numerically?

- ☞ Power counting (Weinberg) valid for self-consistent diagrams
- ☞ At finite temperatures:
Self-consistent scheme rendered finite with local counterterms independent of temperature
- ☞ Analytical properties \Rightarrow subtracted dispersion relations
- ☞ BPHZ-renormalization \Rightarrow Subtracting the integrands
- ☞ Advantage: Clear scheme how to subtract temperature independent sub-divergences
- ☞ Φ -functional \Rightarrow consistency of counterterms

Self-consistent Renormalization

#8

Example: Tadpole-Renormalization

$$\Phi = \text{Diagram with two circles connected by a dot} \Rightarrow -i\Sigma = \text{Diagram with one circle connected to a line}$$

- ☞ Temperature dependent effective mass: $M^2 = m^2 + \Sigma_{\text{ren}}$
- Resummation of the Dyson series

$$\text{Diagram with one circle} + \text{Diagram with two circles} + \text{Diagram with three circles} + \dots$$

- ☞ Renormalized self-energy

$$-i\Sigma_{\text{ren}} = \text{Diagram with red blob} = \frac{l}{\frac{\lambda}{2} G(l)} - \frac{\text{Diagram with red blob and blue loop}}{\frac{\lambda}{2} G_v^2(l) \Sigma_{\text{ren}}} - \frac{\text{Diagram with blue loop}}{\frac{\lambda}{2} G_v(l)}$$

- Renormalization: Subtraction of Vacuum-(sub)divergences
- ☞ Result: Finite “Gap equation”

$$\begin{aligned} M^2 &= m^2 + \Sigma_{\text{ren}} = m^2 + \frac{\lambda}{32\pi^2} \left(M^2 \ln \frac{M^2}{m^2} - \Sigma_{\text{ren}} \right) + \\ &\quad + \underbrace{\frac{\lambda}{2} \int \frac{d^4 p}{(2\pi)^4} 2\pi \delta(p^2 - M^2) n(p_0)}_{\rightarrow 0 \text{ for } T \rightarrow 0} \end{aligned}$$

$n(p_0)$: Bose-Einstein distribution

Self-consistent Renormalization

#9

First step: Vacuum

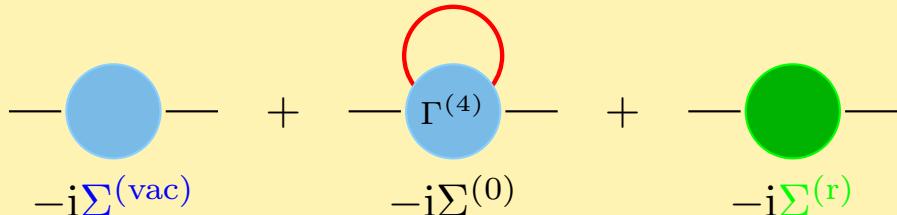
- Power-counting for **self-consistent propagators** as in perturbation theory: $\delta = 4 - E$
- Usual **BPHZ-renormalization** for **wave function, mass and coupling constant renormalization**
- In practice: Use Lehmann-representation and dimensional regularization
- ✓ **Closed self-consistent finite** Dyson-equations of motion
- ✓ **Numerically treatable**

Second step: Finite Temperature

- Split propagator in **vacuum** and **T-dependent** part

$$\overline{iG} = \overline{iG^{(\text{vac})}} + \overline{iG^{(T)}}$$

- Expand self-energy around vacuum part



- Need further splitting of propagator

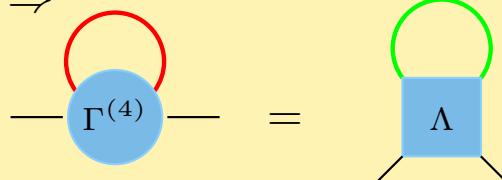
$$\overline{iG^{(T)}} = \overline{iG^{(\text{vac})}} + \overline{iG^{(r)}}$$

Self-consistent Renormalization

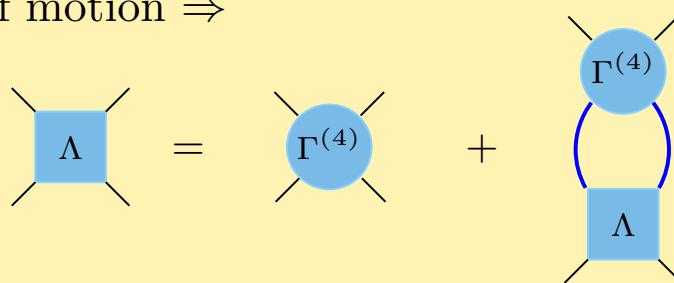
#10

Third step: 4-point vertex renormalization

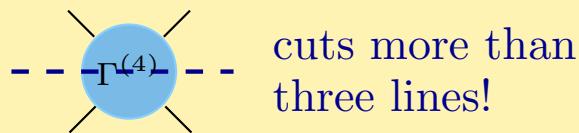
- Σ^0 linear in $G^{(r)}$ \Rightarrow



- Equation of motion \Rightarrow



☞ s-channel Bethe-Salpeter equation



\Rightarrow “BPHZ Boxes” in ladder-diagrams **do not cut inside $\Gamma^{(4)}$.**

\Rightarrow Asymptotics + BPHZ-formalism:

$$\Gamma^{(4)}(l, p) - \Gamma^{(4)}(l, 0) \cong O(l^{-\alpha}) \text{ with } \alpha > 0$$

\Rightarrow Renormalized eq. of motion for Λ :

$$\begin{aligned} \Lambda(p, q) = & \Lambda(0, 0) + \Gamma^{(4)}(p, q) - \Gamma^{(4)}(0, 0) \\ & + i \int \frac{d^4 l}{(2\pi)^4} [\Gamma^{(4)}(p, l) - \Gamma^{(4)}(0, l)] [G^{\text{vac}}]^2(l) \Lambda(l, q) \\ & + i \int \frac{d^4 l}{(2\pi)^4} \Lambda(0, l) [G^{\text{vac}}]^2(l) [\Gamma^{(4)}(l, q) - \Gamma^{(4)}(l, 0)] \end{aligned}$$

✓ Self-energy finite with **vacuum counter terms**

Example: Tadpole+Sunset

#11

Approximation of the Φ -functional

$$\begin{aligned} i\Phi &= \text{(double loop diagram)} + \text{(elliptical loop diagram)} \\ -i\Sigma &= \text{(tadpole diagram)} + \text{(bubble diagram)} \\ -i\Gamma^{(4)} &= \text{(cross diagram)} + \text{(blue loop diagram)} \end{aligned}$$

Renormalization (vacuum)

$$\begin{aligned} -i\Sigma &= \text{(tadpole diagram)} + \text{(loop diagram with dashed box)} \\ &\quad \text{(loop diagram with dashed box)} + \text{(loop diagram with dashed box)} \\ &\quad + \text{(overall)} \end{aligned}$$

- In practice: Use dispersionrelations for propagators
- ☞ Kernels, can be calculated analytically with standard formulae of dimensional regularization
- ✓ Finite Self-consistent integral equations of motion \Rightarrow Solved iteratively
- Calculate also $\Gamma^{(4)}$ and $\Lambda(0, q)$

Example: Tadpole+Sunset

#12

Renormalization (*Finite Temperature*)

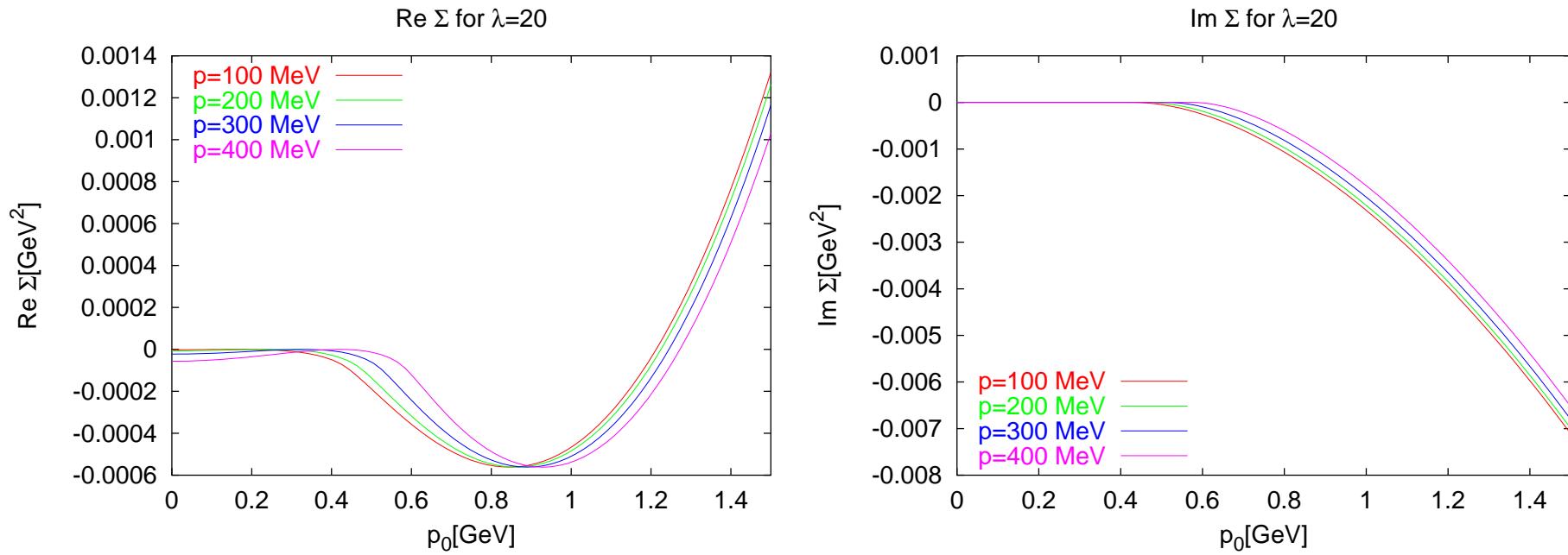
$$-i\Sigma^{(T)}(p) = \text{Diagram 1} - \text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4}$$

The equation shows the renormalization of the self-energy $\Sigma^{(T)}$ at finite temperature. It consists of four terms: a subtraction of two tadpole diagrams (Diagram 1 minus Diagram 2), and additions of a sunset diagram (Diagram 3) and a four-point vertex diagram (Diagram 4). The diagrams are represented by loops with vertices and momenta p or 0 .

- Only finite integrals
- ✓ Numerics for three-dim integrals on a lattice in p_0 and $|\vec{p}|$
- ✓ Equations of motion solved iteratively

