Introduction to Quantum Game Theory

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Motivation: "Institute for Advance Study" in Princeton (1933 -1950)

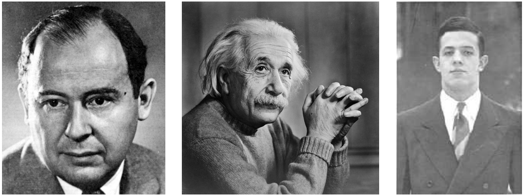

Figure : Johann von Neumann, Albert Einstein und John Forbes Nash Jr.

Johann (John) von Neumann. Zur Theorie der Gesellschaftsspiele. *Mathematische Annalen*, 100:295–300, 1928.

J. von Neumann. *Mathematische Grundlagen der Quantenmechanik*. Springer, 1932.

J. von Neumann and O. Morgenstern. *The Theory of Games and Economic Behaviour*. Princeton University Press, 1947.

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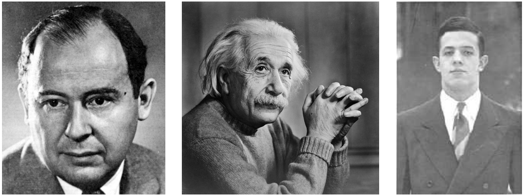

Figure : Johann von Neumann, Albert Einstein und John Forbes Nash Jr.

Quantum Entanglement and the "EPR-Paradoxon":

A. Einstein, B. Podolsky, and N. Rosen. Can Quantum-Mechanical Description of Physical Reality Be Considered Complete? *Physical Review*, 47:777–780, 1935.

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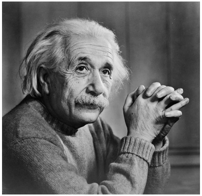


Figure : Johann von Neumann, Albert Einstein und John Forbes Nash Jr.

John F. Nash Jr. Equilibrium Points in N-person Games. *Proceedings of the National Academy of Sciences*, 36:48–49, 1950.

John F. Nash Jr. The Bargaining Problem. *Econometrica*, 18:155–162, 1950.

John F. Nash Jr. Non-Cooperative Games. *The Annals of Mathematics*, 54(2):286–295, 1951.

Quantum Game Theory

Quantum Game Theory is a mathematical and conceptual amplification of classical game theory. It unifies the two mathematical theories of von Neumann (Game Theory and Quantum Theory) with Einstein's quantum entanglement concept and extends the Nash equilibrium definitions in an abstract complex-valued space.

Quote from the german summary of my second phd-thesis: „Das Hauptanliegen dieser Arbeit liegt in einer zusammenfassenden, mathematisch einwandfreien Darstellung der Theorie der Gesellschaftsspiele auf quantentheoretischen, abstrakten Hilbertschen Räumen. “

Research Questions of the Talk

Mathematical description of Quantum Game Theory

What are the main mathematical concepts of quantum game theory? How are the theories (Game Theory and Quantum Theory) unified?

Results for Quantum Games within different game classes

What are the main differences between classical and quantum game theory. Is the underlying Nash equilibrium structure of (2 player)-(2 strategy) games changed within a quantum game theory-based analysis?

Presentation of various applications

How can quantum game theory be applied to real game situations?

Superpositionen von Eigenzuständen

Schrödingers Katze

Figure : Theoretische Versuchsanordnung des Gedankenexperiments.

In einem geschlossenen Kiste befindet sich ein instabiler Atomkern, der innerhalb einer bestimmten Zeitspanne mit einer gewissen Wahrscheinlichkeit zerfällt. Im Falle eines Zerfalls werde Giftgas freigesetzt, was eine im Raum befindliche Katze tötet. Bevor ein Beobachter die Kiste öffnet, schwebt der Zustand ψ der Katze zwischen den Eigenzuständen '$\psi_1 :=$ Lebend' und '$\psi_2 :=$ Tot'.

$$\psi = \frac{1}{\sqrt{2}} (\psi_1 + \psi_2)$$

Quantisierte Messgrößen

Beispiel: Das Wasserstoffatom

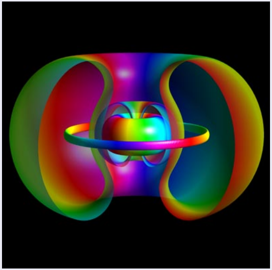


Figure : Aufenthaltswahrscheinlichkeit des Elektrons im Wasserstoffatom (n=4,l=2,m=2). Quelle: Bernd Thaller, Visual Quantum Mechanics

Der Zustand eines Elektrons im Wasserstoffatom wird mit Hilfe der stationären Schrödingergleichung berechnet. Die messbaren Eigenzustände des Elektrons ($\psi_{nlm}(\vec{r})$) sind durch ihre Quantenzahlen (n,l,m) quantisiert, d.h. Messgrößen wie z.B. die Energie können nur diskrete Werte annehmen. Der allgemeine Elektronenzustand ergibt sich durch Überlagerung (Superposition) der Eigenzustände ($a_{nlm} \in \mathbb{C}$).

$$\psi = \sum_{n=1}^{\infty} \sum_{l=0}^{n-1} \sum_{m=-l}^{l} a_{nlm} \, \psi_{nlm}$$

Welle-Teilchen-Dualismus

Das Doppelspaltexperiment

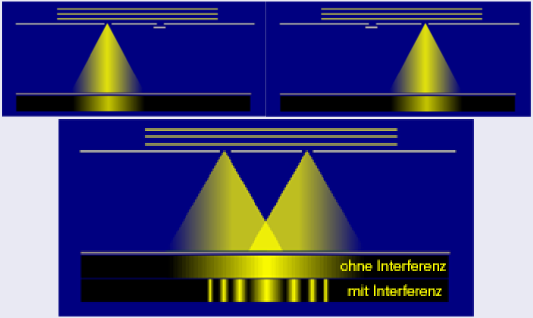


Figure : Beim Doppelspaltexperiment offenbaren Teilchen ihre Welleneigenschaften. Quelle: Michael Craiss

1961 wurde das Doppelspaltexperiment mit Elektronen durch Claus Jönsson durchgeführt und im September 2002 in einer Umfrage der englischen physikalischen Gesellschaft in der Zeitschrift 'Physics World' zum schönsten physikalischen Experiment aller Zeiten gewählt.

Das Einstein-Podolsky-Rosen Paradoxon

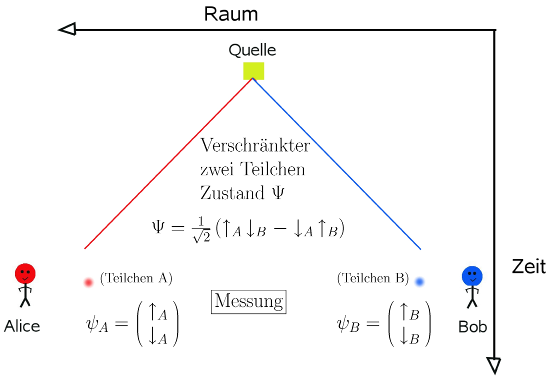


Figure : EPR Gedankenexperiment: Obwohl es keine messbare Wechselwirkung zwischen den Teilchen A und B gibt, sind diese dennoch mittel einer Quantenverschänkung verbunden.

Classical (2 person)-(2 strategy) game

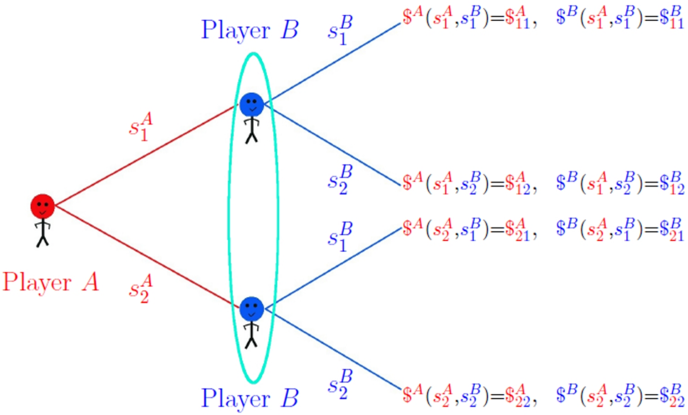


Figure : Game tree of a (2 person)-(2 strategy) game with payoff for player A ($\$^A$) and player B ($\$^B$).

Definition of a (2 player)-(2 strategy) game Γ

An unsymmetric (2×2) game Γ is defined as ...