Results for “Sunset + Tadpole” at $T = 0$

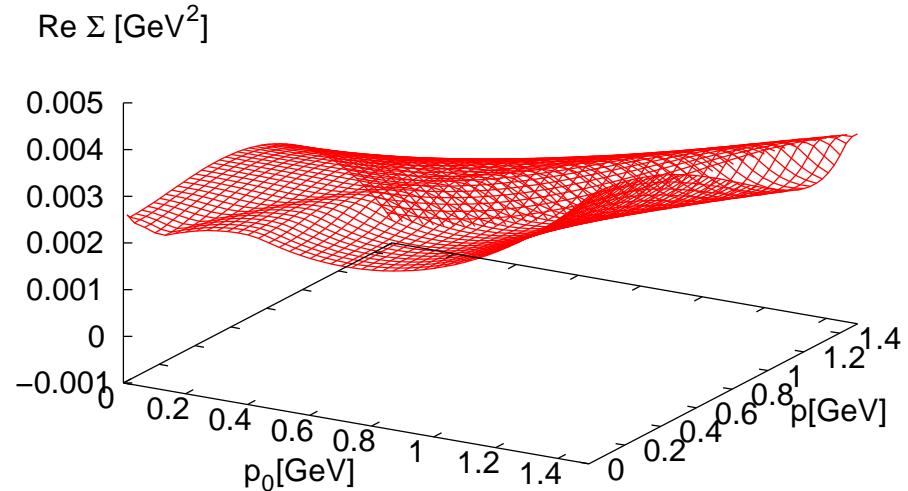
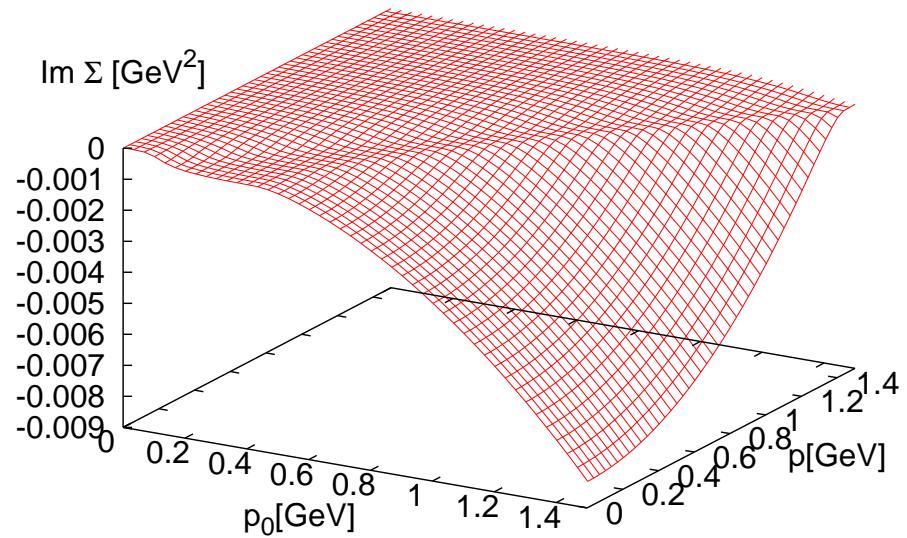
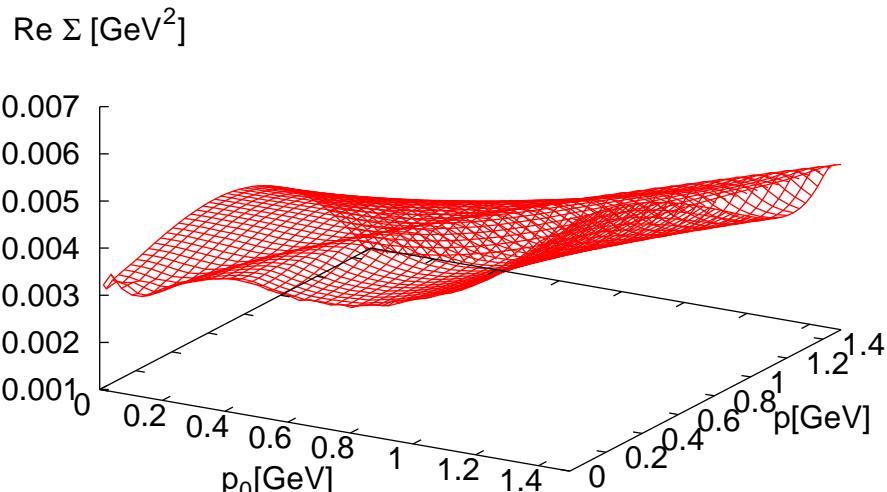
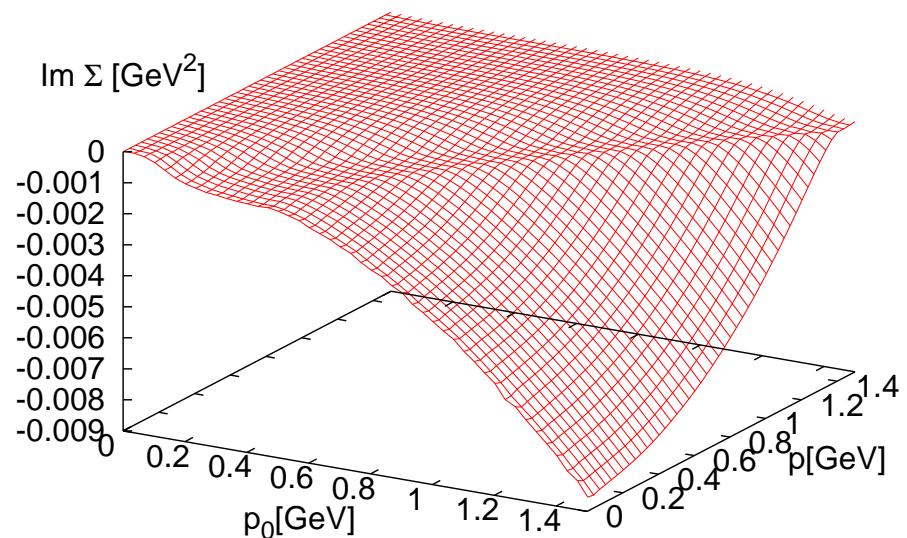
#13



- Difference between perturbative and self-consistent calculation invisible!
- ☞ **Tadpole** contribution “renormalized away” \Rightarrow **on-shell renormalization scheme**
- ☞ Main contribution from the **pole term of the propagator**
- ☞ **Threshold** for continues part of the spectral function $\sqrt{s} = 3m$!

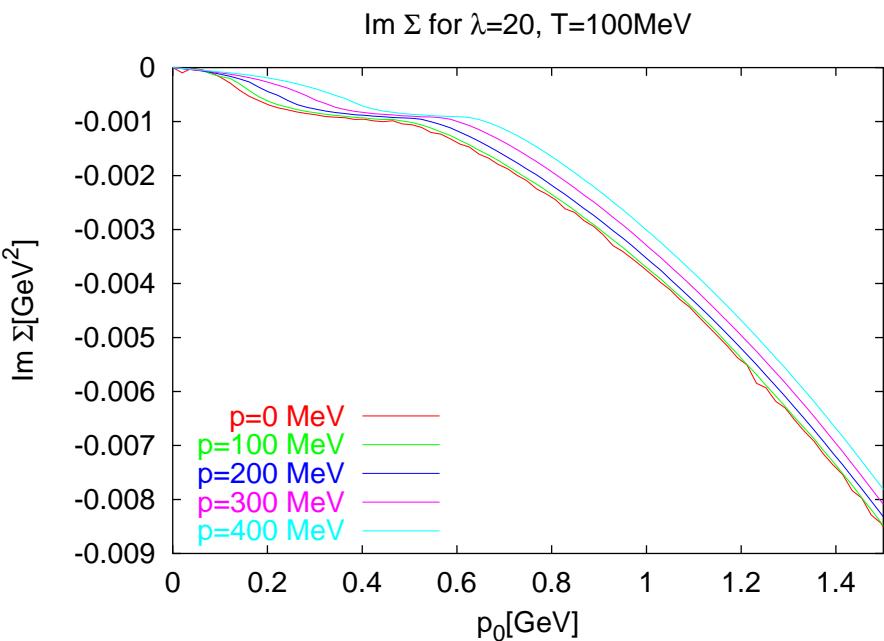
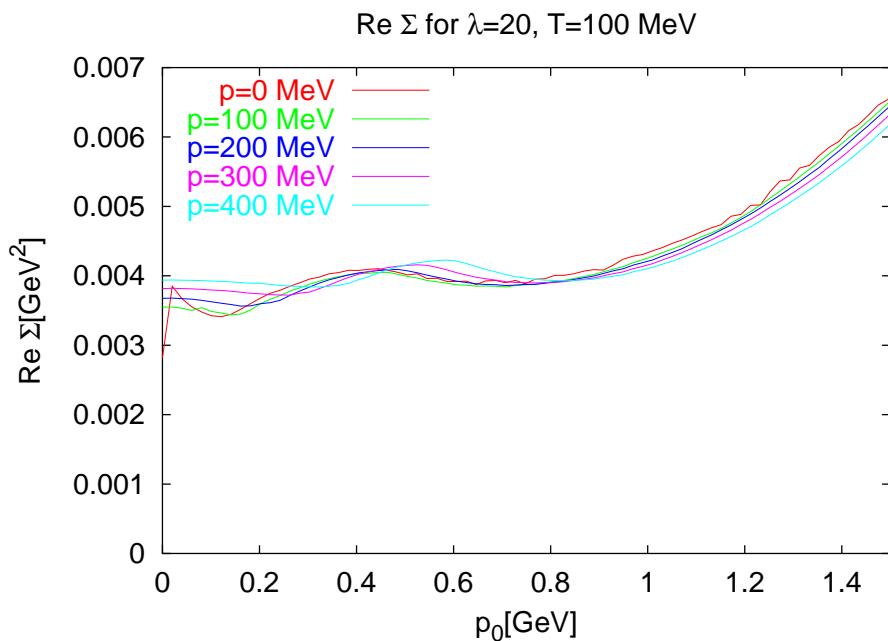
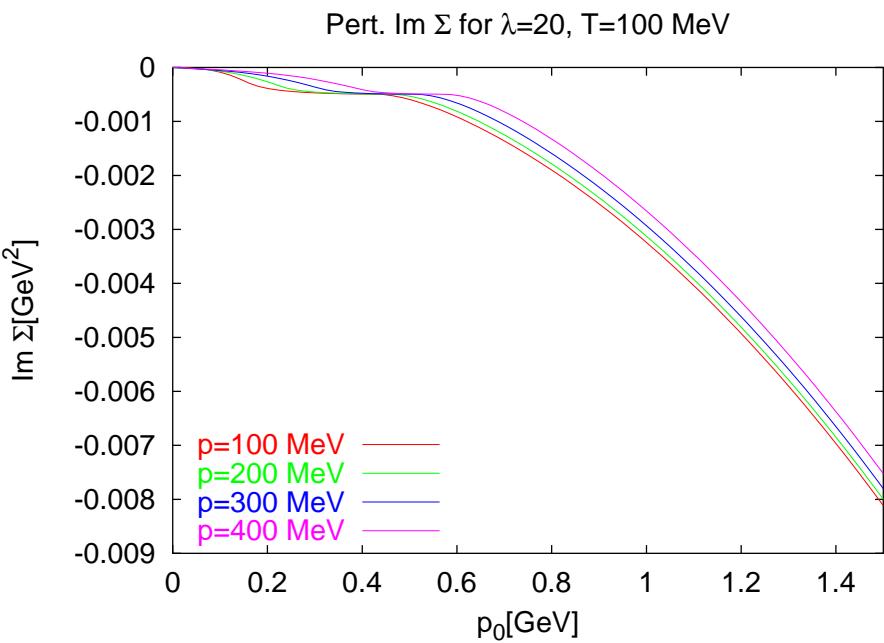
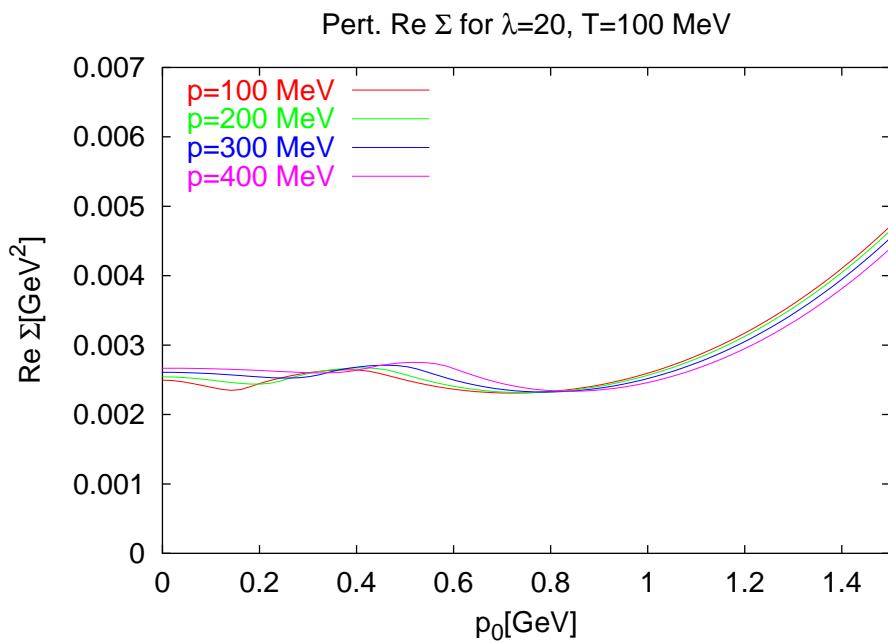
Results for “Sunset + Tadpole” at $T > 0$

#14

Pert. Re Σ for $T=100\text{MeV}$, $\lambda=20$ Pert. Im Σ for $T=100\text{MeV}$, $\lambda=20$ Re Σ for $T=100\text{MeV}$, $\lambda=20$ Im Σ for $T=100\text{MeV}$, $\lambda=20$ 

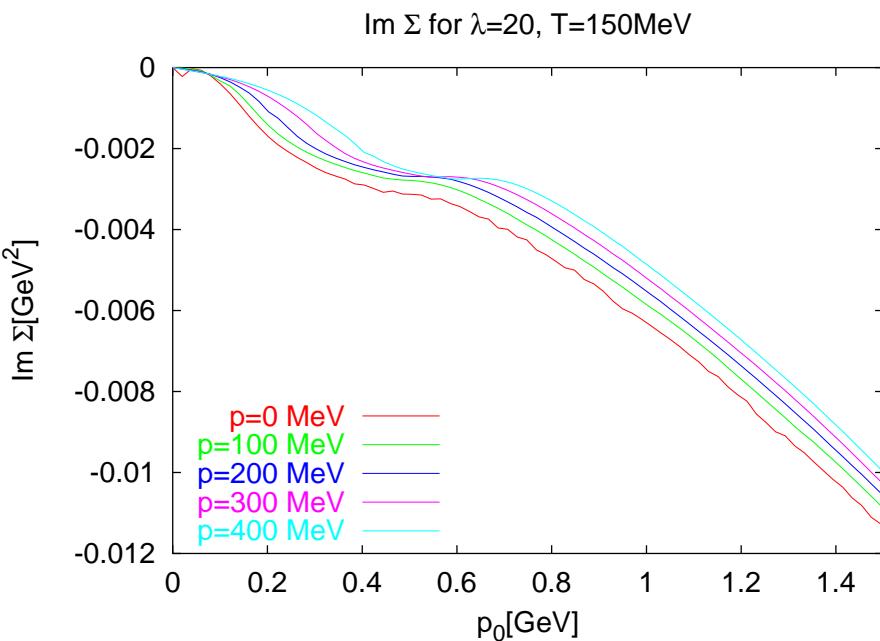
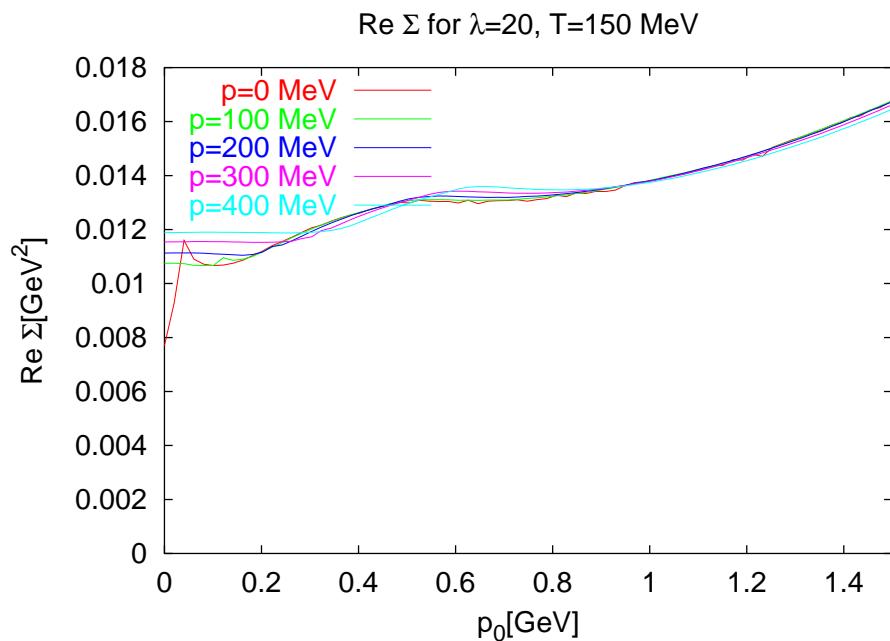
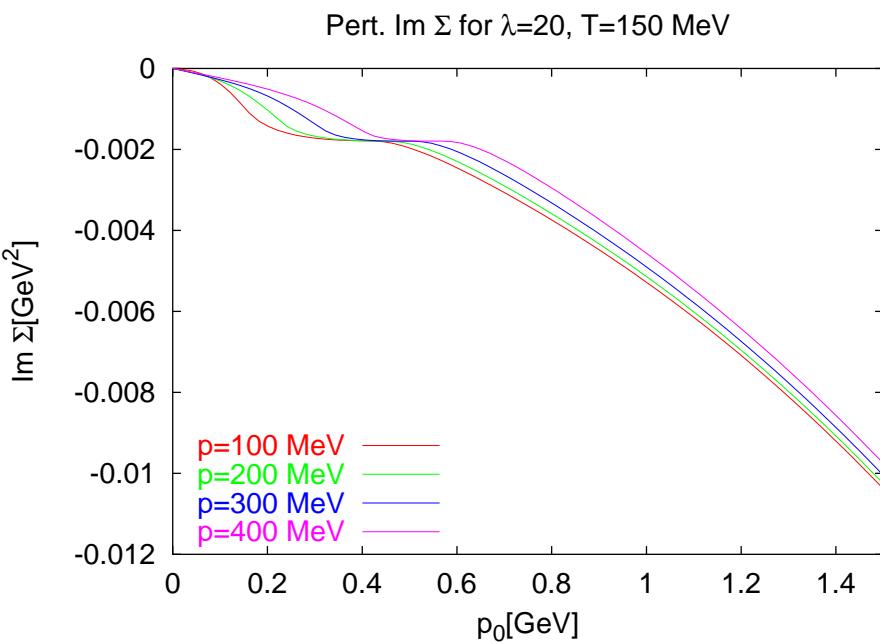
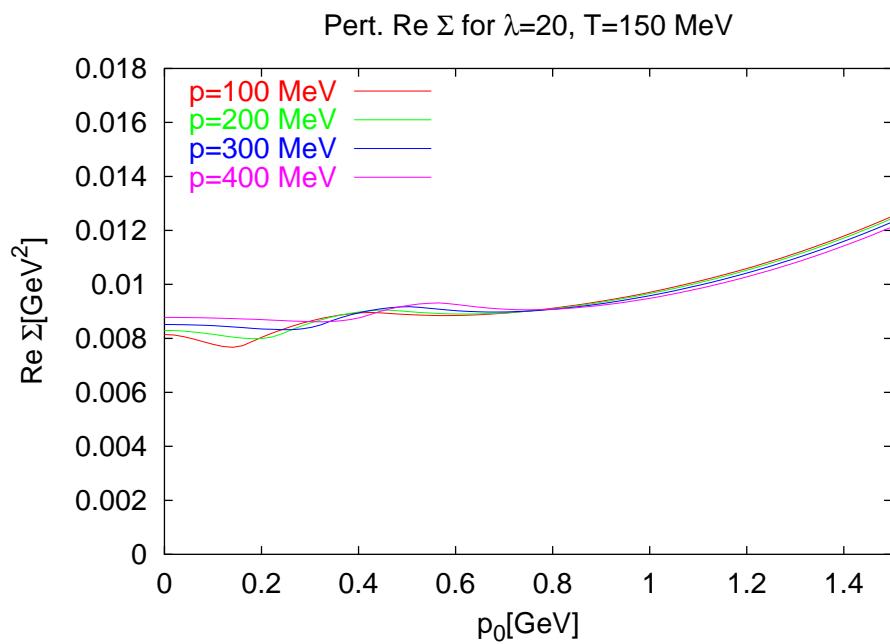
Results for “Sunset + Tadpole” at $T > 0$

#15



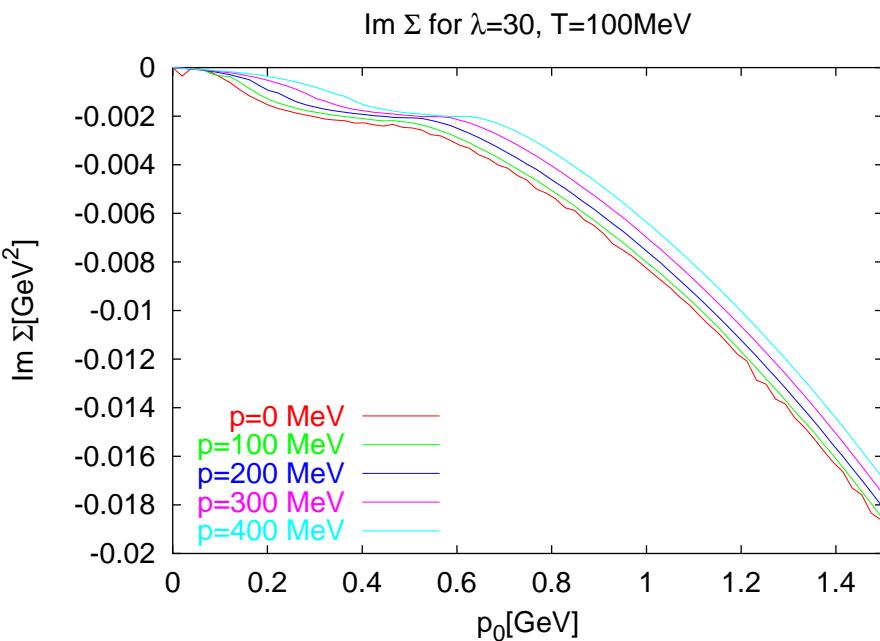
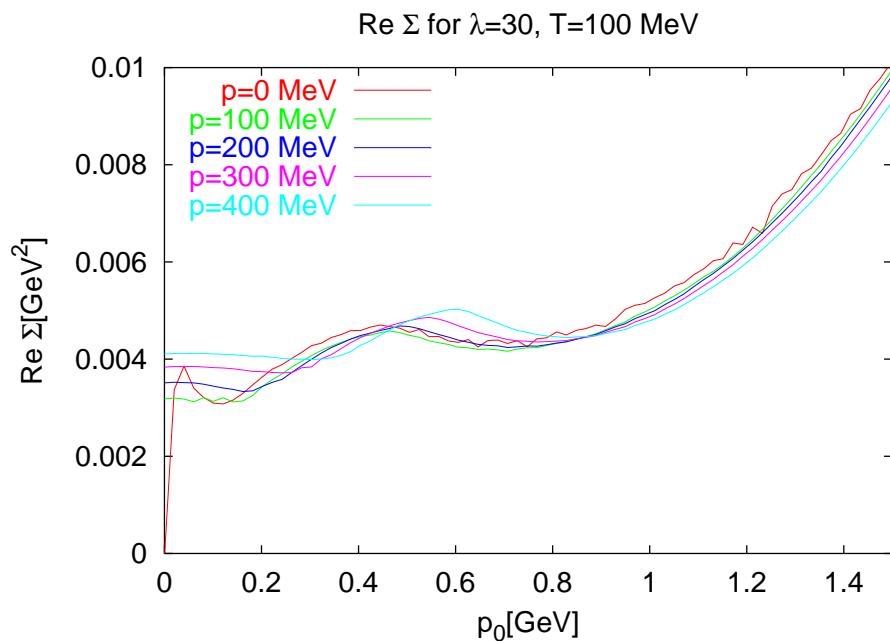
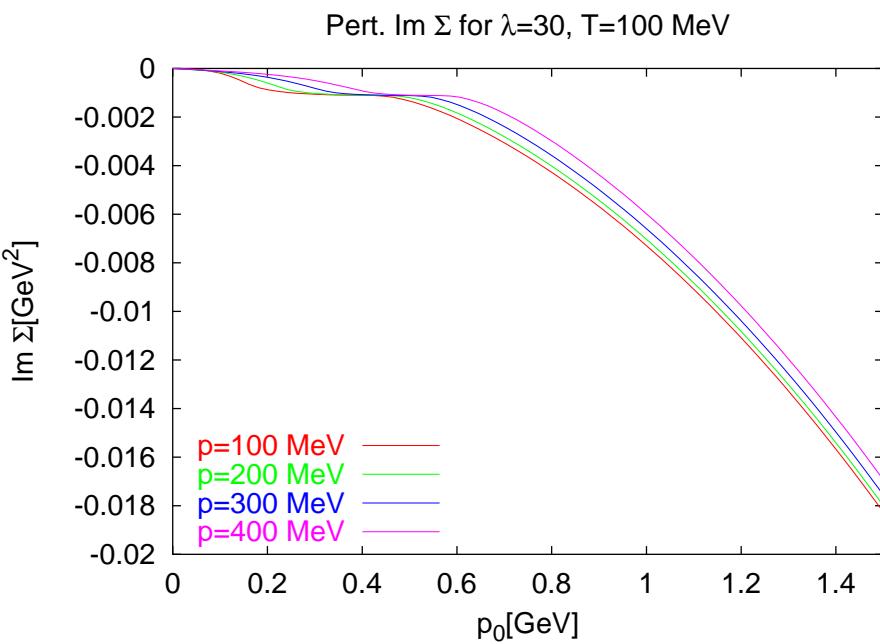
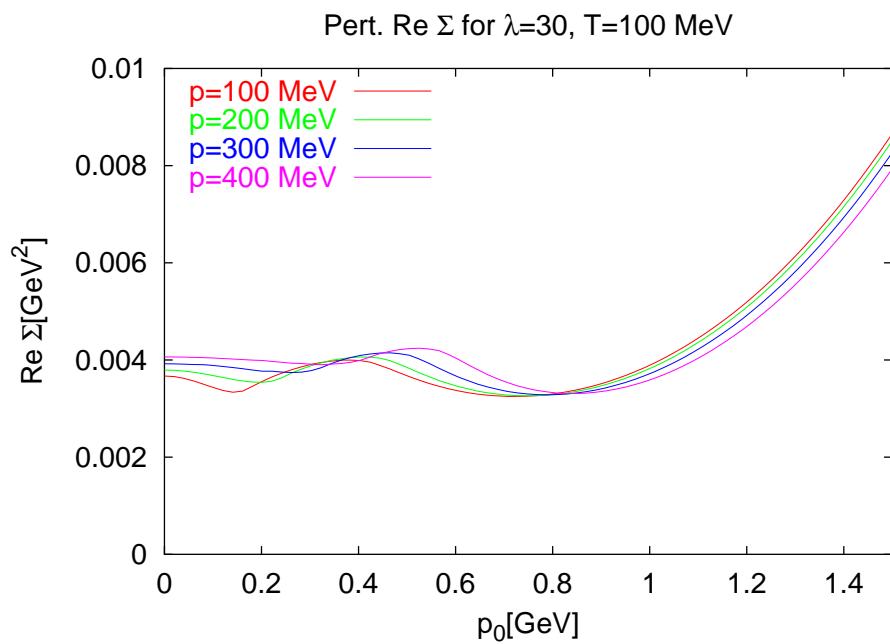
Results for “Sunset + Tadpole” at $T > 0$