(2×2) Game: $\Gamma := \left(\{A, B\}, \mathcal{S}^A \times \mathcal{S}^B, \hat{\$}_A, \hat{\$}_B \right)$

Set of pure strategies of player A and B: $\mathcal{S}^A = \left\{ s_1^A, s_2^A \right\} \;,\; \mathcal{S}^B = \left\{ s_1^B, s_2^B \right\}$

Set of mixed strategies of player A and B: $\tilde{\mathcal{S}}^A = \left\{ \tilde{s}_1^A, \tilde{s}_2^A \right\} \;,\; \tilde{\mathcal{S}}^B = \left\{ \tilde{s}_1^B, \tilde{s}_2^B \right\}$

Payoff matrix for player A: $\hat{\$}_A = \begin{pmatrix} \$_{11}^A & \$_{12}^A \\ \$_{21}^A & \$_{22}^A \end{pmatrix}$

Payoff matrix for player B: $\hat{\$}_B = \begin{pmatrix} \$_{11}^B & \$_{12}^B \\ \$_{21}^B & \$_{22}^B \end{pmatrix}$

The mixed strategy payoff function $\tilde{\$}^\mu$ of player $\mu = A, B$

Normalizing conditions for the mixed strategies of player μ:

$$\tilde{s}_1^\mu + \tilde{s}_2^\mu = 1 \;\forall\, \mu = A, B \quad \tilde{s}_1^\mu, \tilde{s}_2^\mu \in [0,1]$$

The mixed strategy payoff function reduces to:

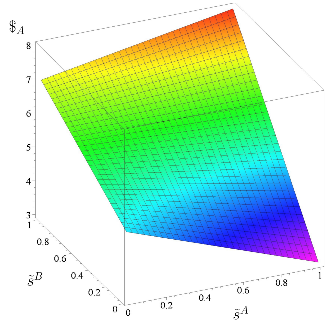
$$\begin{aligned} & \tilde{\$}^\mu : ([0,1] \times [0,1]) \to \mathbb{R} \\ \tilde{\$}^\mu(\tilde{s}^A, \tilde{s}^B) \;=\; & \$_{11}^\mu \tilde{s}^A \tilde{s}^B + \$_{12}^\mu \tilde{s}^A (1 - \tilde{s}^B) + \\ & + \$_{21}^\mu (1 - \tilde{s}^A) \tilde{s}^B + \$_{22}^\mu (1 - \tilde{s}^A)(1 - \tilde{s}^B) \end{aligned}$$

, where $\tilde{s}^A := \tilde{s}_1^A$, $\tilde{s}^B := \tilde{s}_1^B$, $\tilde{s}_2^A = 1 - \tilde{s}_1^A$ and $\tilde{s}_2^B = 1 - \tilde{s}_1^B$

The mixed strategy payoff function $\tilde{\$}^\mu$ of player $\mu = A, B$

Mixed strategy payoff function $\tilde{\$}^A \left(\tilde{s}^A, \tilde{s}^B\right)$ of player A

$(\$^A_{11} = 8,\ \$^A_{12} = 5,\ \$^A_{21} = 7,\ \$^A_{22} = 3)$



Payoff $\tilde{\$}^\mu \left(\tilde{s}^A, \tilde{s}^B\right)$ as a function of $\tilde{s}^A, \tilde{s}^B \in [0, 1]$:

$$\tilde{\$}^\mu : ([0,1] \times [0,1]) \to \mathbb{R}$$

$$\tilde{\$}^\mu(\tilde{s}^A, \tilde{s}^B) = \$^\mu_{11}\tilde{s}^A\tilde{s}^B + \$^\mu_{12}\tilde{s}^A(1-\tilde{s}^B) + $$
$$+\$^\mu_{21}(1-\tilde{s}^A)\tilde{s}^B + \$^\mu_{22}(1-\tilde{s}^A)(1-\tilde{s}^B)$$

, where $\tilde{s}^A := \tilde{s}^A_1$, $\tilde{s}^B := \tilde{s}^B_1$,

$\tilde{s}^A_2 = 1 - \tilde{s}^A_1$ and $\tilde{s}^B_2 = 1 - \tilde{s}^B_1$

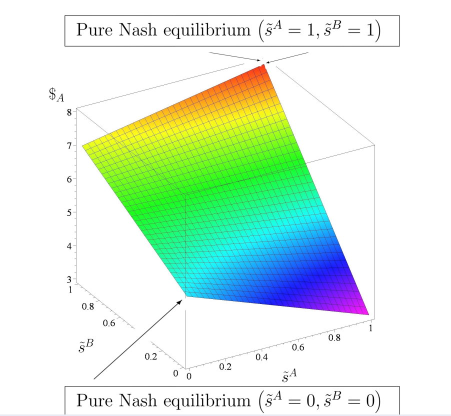
Payoff $\tilde{\$}^\mu \left(\tilde{\mathcal{S}}^A \times \tilde{\mathcal{S}}^B\right)$ as a function of the sets of mixed strategies for player A and B:

$$\tilde{\$}^\mu : \left(\tilde{\mathcal{S}}^A \times \tilde{\mathcal{S}}^B\right) \to \mathbb{R}$$

$$\tilde{\$}^\mu((\tilde{s}^A_1, \tilde{s}^A_2), (\tilde{s}^B_1, \tilde{s}^B_2)) = \$^\mu_{11}\tilde{s}^A_1\tilde{s}^B_1 + \$^\mu_{12}\tilde{s}^A_1\tilde{s}^B_2 + $$
$$+\$^\mu_{21}\tilde{s}^A_2\tilde{s}^B_1 + \$^\mu_{22}\tilde{s}^A_2\tilde{s}^B_2$$

Nash equilibria (NE)

Nash equilibria and $\tilde{\$}^\mu\left(\tilde{s}^A, \tilde{s}^B\right)$



A strategy combination $(\tilde{s}^{A*}, \tilde{s}^{B*})$ is called a Nash equilibrium, if:

$$\tilde{\$}^A(\tilde{s}^{A*}, \tilde{s}^{B*}) \geq \tilde{\$}^A(\tilde{s}^{A}, \tilde{s}^{B*}) \quad \forall\ \tilde{s}^A \in [0,1]$$
$$\tilde{\$}^B(\tilde{s}^{A*}, \tilde{s}^{B*}) \geq \tilde{\$}^B(\tilde{s}^{A*}, \tilde{s}^{B}) \quad \forall\ \tilde{s}^B \in [0,1]$$

A strategy combination $(\tilde{s}^{A\star}, \tilde{s}^{B\star})$ is called an interior (mixed strategy) Nash equilibrium, if:

$$\left.\frac{\partial \tilde{\$}^A(\tilde{s}^A, \tilde{s}^B)}{\partial \tilde{s}^A}\right|_{\tilde{s}^B = \tilde{s}^{B\star}} = 0 \quad \forall\ \tilde{s}^A \in [0,1] \ ,\ \tilde{s}^{B\star} \in\,]0,1[$$
$$\left.\frac{\partial \tilde{\$}^B(\tilde{s}^A, \tilde{s}^B)}{\partial \tilde{s}^B}\right|_{\tilde{s}^A = \tilde{s}^{A\star}} = 0 \quad \forall\ \tilde{s}^B \in [0,1] \ ,\ \tilde{s}^{A\star} \in\,]0,1[$$

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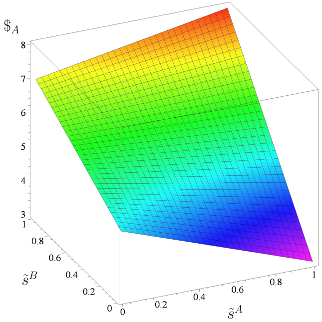

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A strategy combination $(\tilde{s}^{A\star}, \tilde{s}^{B\star})$ is called an interior (mixed strategy) Nash equilibrium, if:

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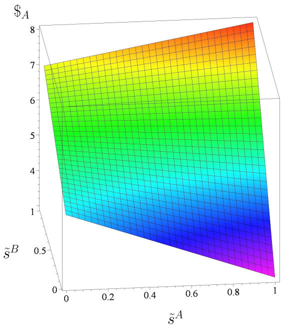

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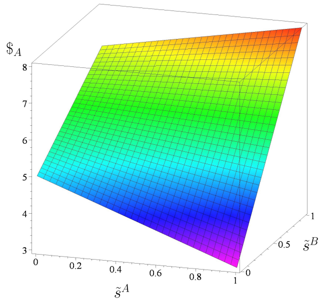

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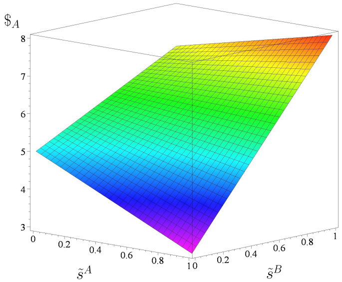

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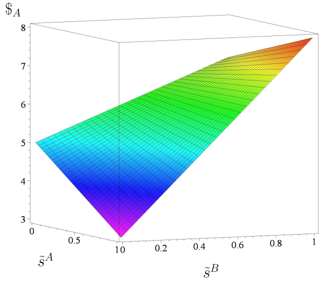

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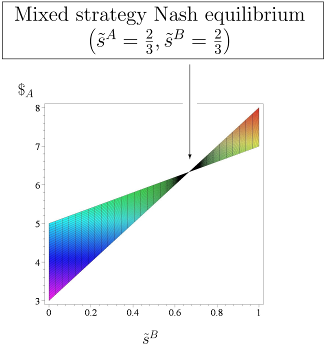
Nash equilibria (NE)

Nash equilibria and $\tilde{\$}^{\mu}\left(\tilde{s}^A, \tilde{s}^B\right)$



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Evolutionary Game Theory

Replicatordynamics: The dynamical behavior of a population of players

$$\frac{dx_i^A(t)}{dt} = x_i^A(t)\left[\sum_{l=1}^{m} \$_{il}^A\, x_l^B(t) - \sum_{l=1}^{m}\sum_{k=1}^{m} \$_{kl}^A\, x_k^A(t)\, x_l^B(t)\right]$$

$$\frac{dx_i^B(t)}{dt} = x_i^B(t)\left[\sum_{l=1}^{m} \$_{li}^B\, x_l^A(t) - \sum_{l=1}^{m}\sum_{k=1}^{m} \$_{lk}^B\, x_l^A(t)\, x_k^B(t)\right]$$

The two population vectors $\vec{x}^A$ and $\vec{x}^B$ have to fulfill the normalizing conditions of a unity vector

$$x_i^\mu(t) \geq 0 \quad \text{and} \quad \sum_{i=1}^{m} x_i^\mu(t) = 1 \quad \forall\, i = 1, 2, ..., m\,,\ \ t \in \mathbb{R},\ \ \mu = A, B$$

Replicatordynamics of (2×2) games

Replicatordynamics of unsymmetric (2×2) games

$$\frac{dx(t)}{dt} = \left(\left(\$^A_{11} + \$^A_{22} - \$^A_{12} - \$^A_{21} \right) \left(x(t) - (x(t))^2 \right) \right) y(t) + \left(\$^A_{12} - \$^A_{22} \right) \left(x(t) - (x(t))^2 \right) =: g_A(x,y)$$

$$\frac{dy(t)}{dt} = \left(\left(\$^B_{11} + \$^B_{22} - \$^B_{12} - \$^B_{21} \right) \left(y(t) - (y(t))^2 \right) \right) x(t) + \left(\$^B_{12} - \$^B_{22} \right) \left(y(t) - (y(t))^2 \right) =: g_B(x,y)$$

Replicatordynamics of symmetric (2×2) games

$$\frac{dx}{dt} = x \left[\$_{11}(x - x^2) + \$_{12}(1 - 2x + x^2) + \$_{21}(x^2 - x) + \$_{22}(2x - x^2 - 1) \right]$$

$$= x \left[(\$_{11} - \$_{21})(x - x^2) + (\$_{12} - \$_{22})(1 - 2x + x^2) \right] =: g(x)$$

with: $x = x(t) := x_1(t) \quad \rightarrow \quad x_2(t) = (1 - x(t))$

Payoff Transformation and Game classes

Nash equivalent games

The set of Nash equilibria, the dynamical behavior of evolutionary games and the existence of evolutionary stable strategies (ESS) are unaffected by positive affine payoff transformations and by additionally added constants, where the strategy choice of the other players are fixed (see e.g. Weibull(1995)[14]). In the following the second kind of payoff transformation will be used to transform the payoff matrices in order to classify the games into different categories.

Symmetric payoff matrix after payoff transformation

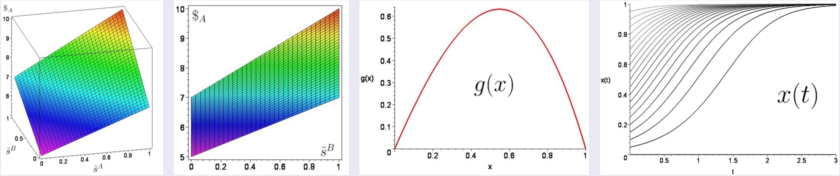
A\B	s_1^B	s_2^B
s_1^A	$(\$_{11},\$_{11})$	$(\$_{12},\$_{21})$
s_2^A	$(\$_{21},\$_{12})$	$(\$_{22},\$_{22})$

$\Longrightarrow$

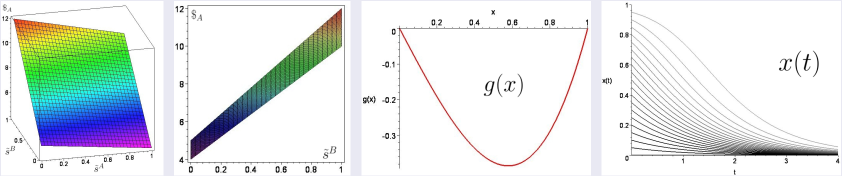
A\B	$Trafo\, s_1^B$	$Trafo\, s_2^B$
$Trafo\, s_1^A$	$(\underbrace{\$_{11}-\$_{21}}_{:=a},\underbrace{\$_{11}-\$_{21}}_{:=a})$	(0,0)
$Trafo\, s_2^A$	(0,0)	$(\underbrace{\$_{22}-\$_{12}}_{:=b},\underbrace{\$_{22}-\$_{12}}_{:=b})$

Symmetric (2 × 2) games: Dominant Class $(a < 0,\ b > 0)$ or $(b < 0,\ a > 0)$

Dominant Game: a=3, b=-2, one pure NE and one ESS (s_1^A, s_1^B)

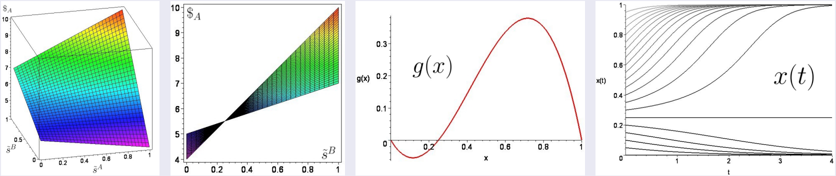


Prisoner's Dilemma: a=-2, b=1, one pure NE and one ESS (s_2^A, s_2^B)

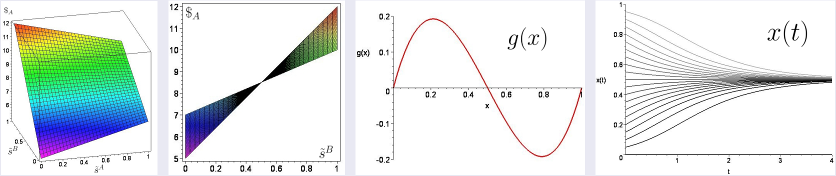


Coordination $(a, b > 0)$ and Anti-Coordination $(a, b < 0)$ Class

Coordination game: a=3, b=1, two pure and one interior NE at $\tilde{s}^\star = \frac{1}{4}$, two ESS $((s_1^A, s_1^B)$ and $(s_2^A, s_2^B))$



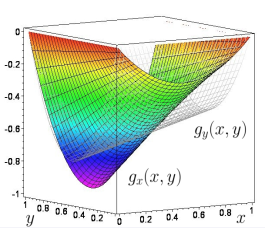
Anti-Coordination game: a=-2, b=-2, two pure asymmetric NE and one interior NE at $\tilde{s}^\star = \frac{1}{2}$, one ESS $(\tilde{s}^{A\star} = \frac{1}{2}, \tilde{s}^{B\star} = \frac{1}{2})$



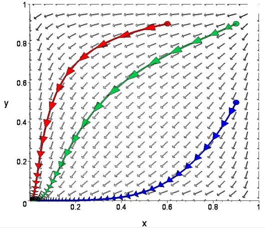
Game classes of unsymmetric (2 player)-(2 strategy) games

Corner Class (one ESS)