#16



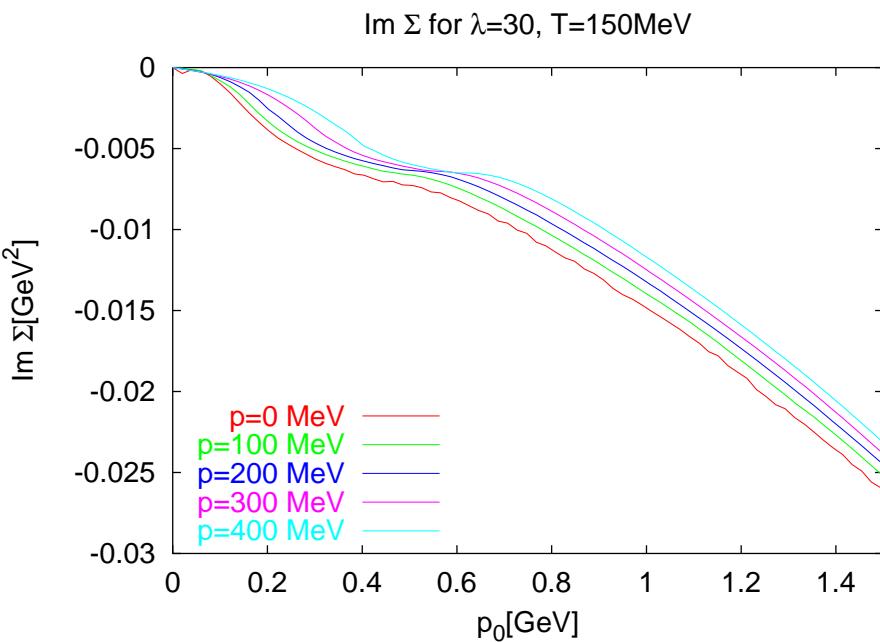
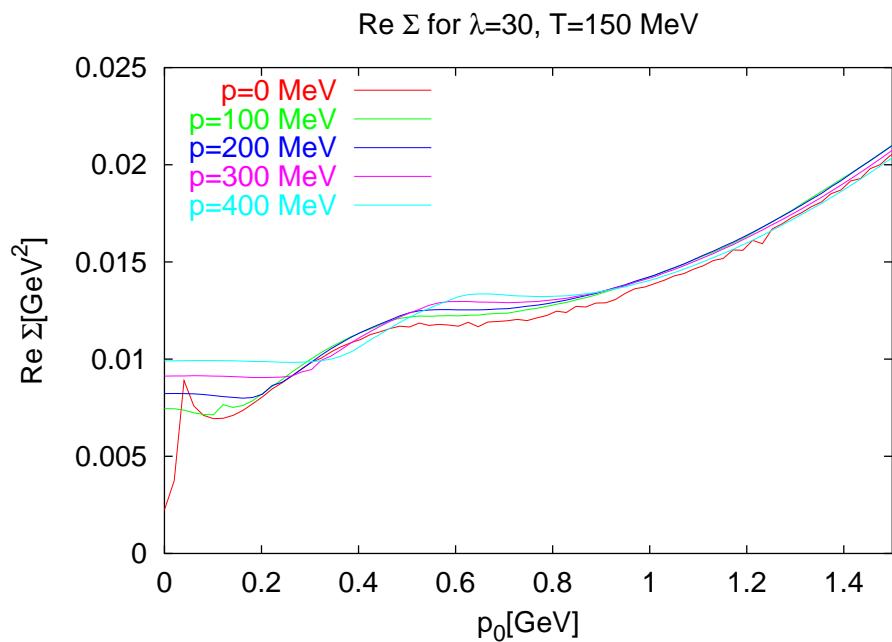
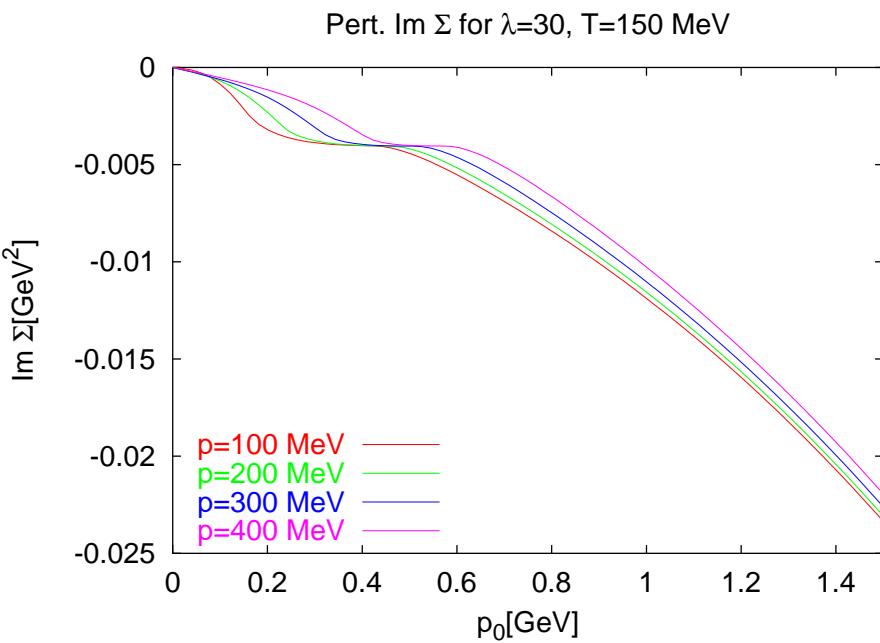
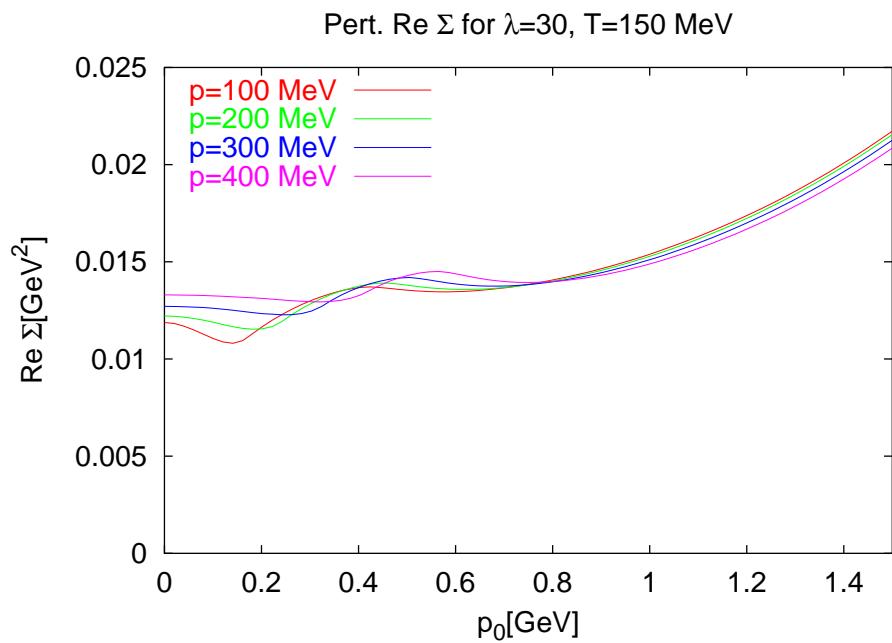
Results for “Sunset + Tadpole” at $T > 0$