$g_x(x, y)$ (colored) and $g_y(x, y)$ (wired):

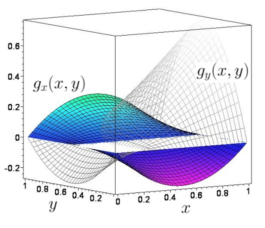


Phase diagram of xy-trajectories:

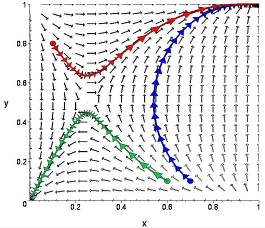


Saddle Class (two ESS)

$g_x(x, y)$ (colored) and $g_y(x, y)$ (wired):

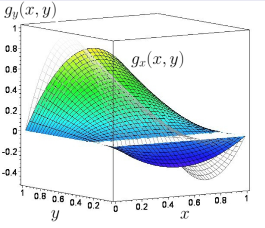


Phase diagram of xy-trajectories:

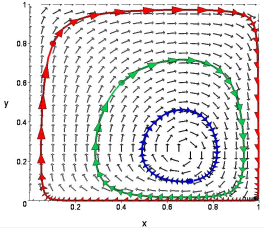


Center Class (no ESS)

$g_x(x, y)$ (colored) and $g_y(x, y)$ (wired):



Phase diagram of xy-trajectories:



Related Literature (I): Different Quantum Games

- The *Quantum* Penny Flip Game
  1999, D. A. Meyer, *Quantum strategies*, PRL 82 (1052)
- The *Quantum* Prisoner's Dilemma
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- The *Quantum* Battle of Sexes
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- The *Quantum* Coordination Game
  2003, B. A. Huberman and T. Hogg, *Quantum Solution of Coordination Problems*, Quantum Information Processing 2(6)
- The *Quantum* Ultimatum Game
  2005, R. Vilela Mendes, *The Quantum Ultimatum Game*, Quantum Information Processing 4(1)

Related Literature (II)

- Economics and Quantum Game Theory

  2002, E. W. Piotrowski and J. Sladkowski, *Quantum Market Games*, Physica A (312) 208

  2002, Kay-Yut Chen, T. Hogg and R. Beaulsoleil *A Quantum Treatment of Public Goods Economics*, Quantum Information Processing 1(6)

  2004, E. W. Piotrowski and J. Sladkowski *Quantum Game Theory in Finance*, Quantitative Finance 4 (1-7)

  2007, T. Hogg, P. Harsha and Kay-Yut Chen *Quantum Auctions*, Int. J. of Quantum Information 5:751-780

  2007, M. Hanauske, S. Bernius and B. Dugall, *Quantum Game Theory and Open Access Publishing*, Physica A, Vol.382 (2007), p.650-664 (physics/0612234)

Related Literature (III)

- Quantum Computer and Quantum Game Theory
  2002, J. Du, H. Li, X. Xu, M. Shi, J. Wu, X. Zhou and R. Han *Experimental realization of quantum games on a quantum computer*, PRL 88 (137902)

  2007, R. Prevedel, A. Stefanov, P. Walther and A. Zeilinger *Experimental realization of a quantum game on a one-way quantum computer*, New Journal of Physics 9 (205)

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- Extensions of Quantum Game Theory
  2001, S. C. Benjamin and P. M. Hayden, *Multi-Player Quantum Games*, PRA 64 (030301) ...

Related Literature (IV)

- Experimental Economics and Quantum Game Theory

  2006, Kay-Yut Chen and Tad Hogg *How well do people play a quantum prisoner's dilemma?*, Quantum Information Processing 5(43)

  2007, Kay-Yut Chen and Tad Hogg *Experiments with Probabilistic Quantum Auctions*, arXiv:0707.4195v1

  2007, M. Hanauske, S. Bernius, W. König and B. Dugall *Experimental Validation of Quantum Game Theory*, Accepted Paper at the Conference *LOFT 2008*

- and reviews in Physics World and Nature ...

A 2-Player–2-Strategy-Quantumgame

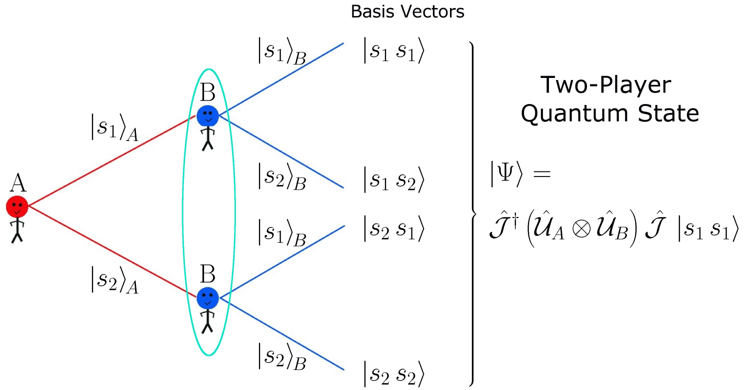


Figure : $|\Psi\rangle$: Two-Player State, $\hat{\mathcal{J}}(\gamma)$: Entangling Operator, γ: Strength of Entanglement, $\widehat{\mathcal{U}}_A, \widehat{\mathcal{U}}_B$: Strategy Decision Operator of Player A and B

The Two-Player Quantum Wavefunction $|\Psi\rangle$

The Two-Player Quantum State $|\Psi\rangle$

$$|\Psi\rangle = \hat{\mathcal{J}}^{\dagger} \left(\hat{\mathcal{U}}_A \otimes \hat{\mathcal{U}}_B\right) \hat{\mathcal{J}} \; |s_1 \, s_1\rangle$$

$$\begin{aligned} \hat{\mathcal{U}}_A \quad &: \quad \text{Decision Operator of Player A} \\ \hat{\mathcal{U}}_B \quad &: \quad \text{Decision Operator of Player B} \\ \hat{\mathcal{J}} \quad &: \quad \text{Entangling Operator} \\ \hat{\mathcal{J}}^{\dagger} \quad &: \quad \text{Disentangling Operator} \\ \hat{\mathcal{J}} \; |s_1 \, s_1\rangle \quad &: \quad \text{Two-Player Initial State } (|\Psi_0\rangle) \end{aligned}$$

In words ...

The setup of the quantum game begins with the choice of the initial state $|\Psi_0\rangle$. After the two players have chosen their individual quantum strategies ($\hat{\mathcal{U}}_A := \hat{\mathcal{U}}(\theta_A, \varphi_A)$ and $\hat{\mathcal{U}}_B := \hat{\mathcal{U}}(\theta_B, \varphi_B)$) the disentangling operator $\hat{\mathcal{J}}^{\dagger}$ is acting to prepare the measurement.

The quantum decision state $|\psi\rangle_\mu$ of player $\mu = A, B$

To illustrate the operator formalism of quantum game theory and the concept of quantum strategies, we want to focus at first on the real and imaginary values of the two spinor components ψ_1^A and ψ_2^A of the of the state $|\psi\rangle_A$ of player A:

$$|\psi\rangle_A = \psi_1^A \left|s_1^A\right\rangle + \psi_2^A \left|s_2^A\right\rangle = \begin{pmatrix} \psi_1^A \\ -\psi_2^A \end{pmatrix} \in \mathcal{H}_A$$

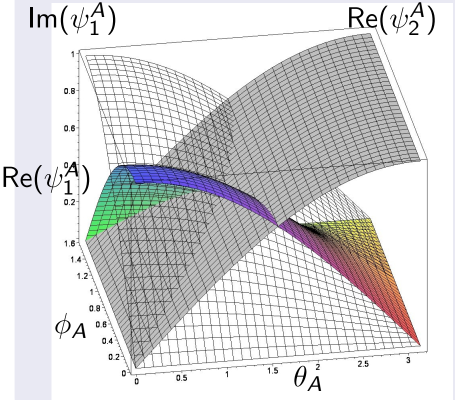
$$\left|s_1^A\right\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} , \left|s_2^A\right\rangle = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$|\psi\rangle_A = \widehat{\mathcal{U}}(\theta_A, \varphi_A) \left|s_1^A\right\rangle = \begin{pmatrix} e^{i\,\varphi_A} \cos(\frac{\theta_A}{2}) \\ -\sin(\frac{\theta_A}{2}) \end{pmatrix}$$

$$\widehat{\mathcal{U}}(\theta, \varphi) := \begin{pmatrix} e^{i\,\varphi} \cos(\frac{\theta}{2}) & \sin(\frac{\theta}{2}) \\ -\sin(\frac{\theta}{2}) & e^{-i\,\varphi} \cos(\frac{\theta}{2}) \end{pmatrix} \;\forall\; \theta \in [0, \pi] \wedge \varphi \in [0, \frac{\pi}{2}]$$

The quantum decision state $|\psi\rangle_\mu$ of player $\mu = A, B$

Real and imaginary parts of $|\psi\rangle_A$



Quantum state of player A:

$$|\psi\rangle_A = \psi_1^A \left|s_1^A\right\rangle + \psi_2^A \left|s_2^A\right\rangle = \begin{pmatrix} \psi_1^A \\ -\psi_2^A \end{pmatrix} \in \mathcal{H}_A$$

$$\text{with:} \quad \left|s_1^A\right\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} , \quad \left|s_2^A\right\rangle = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

s_1-quantum strategies and the decision operator $\widehat{\mathcal{U}}(\theta, \varphi)$:

$$|\psi\rangle_A = \widehat{\mathcal{U}}(\theta_A, \varphi_A) \left|s_1^A\right\rangle = \begin{pmatrix} e^{i\,\varphi_A} \cos(\frac{\theta_A}{2}) \\ -\sin(\frac{\theta_A}{2}) \end{pmatrix}$$

$$\widehat{\mathcal{U}}(\theta, \varphi) := \begin{pmatrix} e^{i\,\varphi} \cos(\frac{\theta}{2}) & \sin(\frac{\theta}{2}) \\ -\sin(\frac{\theta}{2}) & e^{-i\,\varphi} \cos(\frac{\theta}{2}) \end{pmatrix}$$

$$\forall \quad \theta \in [0, \pi] \ \wedge \ \varphi \in [0, \frac{\pi}{2}]$$

The 2-player state $|\Psi\rangle$ and the entangling operator $\widehat{\mathcal{J}}(\gamma)$

$$|\Psi\rangle = \hat{\mathcal{J}}^{\dagger} \left(\hat{\mathcal{U}}_A \otimes \hat{\mathcal{U}}_B \right) \hat{\mathcal{J}} \left| s_1^A s_1^B \right\rangle$$

$$\widehat{\mathcal{J}} := e^{i \frac{\gamma}{2} (\widehat{s_1} \otimes \widehat{s_1})} = \begin{pmatrix} \cos\left(\frac{\gamma}{2}\right) & 0 & 0 & i\sin\left(\frac{\gamma}{2}\right) \\ 0 & \cos\left(\frac{\gamma}{2}\right) & -i\sin\left(\frac{\gamma}{2}\right) & 0 \\ 0 & -i\sin\left(\frac{\gamma}{2}\right) & \cos\left(\frac{\gamma}{2}\right) & 0 \\ i\sin\left(\frac{\gamma}{2}\right) & 0 & 0 & \cos\left(\frac{\gamma}{2}\right) \end{pmatrix}$$

$$\gamma \in [0, \frac{\pi}{2}] \, , \quad \left| s_1^A s_1^B \right\rangle := \left| s_1^A \right\rangle \otimes \left| s_1^B \right\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Interpretation

The most important, but also most difficult mathematical concept in QGT is the two player quantum state $|\Psi\rangle$. It is formally constructed with the use of the decision operators $\hat{\mathcal{U}}_A$ and $\hat{\mathcal{U}}_B$ of player A and B and the entangling and disentangling operator $\hat{\mathcal{J}}$ and $\hat{\mathcal{J}}^\dagger$. $|\Psi\rangle$ is an spinor in a complex valued, 4-dimensional, abstract mathematical space called the 2-player "Hilbertspace" $\mathcal{H}$. The space of all conceivable decision paths is extended from the purely rational, measurable space in the Hilbertspace of complex numbers. Trough the concept of a potential entanglement of the imaginary quantum strategy parts, it is possible to include cooperate decision path, caused by cultural or moral standards. QGT is therefore a model which goes beyond *Homo Economicus* and the parameter γ, which is a measure for the strength of entanglement and fellow feeling, describes how strongly the players behave as a collektive state (*Homo Sociologicus* or *Homo Transzendentalis*).

The extended payoff $\$_\mu(\theta_A, \varphi_A, \theta_B, \varphi_B)$ of player $\mu = A, B$

The extended payoff $\$_\mu(\theta_A, \varphi_A, \theta_B, \varphi_B, \gamma)$ of player $\mu = A, B$ is an amplification of the classical mixed strategy payoff function $\tilde{\$}^\mu(\tilde{s}^A, \tilde{s}^B)$:

$$\begin{aligned} \$_A &= \$^A_{11}\, P_{11} + \$^A_{12}\, P_{12} + \$^A_{21}\, P_{21} + \$^A_{22}\, P_{22} \\ \$_B &= \$^B_{11}\, P_{11} + \$^B_{12}\, P_{12} + \$^B_{21}\, P_{21} + \$^B_{22}\, P_{22} \\ \text{with:} \quad & P_{\sigma\sigma'} = |\langle \sigma\sigma' | \Psi \rangle|^2 \;, \quad \sigma = \left\{ s^A_1, s^A_2 \right\} \text{ and } \sigma' = \left\{ s^B_1, s^B_2 \right\} \end{aligned}$$

$P_{\sigma\sigma'}$ are the real valued probabilities of finding the two player state $|\Psi\rangle$ in the pure strategy Eigenstate $|\sigma\sigma'\rangle$, e.g.

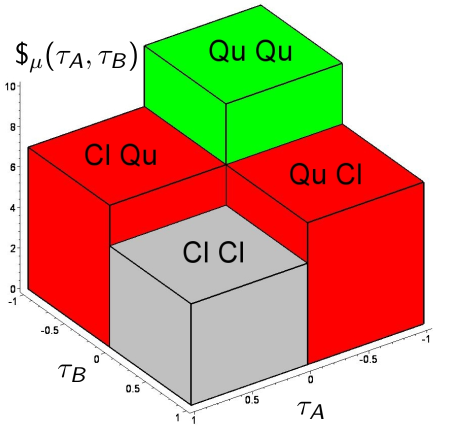
$$P_{12} := P_{s^A_1 s^B_2} = \left| \left\langle s^A_1 s^B_2 | \Psi \right\rangle \right|^2$$

The extended payoff $\$_\mu(\tau_A, \tau_B)$ of player $\mu = A, B$

In contrast to the classical mixed payoff functions ($\tilde{\$}^A(\tilde{s}^A, \tilde{s}^B)$ and $\tilde{\$}^B(\tilde{s}^A, \tilde{s}^B)$), which depend only on the two parameters $\tilde{s}^A$ and $\tilde{s}^B$, the quantum version of the mixed strategy payoff function depends on five parameters; namely the four decision angles ($\theta_A, \varphi_A, \theta_B$ and φ_B) and the entangling parameter γ. In order to visualize the payoff function as a surface in a three dimensional space it is necessary to reduce the set of parameters in the final state:
$|\Psi\rangle = |\Psi_f(\theta_A, \varphi_A, \theta_B, \varphi_B)\rangle \to |\Psi(\tau_A, \tau_B)\rangle$. The two strategy angles θ and φ depend only on a single parameter $\tau \in [-1, 1]$. Positive τ-values represent pure and mixed classical strategies, whereas negative τ-values correspond to quantum strategies, where $\theta = 0$ and $\varphi > 0$. The whole strategy space is separated into four regions, namely the absolute classical region (*ClCl*: $\tau_A, \tau_B \geq 0$), the absolute quantum region (*QuQu*: $\tau_A, \tau_B < 0$) and the two partially classical-quantum regions (*ClQu*: $\tau_A \geq 0 \wedge \tau_B < 0$ and *QuCl*: $\tau_A < 0 \wedge \tau_B \geq 0$).