#17



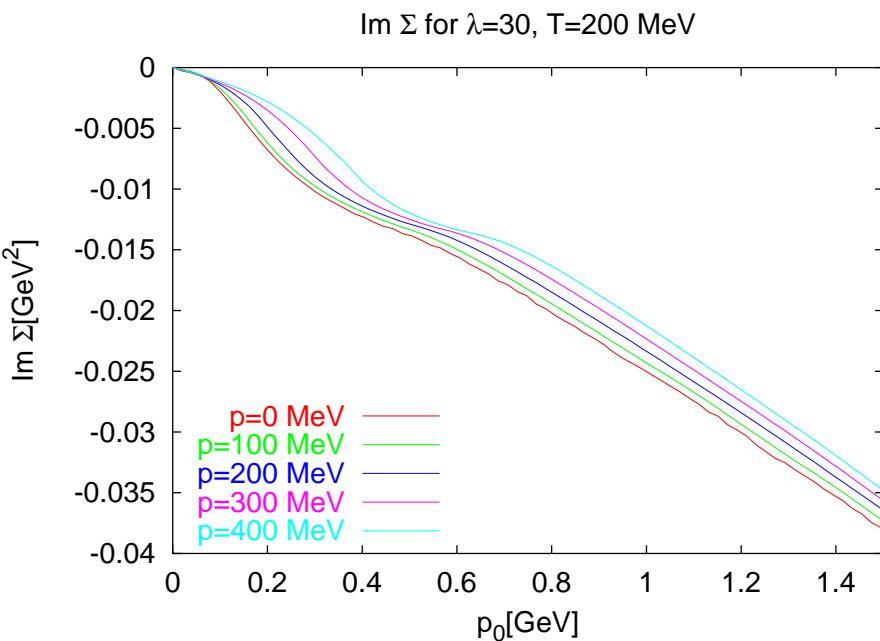
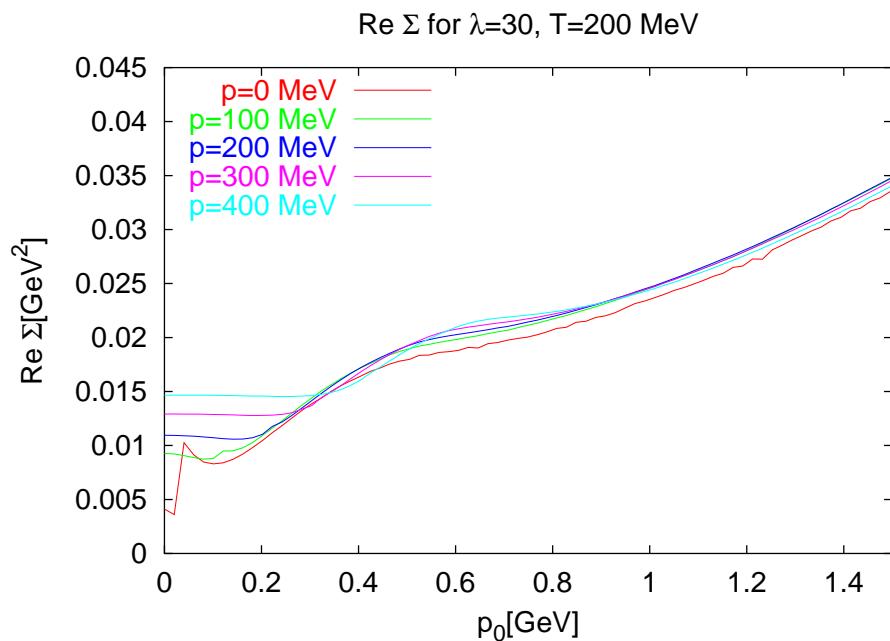
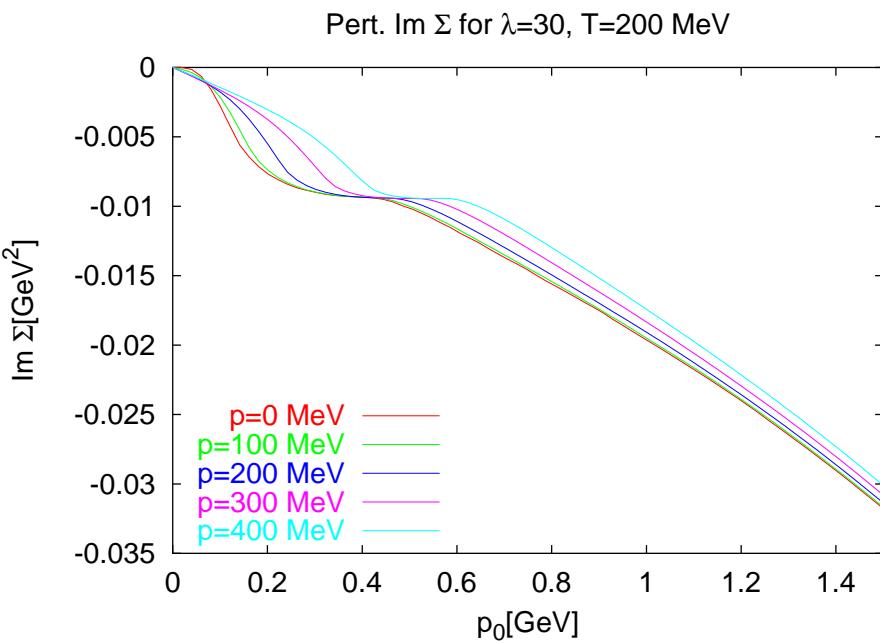
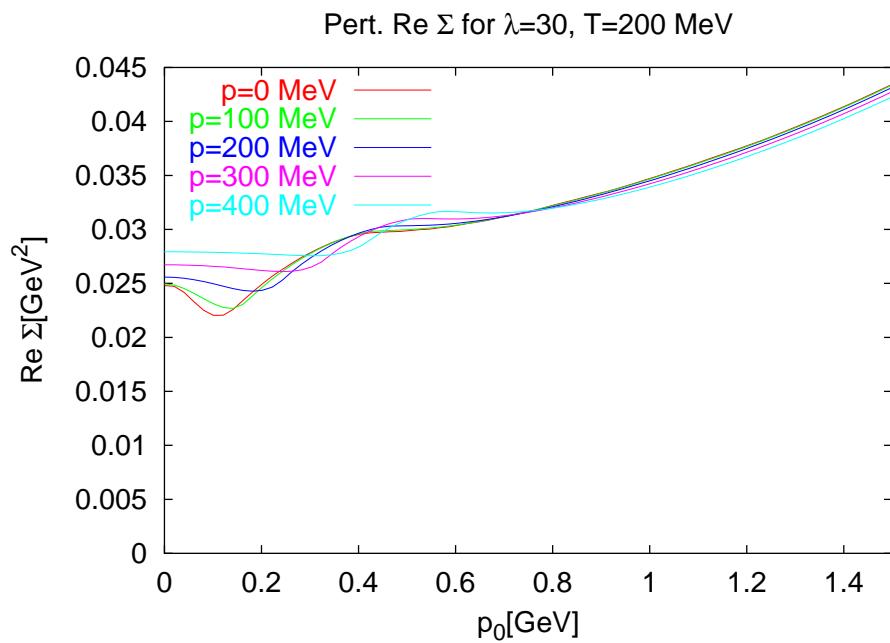
Results for “Sunset + Tadpole” at $T > 0$

#18



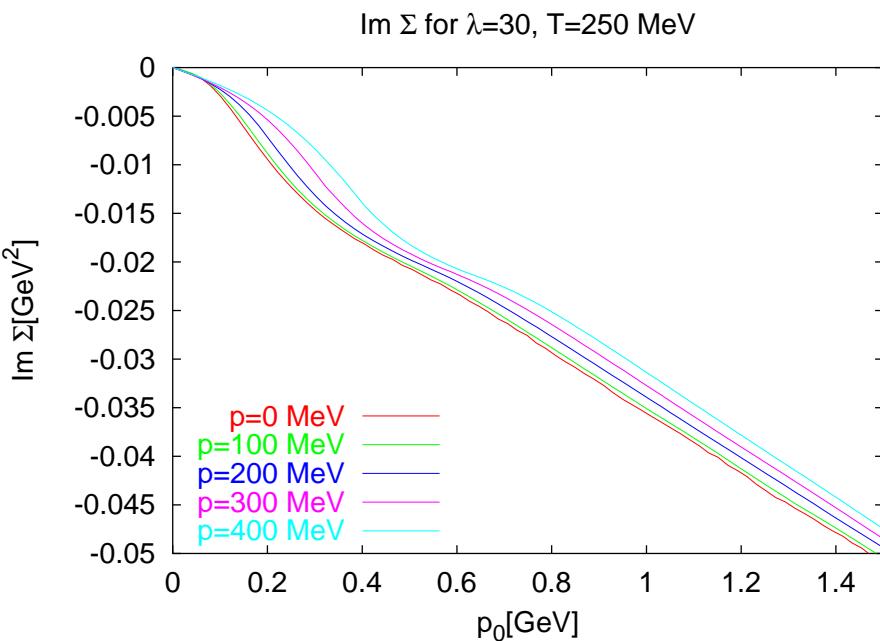
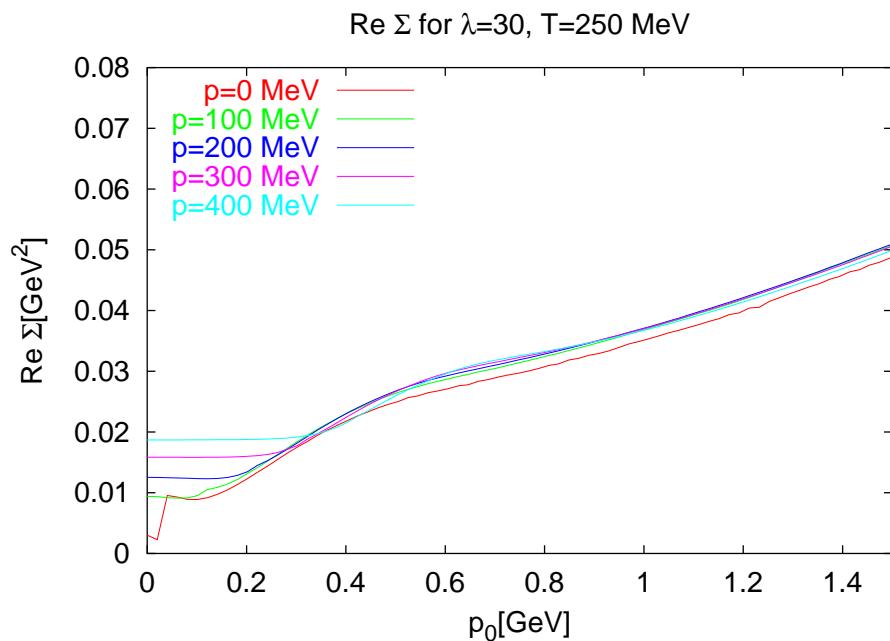
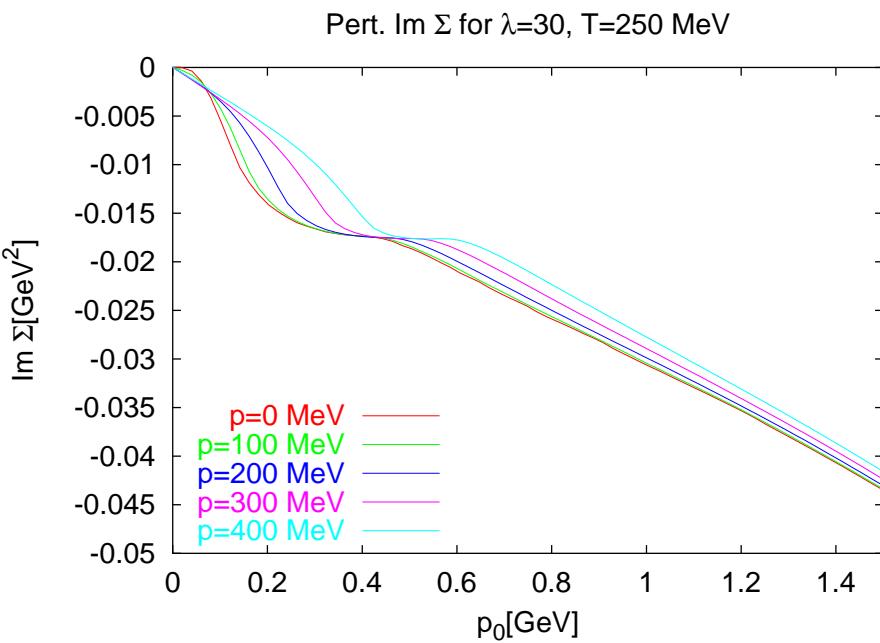
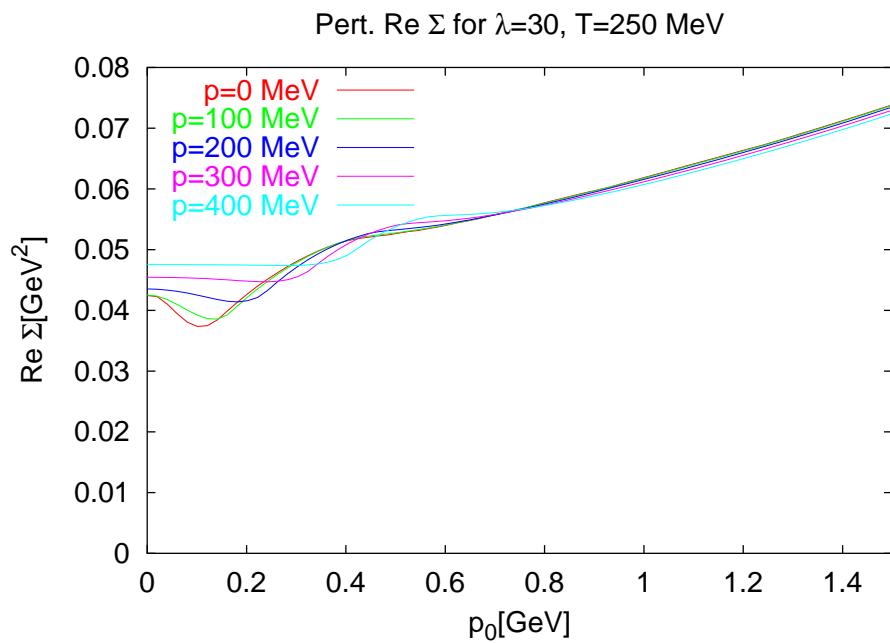
Results for “Sunset + Tadpole” at $T > 0$

#19



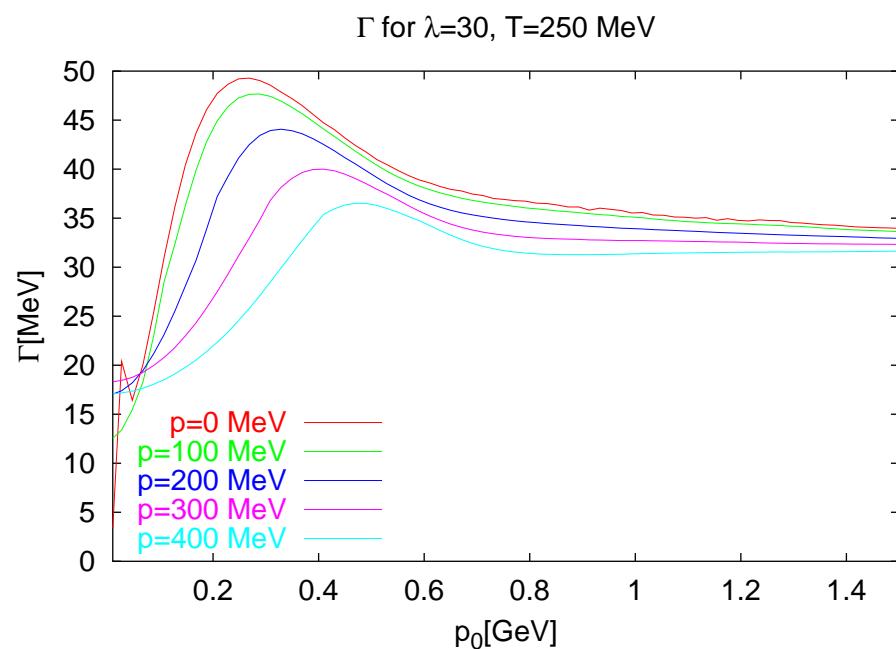
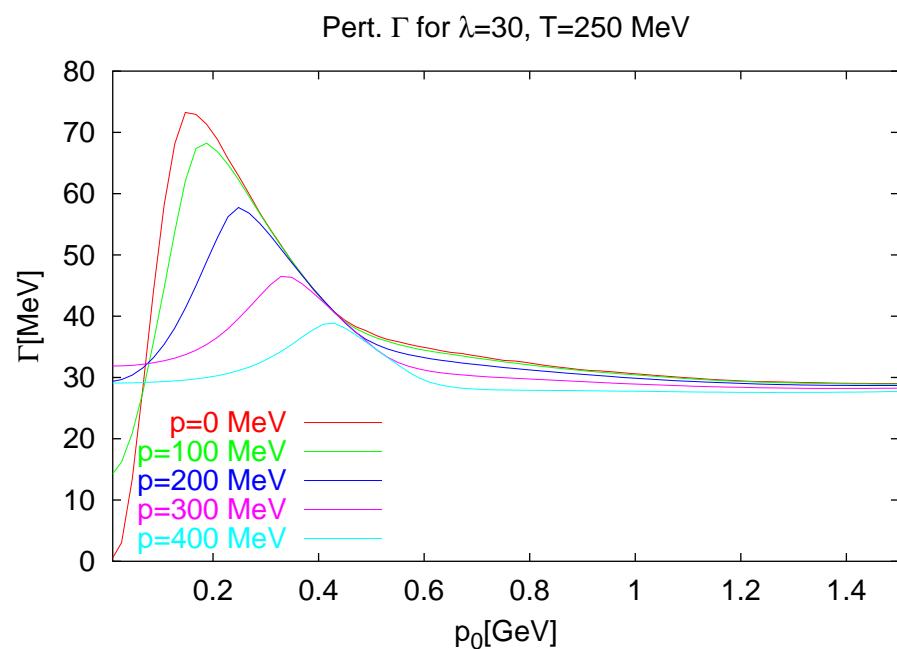
Results for “Sunset + Tadpole” at $T > 0$

#20



Results for “Sunset + Tadpole” at $T > 0$

#21



Symmetries at the correlator level

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Symmetry restoration

- Problem with Φ -Functional: Most approximations break symmetry!
- Reason: Only conserving for Expectation values for currents, not for correlation functions
- Dyson's equation as functional of φ :

$$\frac{\delta \Gamma[\varphi, G]}{\delta G} \Big|_{G=G_{\text{eff}}[\varphi]} \equiv 0$$

- Define new effective action functional

$$\Gamma_{\text{eff}}[\varphi] = \Gamma[\varphi, G_{\text{eff}}[\varphi]]$$

- Symmetry analysis $\Rightarrow \Gamma_{\text{eff}}[\varphi]$ symmetric functional in φ
- Stationary point

$$\frac{\delta \Gamma_{\text{eff}}}{\delta \phi} \Big|_{\varphi=\varphi_0} = 0$$

- ☞ φ_0 and $G = G_{\text{eff}}[\varphi_0]$: self-consistent Φ -Functional solutions!
- ☞ Γ_{eff} generates external vertex functions fulfilling Ward-Takahashi identities of symmetries
- ☞ External Propagator

$$(G_{\text{ext}}^{-1})_{12} = \frac{\delta^2 \Gamma_{\text{eff}}[\varphi]}{\delta \varphi_1 \delta \varphi_2} \Big|_{\varphi=\varphi_0}$$

- ☞ G_{ext} generally not identical with Dyson resummed propagator

Example: Hartree approximation

#23

External self-energy

- Hartree approximation:

$$i\Phi = \text{Diagram A} + \text{Diagram B} + \text{Diagram C}$$

- External self-energy defined on top of Hartree approximation

$$-i\Sigma_{\text{ext}} = \underbrace{\text{Diagram D} + \text{Diagram E} + \dots}_{\Sigma_{\text{int}}} + \text{Diagram F} + \text{Diagram G} + \dots$$

☞ RPA–Resummation restores symmetry

Diagrammar for external vertices I

#24

1st step: define Φ and internal propagator

$$\begin{aligned}
 i\Phi &= \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} + \frac{1}{2} \text{Diagram 4} + \frac{1}{2} \text{Diagram 5} \\
 i(\square - \tilde{m}^2)\varphi &= \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 6} \\
 -i\Sigma &= \text{Diagram 7} + \text{Diagram 8} + \text{Diagram 9} + \text{Diagram 10}
 \end{aligned}$$

☞ Defines mean field and Dyson resummend **internal** propagator

2nd step: Derivatives

$$\frac{\delta G_{\text{eff}}}{\delta \varphi} = i\Gamma^{(3)} = \text{Diagram 11} = \text{Diagram 12} + \text{Diagram 13}$$

External self-energy

$$\text{Diagram 14} = \text{Diagram 15} + \text{Diagram 16}$$

Diagrammar for external vertices II

#25

Definition of Bethe–Salpeter equation elements

$$i\Phi_{\varphi,\varphi} = \text{Diagram with two external lines and one internal loop} + \text{Diagram with two external lines and a horizontal mean field line}$$

$$iI^{(3)} = i\Phi_{iG,\varphi} = \text{Diagram with two external lines and one internal loop} + \text{Diagram with two external lines and a vertical mean field line}$$

$$iK = i\Phi_{iG,iG} = \text{Diagram with two external lines and two internal loops} + \text{Diagram with two external lines and a vertical mean field line}$$

- Here: Green's function lines and mean fields: fixed from self-consistent Φ -Functional solution

Application to the π - ρ -System

#26

The free vector meson

- Gauge invariant classical Lagrangian:

$$\mathcal{L} = -\frac{1}{4}V_{\mu\nu}V^{\mu\nu} + \frac{1}{2}m^2V_\mu V^\mu + \frac{1}{2}(\partial^\mu\varphi)(\partial_\mu\varphi) + m\varphi\partial_\mu V^\mu$$

- Gauge invariance:

$$\delta V_\mu(x) = \partial_\mu\chi(x), \quad \delta\varphi = m\chi(x)$$

- Quantisation: Gauge fixing and ghosts

$$\begin{aligned}\mathcal{L}_V = & -\frac{1}{4}V_{\mu\nu}V^{\mu\nu} + \frac{m}{2}V_\mu V^\mu - \frac{1}{2\xi}(\partial_\mu V^\mu)^2 + \\ & + \frac{1}{2}(\partial_\mu\varphi)(\partial^\mu\varphi) - \frac{\xi m^2}{2}\varphi^2 + \\ & + (\partial_\mu\eta^*)(\partial_\mu\eta) - \xi m^2\eta^*\eta.\end{aligned}$$

- Free vacuum propagators

$$\begin{aligned}\Delta_V^{\mu\nu}(p) &= -\frac{g^{\mu\nu}}{p^2 - m^2 + i\eta} + \frac{(1 - \xi)p^\mu p^\nu}{(p^2 - m^2 + i\eta)(p^2 - \xi m^2 + i\eta)} \\ \Delta_\varphi(p) &= \frac{1}{p^2 - \xi m^2 + i\eta} \\ \Delta_\eta(p) &= \frac{1}{p^2 - \xi m^2 + i\eta}.\end{aligned}$$

☞ Usual power counting \Rightarrow renormalisable

☞ Partition sum: Three bosonic degrees of freedom!

Application to the π - ρ -System

#27

Adding π^\pm and γ

- Gauge-covariant derivative

$$D_\mu \pi = \partial_\mu \pi + ig V_\mu \pi + ie A_\mu$$

☞ Quantisation of free photon as usual

- Minimal coupling:

$$\mathcal{L}_{\pi V} = \mathcal{L}_V + (D_\mu \pi)^* (D^\mu \pi) - m_\pi^2 \pi^* \pi - \frac{\lambda}{8} (\pi^* \pi)^2 - \frac{e}{2g_{\rho\gamma}} A_{\mu\nu} V^{\mu\nu}$$

☞ Eqs. of motion: Vector meson dominance (Kroll, Lee, Zumino)

- Adding Leptons like in QED:

$$\mathcal{L}_{e\gamma} = \bar{\psi} (iD - m_e) \psi$$

with

$$D_\mu \psi = \partial_\mu \psi + ie \psi \quad (1)$$

Application to the π - ρ -System

#28

The Propagators

$$\mu \text{ } \begin{array}{c} p \\ \text{○○○○} \end{array} \nu = -\frac{\mathrm{i}g^{\mu\nu}}{p^2 - m_\rho^2 + \mathrm{i}\eta} + \frac{\mathrm{i}(1 - \xi_\rho)p^\mu p^\nu}{(p^2 - m_\rho^2 + \mathrm{i}\eta)(p^2 - \xi m_\rho^2 + \mathrm{i}\eta)}$$

$$\mu \text{ } \begin{array}{c} p \\ \sim\sim\sim\sim \end{array} \nu = -\frac{\mathrm{i}g^{\mu\nu}}{p^2 + \mathrm{i}\eta} + \frac{\mathrm{i}(1 - \xi_\gamma)p^\mu p^\nu}{(p^2 + \mathrm{i}\eta)^2}$$

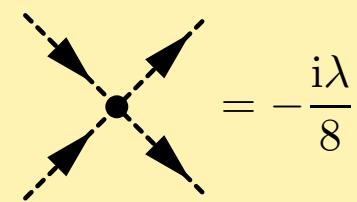
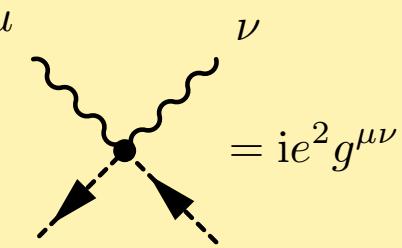
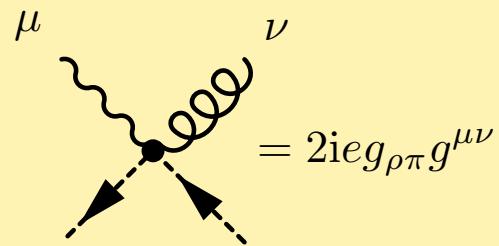
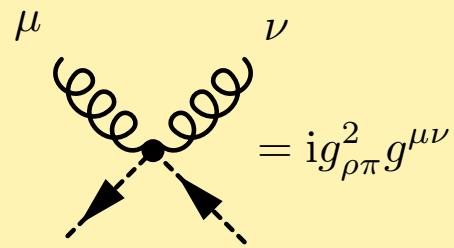
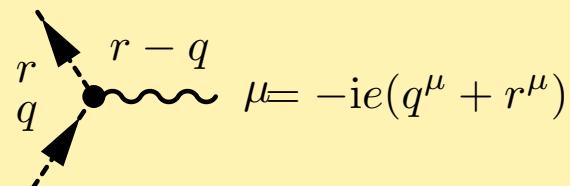
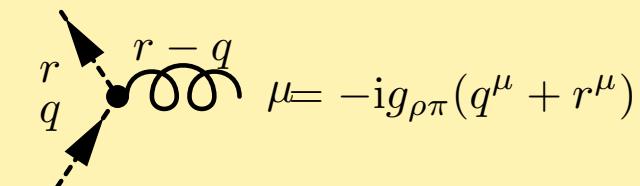
$$\text{---} \begin{array}{c} p \\ \blacktriangleleft \end{array} = \frac{\mathrm{i}}{p^2 - m_\pi^2 + \mathrm{i}\eta}$$

$$\text{---} \begin{array}{c} p \\ \blacktriangleright \end{array} = \frac{\mathrm{i}(\not{p} + m)}{p^2 - m_l^2 + \mathrm{i}\eta}$$

Application to the π - ρ -System

#29

The Vertices



$$\mu \sim \text{---} \bullet \text{---} \quad \nu = i \frac{e}{g_{\rho\gamma}} p^2 \Theta^{\mu\nu}(p)$$