The extended payoff $\$_\mu(\tau_A, \tau_B)$ of player $\mu = A, B$

Visualisationspace of $\$_\mu(\tau_A, \tau_B)$



The expected payoff within a quantum version of a general 2-player game:

$$
\begin{aligned}
\$_A &= \$^A_{11}\, P_{11} + \$^A_{12}\, P_{12} + \$^A_{21}\, P_{21} + \$^A_{22}\, P_{22} \\
\$_B &= \$^B_{11}\, P_{11} + \$^B_{12}\, P_{12} + \$^B_{21}\, P_{21} + \$^B_{22}\, P_{22} \\
\text{with:} \quad P_{\sigma\sigma'} &= |\langle \sigma\sigma' | \Psi \rangle|^2 \quad , \quad \sigma, \sigma' = \{s_1, s_2\}
\end{aligned}
$$

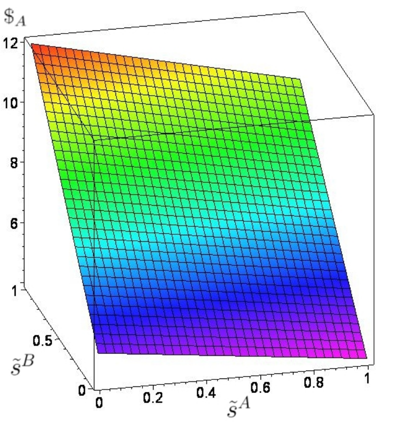
Reduction of quantum strategies:

$|\Psi\rangle = |\Psi(\theta_A, \varphi_A, \theta_B, \varphi_B)\rangle \rightarrow |\Psi(\tau_A, \tau_B)\rangle$

$$
\underbrace{\{(\tau\,\pi, 0)\mid \tau \in [0,1]\}}_{\text{classical region } Cl} \;\wedge\; \underbrace{\{(0, \tau\,\tfrac{\pi}{2})\mid \tau \in [-1,0[\}}_{\text{quantum region } Qu}
$$

Quantum extension of dominant class games

Classical payoff for player A



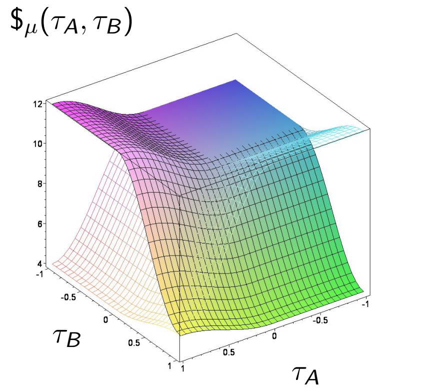
A\B	s_1^B	s_2^B
s_1^A	(10,10)	(4,12)
s_2^A	(12,4)	(5,5)

Table : Payoffmatrix of a dominant, prisoners dilemma like game.

This dominant, prisoners dilemma like game has only one pure, symmetric Nash equilibrium (s_2^A, s_2^B) which is the only ESS of the evolutionary game.

Quantum extension of dominant class games

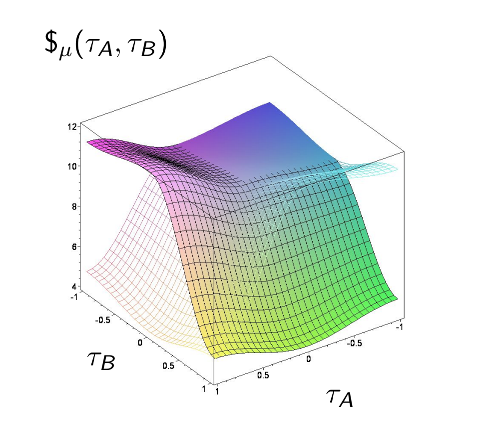
Payoff of player A (colored) and player B (wired) for $\gamma = 0$ (no entanglement)



The diagram clearly exhibits that the non-entangled quantum game simply describes the classical version of the prisoner's dilemma game. For the case, that both players decide to play a quantum strategy $(\tau_A < 0 \wedge \tau_B < 0)$ their payoff is equal to the case where both players choose the classical pure strategy s_1 $(\$_A(\tau_A = 0, \tau_B = 0) = 10)$. The classical Nash equilibrium $((s_2^A, s_2^B)$, the dominant strategy) corresponds to the following τ-values:$(s_2^A, s_2^B) \hat{=} (\tau_A = 1, \tau_B = 1)$.

Quantum extension of dominant class games

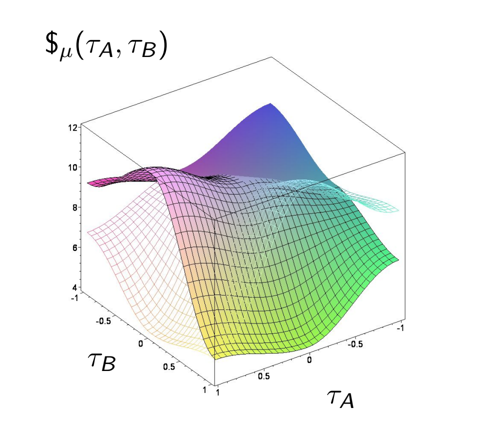
Payoff of player A (colored) and player B (wired) for $\gamma = \frac{\pi}{10} \approx 0.31$



For the absolute classical region *ClCl* the shape of the surfaces does not change, whereas for the partially classical-quantum (*ClQu* and *QuCl*) and absolute quantum region regions *QuQu* the payoff structure changes due to a possible interference of quantum strategies within Hilbertspace. The structure of Nash-equilibria does not change for the left picture, whereas for the following pictures the previously present dominant strategy of the prisoner's dilemma game has disappeared and a new, advisable quantum Nash-equilibrium will appear at $(\widehat{Q}, \widehat{Q} \hat{=} (\tau_A = -1, \tau_B = -1))$. During the transition from this figure to the next picture two separate phenomena occur. At first, for an entanglement value $\gamma_1 \approx 0.37$, the best response for player A to the strategy $s_2^B \hat{=} \tau_B = 1$ is no longer the strategy $s_2^A \hat{=} \tau_A = 1$, as $\$_A(\tau_A = -1, \tau_B = 1) \approx 5.05$ is now higher than $\$_A(\tau_A = 1, \tau_B = 1) = 5$. Secondly, for an entanglement value $\gamma_2 \approx 0.53$, the best response for player A to the strategy $\widehat{Q}_B \hat{=} \tau_B = -1$ is no longer the strategy $s_2^A \hat{=} \tau_A = 1$, as $\$_A(\tau_A = 1, \tau_B = -1) \approx 9.96$ is for $\gamma_2 = 0.53$ lower than $\$_A(\tau_A = -1, \tau_B = -1) = 10$.

Quantum extension of dominant class games

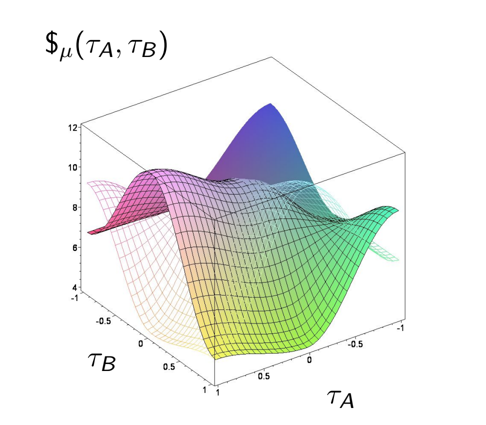
Payoff of player A (colored) and player B (wired) for $\gamma = \frac{\pi}{8} \approx 0.52$



For the absolute classical region *ClCl* the shape of the surfaces does not change, whereas for the partially classical-quantum (*ClQu* and *QuCl*) and absolute quantum region regions *QuQu* the payoff structure changes due to a possible interference of quantum strategies within Hilbertspace. The structure of Nash-equilibria did not change for the last figure, whereas for this and thee following pictures the previously present dominant strategy of the prisoner's dilemma game has disappeared and a new, advisable quantum Nash-equilibrium has appeared $(\widehat{Q}, \widehat{Q} \hat{=} (\tau_A = -1, \tau_B = -1))$. During the transition from the last picture to this figure two separate phenomena occurred. At first, for an entanglement value $\gamma_1 \approx 0.37$, the best response for player A to the strategy $s_2^B \hat{=} \tau_B = 1$ is no longer the strategy $s_2^A \hat{=} \tau_A = 1$, as $\$_A(\tau_A = -1, \tau_B = 1) \approx 5.05$ is now higher than $\$_A(\tau_A = 1, \tau_B = 1) = 5$. Secondly, for an entanglement value $\gamma_2 \approx 0.53$, the best response for player A to the strategy $\widehat{Q}_B \hat{=} \tau_B = -1$ is no longer the strategy $s_2^A \hat{=} \tau_A = 1$, as $\$_A(\tau_A = 1, \tau_B = -1) \approx 9.96$ is for $\gamma_2 = 0.53$ lower than $\$_A(\tau_A = -1, \tau_B = -1) = 10$.