Application to Vector bosons

#30

- Kroll–Lee–Zumino interaction: Coupling of massive vector bosons to conserved currents \Rightarrow gauge theory
- Symmetry breaking at correlator level

Problems:

- ☞ Internal propagators contain spurious degrees of freedom
- ☞ Negative norm states
- ☞ Numerically unstable due to light cone singularities

- Classical picture (Fokker–Planck–equation):

$$\Pi^{\mu\nu}(\tau, \vec{p} = 0) \propto \langle v^\mu(\tau) v^\nu(0) \rangle$$

- „One-loop” approximation in the classical limit

$$\Pi^{\mu\nu}(\tau, \vec{p} = 0) \propto \exp(-\Gamma\tau)$$

- ☞ $1/\Gamma$: Relaxation time scale due to scattering

- Exact behaviour:

$$\Pi^{00}(\tau, \vec{p} = 0) \propto \langle 1 \cdot 1 \rangle = \text{const}$$

$$\Pi^{jk}(\tau, \vec{p} = 0) \propto \langle v^j v^k \rangle \propto \exp(-\Gamma_x \tau)$$

- ☞ For Π^{jk} : If $\Gamma \approx \Gamma_x \Rightarrow$ 1-loop approximation justified

- ☞ Classical limit also shows:

Π^{jk} only slightly modified by ladder resummation

- In self-consistent approximations:

Use only $p_j p_k \Pi^{jk}$ and $g_{jk} \Pi^{jk}$

- ☞ Construct Π_T and Π_L

The interacting π - ρ - a_1 system

#31

The self-consistent approximation

Lagrangian:

$$\mathcal{L}_{\text{int}} = \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3}$$

Φ -Funktional:

$$\Phi = \text{Diagram 4} + \text{Diagram 5} + \text{Diagram 6}$$

Self-energies:

$$\Pi_{\rho} = \text{Diagram 7} + \text{Diagram 8}$$

$$\Pi_{a_1} = \text{Diagram 9}$$

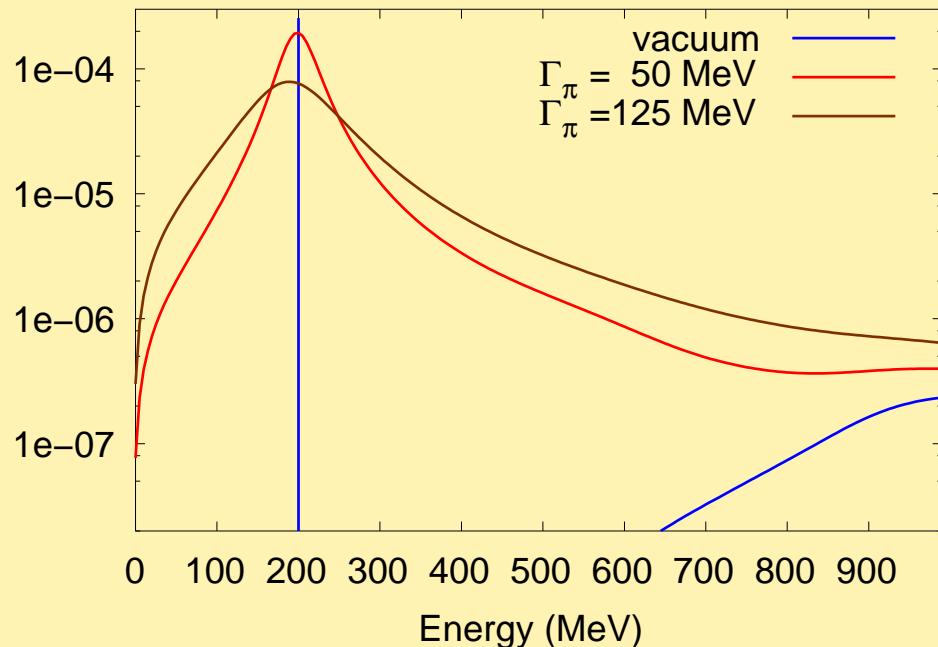
$$\Sigma_{\pi} = \text{Diagram 10} + \text{Diagram 11} + \text{Diagram 12}$$

Results for the $\pi\rho a_1$ -System

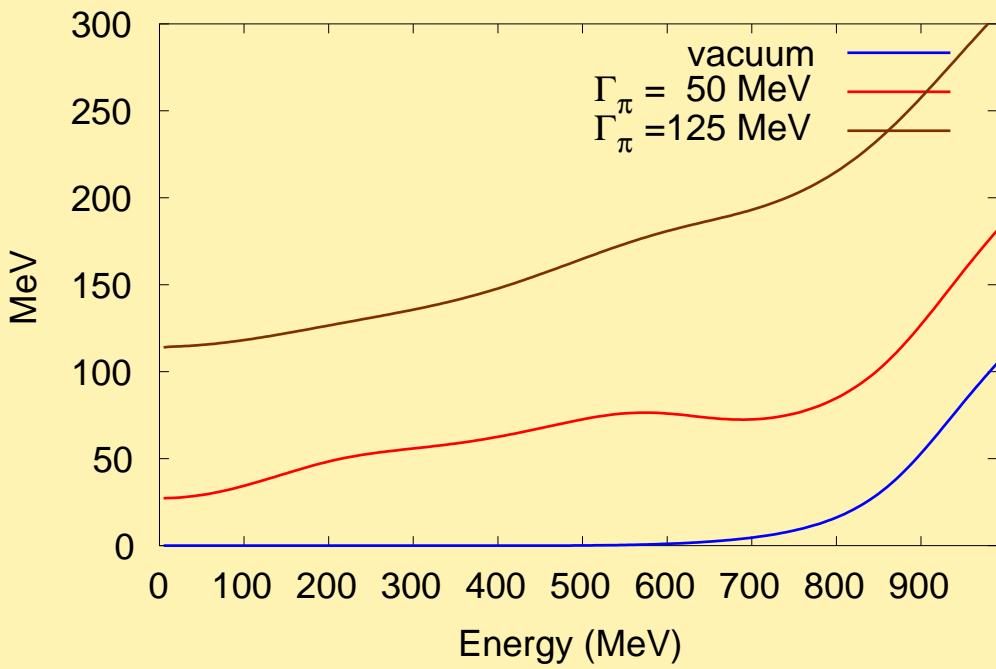
#32

Broad pions in the medium

pi-Meson Spectral function, T=110 MeV; p=150 MeV/c



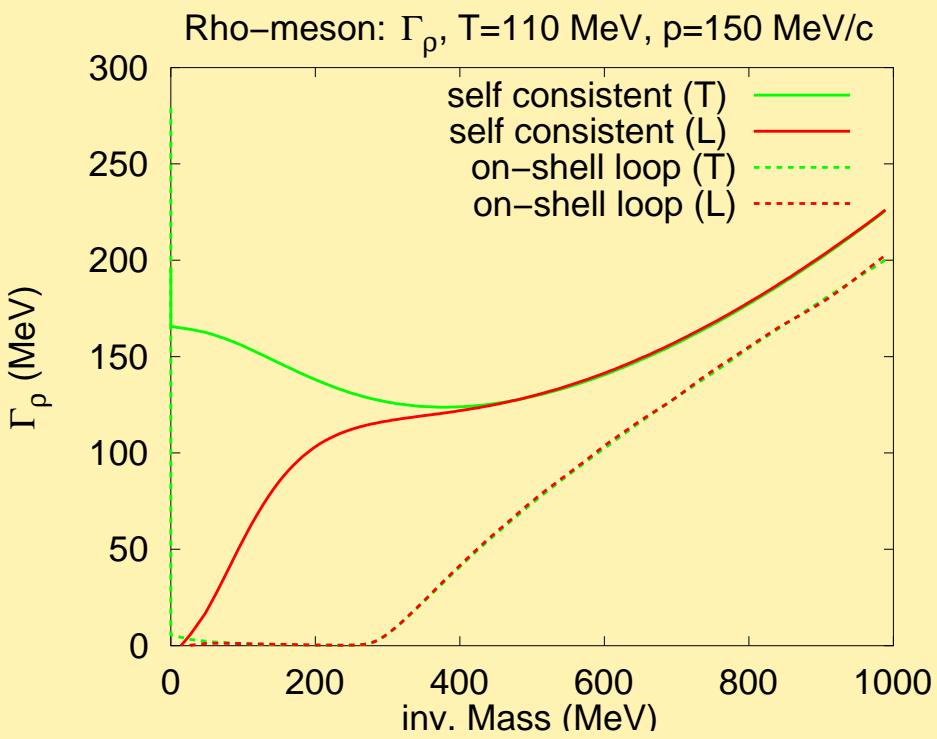
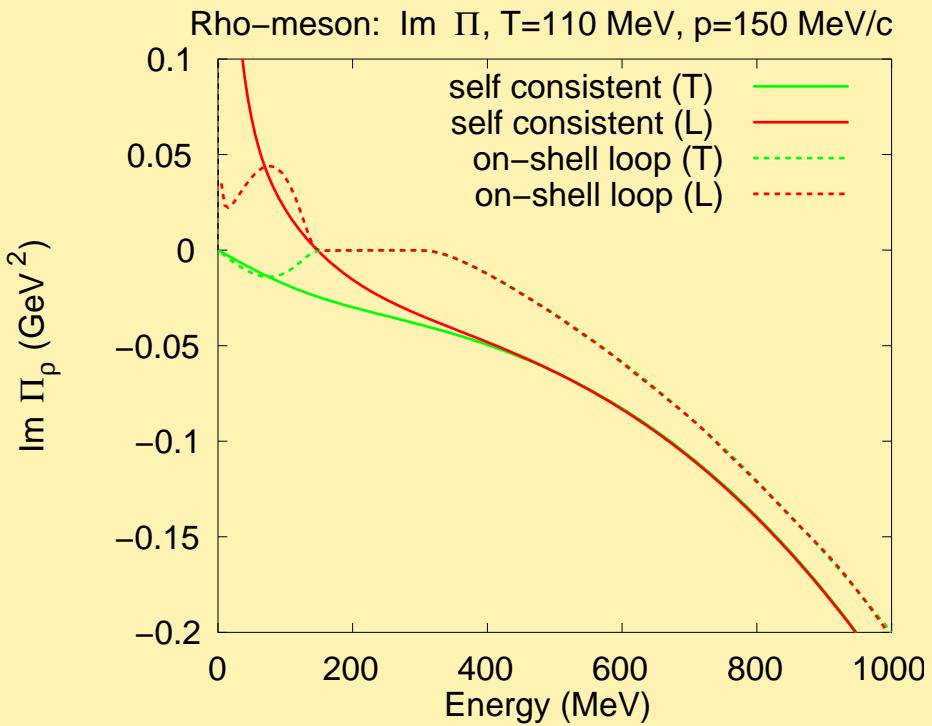
pi-Meson Width, T=110 MeV; p=150 MeV/c



Results for the $\pi\rho a_1$ -System

#33

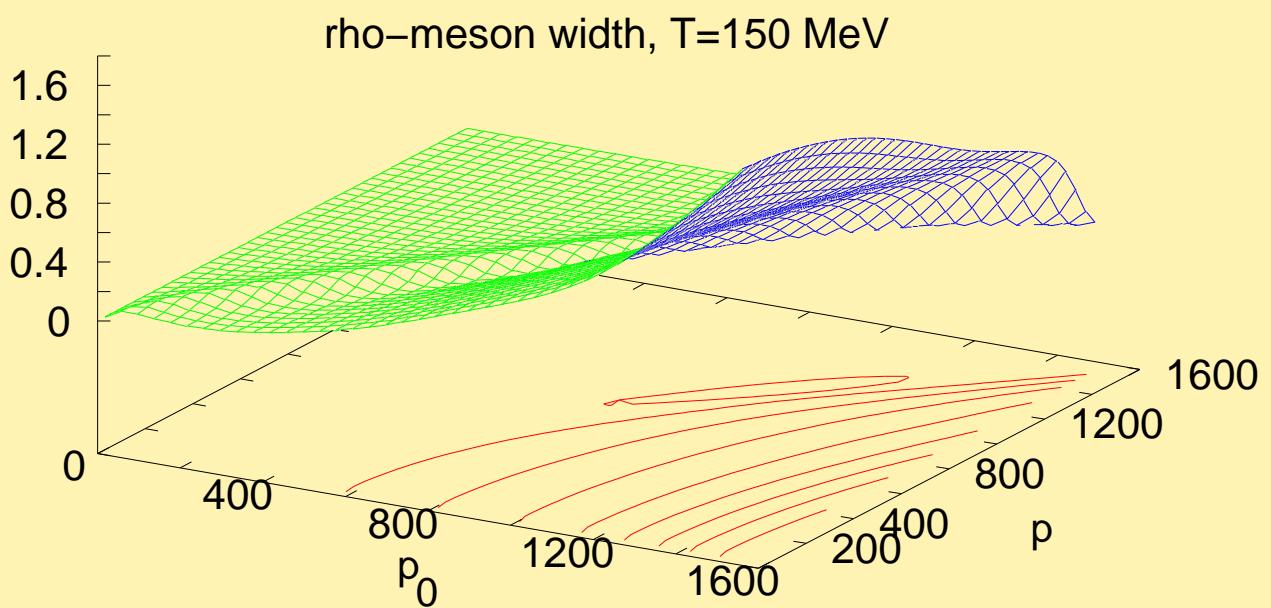
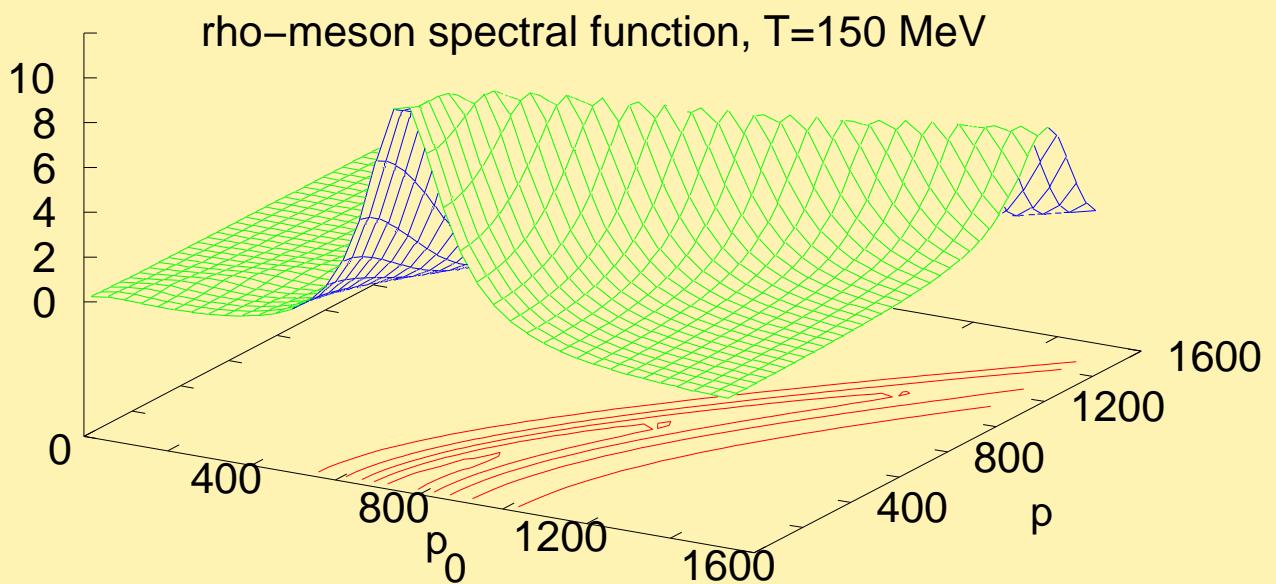
ρ -meson Polarisation tensor



Results for the $\pi\rho a_1$ -System

#34

ρ -meson properties I

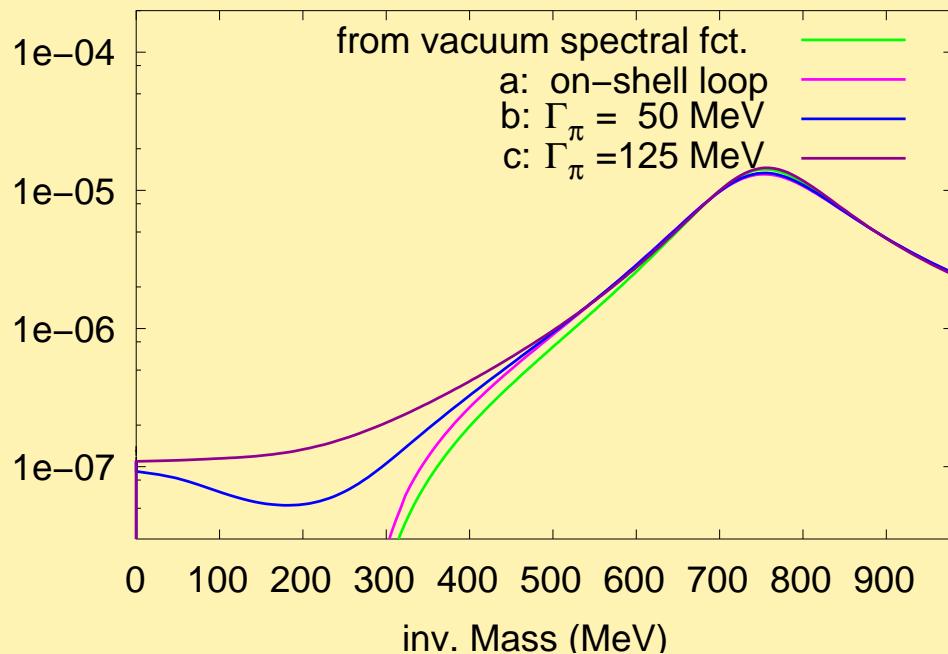


Results for the $\pi\rho a_1$ -System

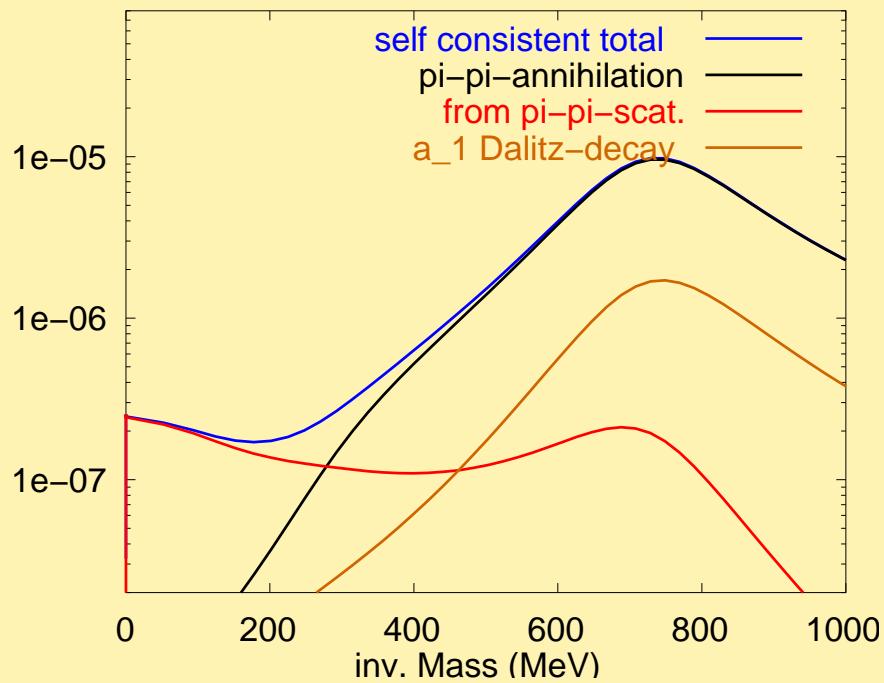
#35

ρ -meson properties II

Rho-meson Spectral fct., T=110 MeV, p=150 MeV/c



Rho-Meson Spectral Fnct., T=150 MeV, p=150 MeV/c

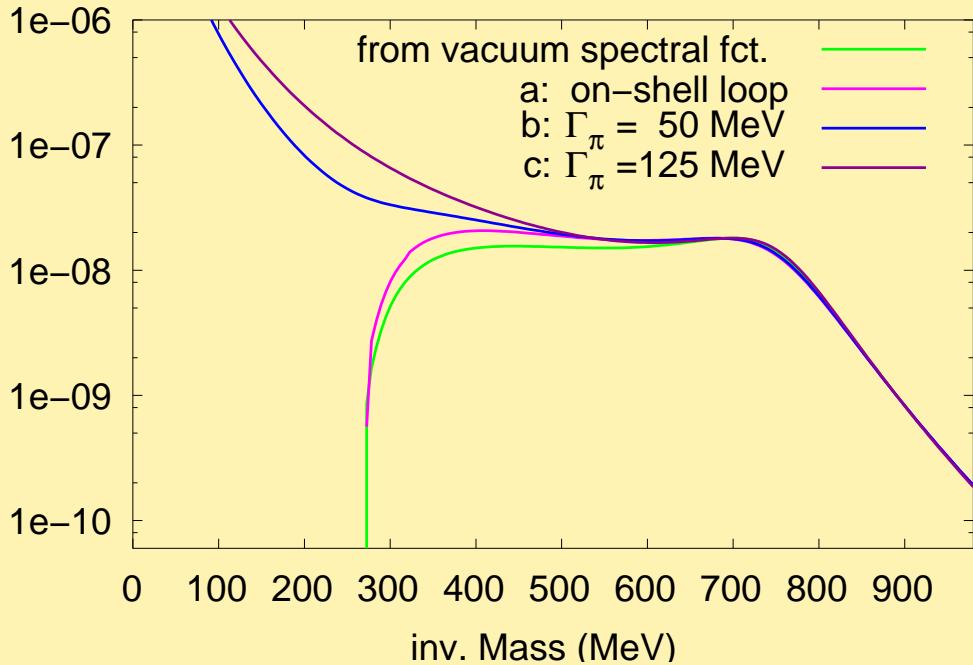


Results for the $\pi\rho a_1$ -System

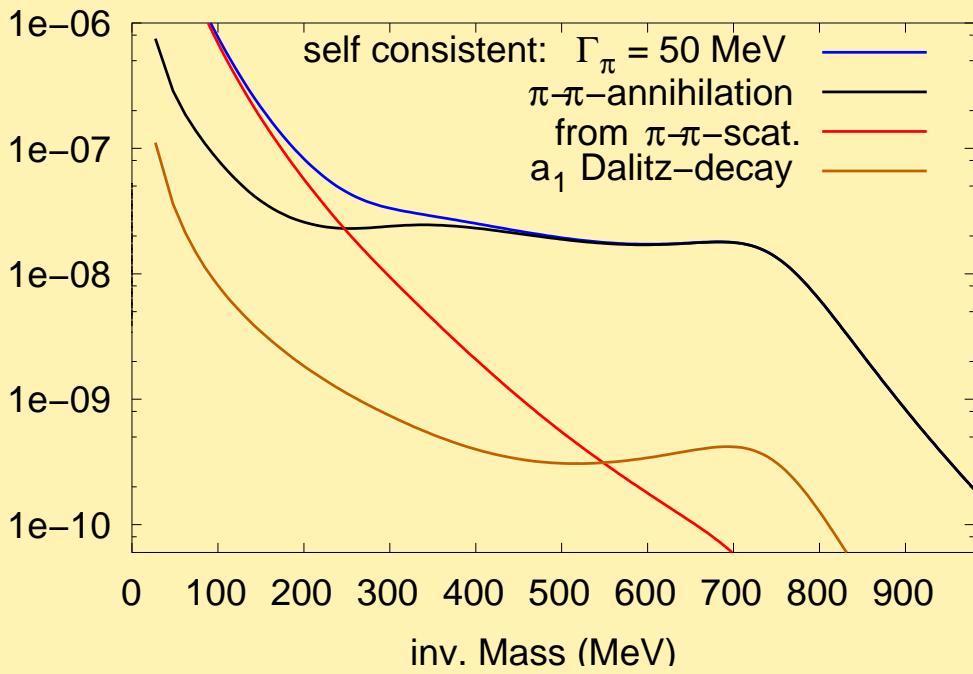
#36

Dilepton rate

Dilepton rate (arb.units), T=110 MeV, p=150 MeV/c



Dilepton rate (arb.units), T=110 MeV, p=150 MeV/c



Conclusions

#37

Summary

- Self-consistent Φ -derivable schemes
- Renormalization
- Symmetry analysis
- Scheme for vector particles
- Numerical treatment

Outlook

- “Toolbox” for application to realistic models
- Perspectives for self-consistent treatment of gauge theories
- QCD e.g. beyond HTL?
- Transport equations for particles with finite width