Quantum extension of dominant class games

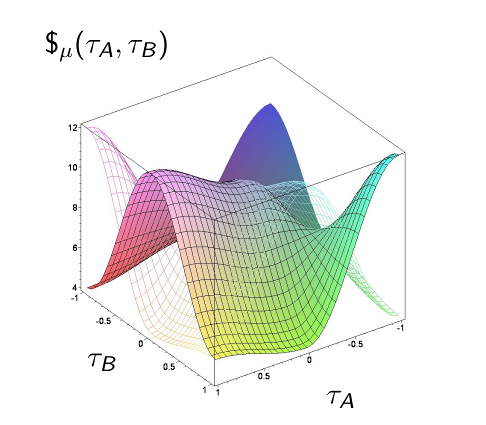
Payoff of player A (colored) and player B (wired) for $\gamma = \frac{\pi}{6} \approx 0.94$



The results show, that a quantum extension of a classical prisoner's dilemma game is able to change the structure of Nash-equilibria, and even previously present dominant strategies could become nonexistent, if the value of entanglement increases further than a defined γ-threshold. Players with a higher strategic entanglement value γ escape the dilemma as they see the advantage of the quantum strategy combination $(\widehat{Q}_A, \widehat{Q}_B)$, which is measured as if both are playing the classical strategy s_2.

Quantum extension of dominant class games

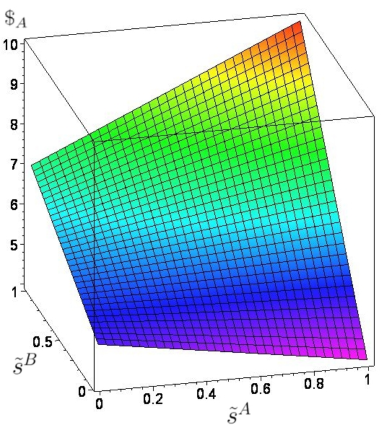
Payoff of player A (colored) and player B (wired) for $\gamma = \frac{\pi}{2} \approx 1.57$



The results show, that a quantum extension of a classical prisoner's dilemma game is able to change the structure of Nash-equilibria, and even previously present dominant strategies could become nonexistent, if the value of entanglement increases further than a defined γ-threshold. Players with a higher strategic entanglement value γ escape the dilemma as they see the advantage of the quantum strategy combination $(\widehat{Q}_A, \widehat{Q}_B)$, which is measured as if both are playing the classical strategy s_2.

Quantum extension of coordination class games

Classical payoff for player A



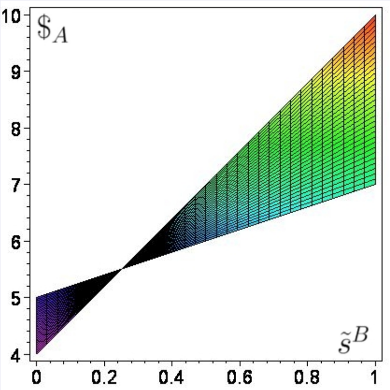
A\B	s_1^B	s_2^B
s_1^A	(10,10)	(4,7)
s_2^A	(7,4)	(5,5)

Table : Payoffmatrix of a coordination game.

This coordination game has two pure, symmetric Nash equilibria and one interior NE at $s^\star = \frac{1}{4}$. The evolutionary game game has two ESSs.

Quantum extension of coordination class games

Classical payoff for player A (projected)



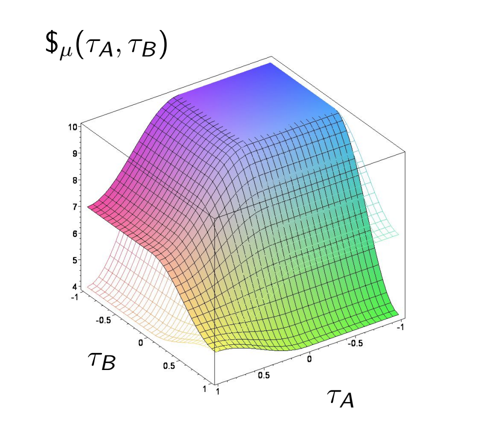
A\B	s_1^B	s_2^B
s_1^A	(10,10)	(4,7)
s_2^A	(7,4)	(5,5)

Table : Payoffmatrix of a coordination game.

This coordination game has two pure, symmetric Nash equilibria and one interior NE at $s^\star = \frac{1}{4}$. The evolutionary game game has two ESSs.

Quantum extension of coordination class games

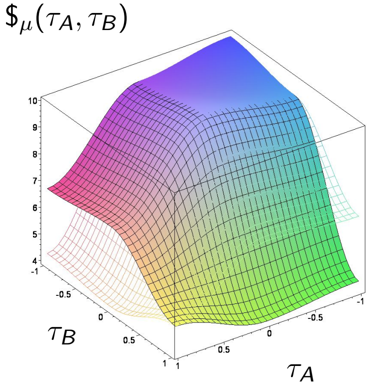
Payoff of player A (colored) and player B (wired) for $\gamma = 0$ (no entanglement)



Again, the diagram clearly indicates that the non-entangled quantum game is identical to the classical version of the underlying coordination game. For the case, that both players decide to play a quantum strategy ($\tau_A < 0 \wedge \tau_B < 0$) their payoff is equal to the case where both players choose the classical pure strategy s_1 ($\$_A(\tau_A = 0, \tau_B = 0) = 10$), with the overall highest possible payoff. The classical pure Nash equilibria correspond to the following τ-values: $(s_1^A, s_1^B) \hat{=} (\tau_A = 0, \tau_B = 0)$ and $(s_2^A, s_2^B) \hat{=} (\tau_A = 1, \tau_B = 1)$, whereas the classical mixed strategy equilibrium is at: $\tau^\star = \frac{2}{\pi} \arccos(\sqrt{\frac{1}{4}}) = \frac{2}{3}$.

Quantum extension of coordination class games

Payoff of player A (colored) and player B (wired) for $\gamma = \frac{\pi}{10} \approx 0.31$



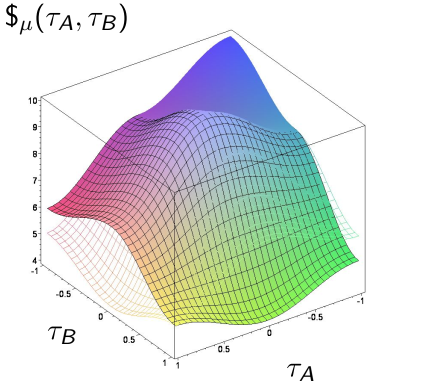
Even for tiny values of γ a new quantum Nash-equilibrium appears $(\tau_A = -1, \tau_B = -1)$.

At moderate values of γ the low payoff evolutionary stable strategy $(\tau_A = 1, \tau_B = 1)$ disappears.

The specific γ-value at which this disappearance happens, depends on the whole set of payoff parameters and not only on a and b.

Quantum extension of coordination class games

Payoff of player A (colored) and player B (wired) for $\gamma = \frac{\pi}{8} \approx 0.52$



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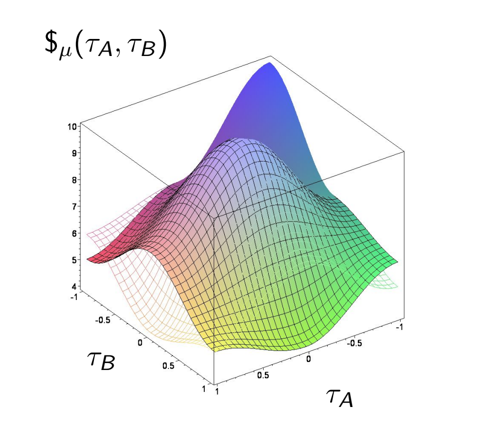
Quantum extension of coordination class games

Payoff of player A (colored) and player B (wired) for $\gamma = \frac{\pi}{6} \approx 0.94$



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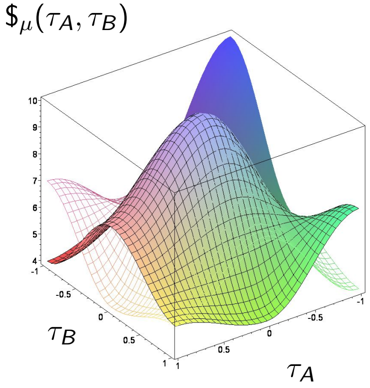
Quantum extension of coordination class games

Payoff of player A (colored) and player B (wired) for $\gamma = \frac{\pi}{2} \approx 1.57$



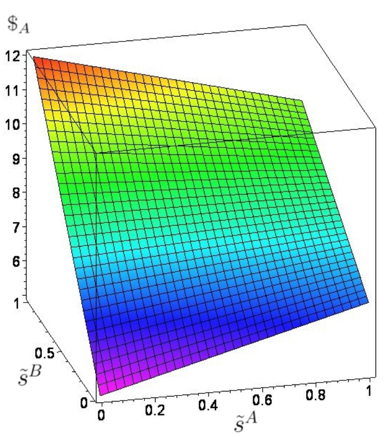
Even for tiny values of γ a new quantum Nash-equilibrium appears $(\tau_A = -1, \tau_B = -1)$.

At moderate values of γ the low payoff evolutionary stable strategy $(\tau_A = 1, \tau_B = 1)$ disappears.

The specific γ-value at which this disappearance happens, depends on the whole set of payoff parameters and not only on a and b.

Quantum extension of anti-coordination class games

Classical payoff for player A



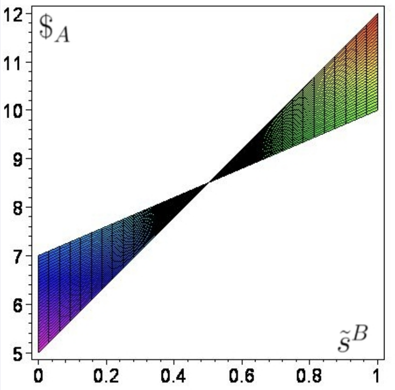
A\B	s_1^B	s_2^B
s_1^A	(10,10)	(7,12)
s_2^A	(12,7)	(5,5)

Table : Payoffmatrix of a coordination game.

This anti-coordination game has two pure, unsymmetric Nash equilibria and one interior NE at $s^\star = \frac{1}{2}$. The evolutionary game game has one mixed strategy ESS.

Quantum extension of anti-coordination class games

Classical payoff for player A (projected)



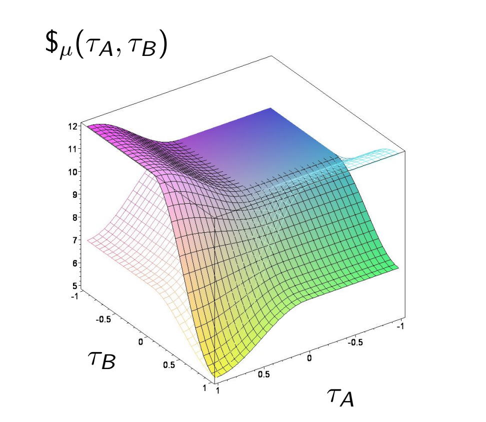
A\B	s_1^B	s_2^B
s_1^A	(10,10)	(7,12)
s_2^A	(12,7)	(5,5)

Table : Payoffmatrix of a coordination game.

This anti-coordination game has two pure, unsymmetric Nash equilibria and one interior NE at $s^\star = \frac{1}{2}$. The evolutionary game game has one mixed strategy ESS.

Quantum extension of anti-coordination class games

Payoff of player A (colored) and player B (wired) for $\gamma = 0$



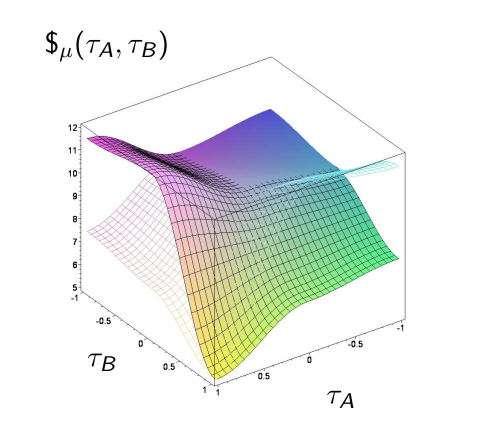
Beside the mixed strategy evolutionary stable strategy, a new quantum ESS appears at a specific γ-value.

For details see:

- M. Hanauske, *Advances in Evolutionary Game Theory*, 2009, *Lecture at the 'Université Lumière Lyon 2' in Lyon, France (MINERVE Exchange Program); Slides and additional material*
- *M. Hanauske, J. Kunz, S. Bernius, and W. König. ,Doves and hawks in economics revisited: An evolutionary quantum game theory-based analysis of financial crises. , 2009, to appear in Physica A, arXiv:0904.2113, RePEc:pra:mprapa:14680 and SSRN$_{id}$:1597735 .*

Quantum extension of anti-coordination class games

Payoff of player A (colored) and player B (wired) for $\gamma = \frac{\pi}{10} \approx 0.31$



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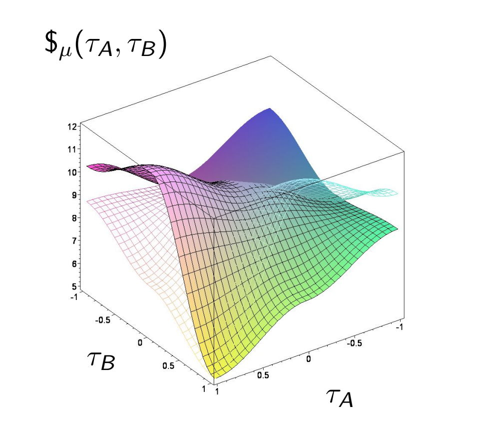

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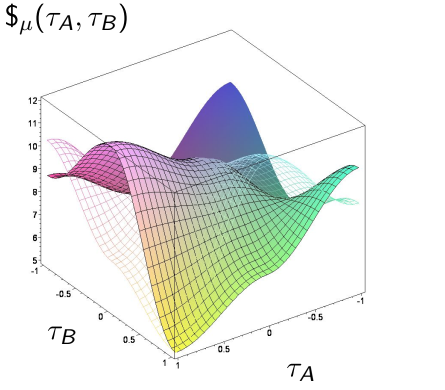

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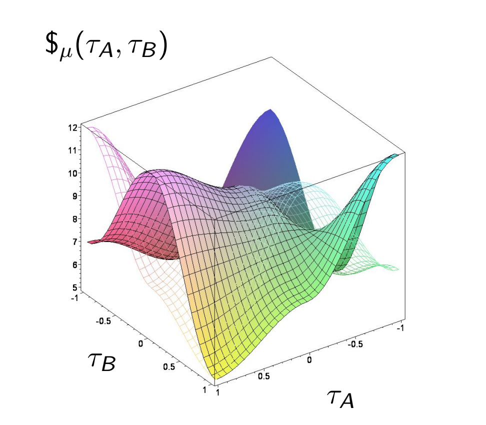
Payoff of player A (colored) and player B (wired) for $\gamma = \frac{\pi}{2} \approx 1.57$



Beside the mixed strategy evolutionary stable strategy, a new quantum ESS appears at a specific γ-value.

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PHD-Thesis: Evolutionäre Quanten Spieltheorie im Kontext sozio-ökonomischer Systeme

Articles of my cumulative PHD-Thesis

- Article 0: Evolutionary Quantum Game Theory
- Article 1: Quantum Game Theory and Open Access Publishing
- Article 2: Evolutionary Quantum Game Theory and Scientific Communication
- Article 3: Doves and hawks in economics revisited: *An evolutionary quantum game theory-based analysis of financial crises*
- Article 4: Experimental Validation of Quantum Game Theory
- Article 5: Evolutionary Game Theory and Complex Networks of Scientific Information

Summary

Summary of the talk

Quantum game theory is a mathematical and conceptual amplification of classical game theory. The space of all conceivable decision paths is extended from the purely rational, measurable space in the Hilbertspace of complex numbers. Trough the concept of a potential entanglement of the imaginary quantum strategy parts, it is possible to include corporate decision path, caused by cultural or moral standards. If this strategy entanglement is large enough, then, additional Nash-equilibria can occur, previously present dominant strategies could become nonexistent and new evolutionary stable strategies can appear.

Within this talk the framework of Quantum Game Theory was described in detail. The formal mathematical model, the different concepts of equilibria and the various classes of quantum games have been defined, explained and visualized to understand the main ideas of Quantum Game Theory. Additionally some applications where discussed at the end of the talk.

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Matthias Hanauske.

Evolutionary Game Theory and Complex Networks of Scientific Information.

Article is going to be published as a chapter in the book *"Models of science dynamics - Encounters between complexity theory and information science", Spinger book in the Complexity series, Editors: Andrea Scharnhorst, Katy Börner and Peter van den Besselaar*, year = 2011.

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