Introduction to Quantum Game Theory

Dr. phil.nat. Matthias Hanauske

Johann Wolfgang Goethe-University
Institute for Information Systems
Mertonstr. 17, 60054 Frankfurt/Main

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Motivation (I)

In 1928, Johann (John) von Neumann published the first article on *Game Theory* ("Zur Theorie der Gesellschaftsspiele", [13]). Beside this research field Mr.von Neumann was mainly working on the mathematical foundations of *Quantum Theory*. In the year 1932 he published a book ("Mathematische Grundlagen der Quantenmechanik", [11]), which was one of the first works formulating the mathematics of quantum theory, which was (from that time on) used as the mathematical toolbox to describe all of the known physical interactions between elementary particles. In the year 1933 Mr. von Neumann emigrated from Germany to the USA and started to work at the "Institute for Advance Study" in Princeton. The first book about game theory was published in 1944 by von Neumann and Morgenstern [12].

Motivation (II)

In the same year (1933) as Mr. von Neumann emigrated, Mr. Albert Einstein emigrated as well an both of them where working at the "Institute for Advance Study" in Princeton. Beside his main research field (the unification of "General Relativity" with the other three known fundamental forces) Mr. Einstein also worked on the interpretation of quantum theory. In the year 1935, together with Mr. Boris Podolsky and Mr. Nathan Rosen, he published an article ("Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?", [1]), which introduced a new quantum property called *Quantum Entanglement*. Mr. Einstein was very skeptical about quantum theory and specially about this strange property which he used to call "Spukhafte Fernwirkung". However the article was later extensively discussed as the "EPR-Paradoxon" and much later, in the 70th (after the dead of Mr. Einstein) this strange quantum property was experimentally proven.

Motivation (III)

Today, quantum entanglement is used in several practical applications (e.g. quantum cryptography, quantum computation) and it is also a basic idea in *Quantum Game Theory*. In the year 1999 Mr. Jens Eisert wrote the first article about quantum game theory and presented a a new theory, which unifies the two mathematical theories of von Neumann (Game Theory and Quantum Theory) with the Einstein's quantum entanglement concept.

Research Questions

Mathematical description of Quantum Game Theory

What are the main mathematical concepts of quantum game theory? How are the theories (Game Theory and Quantum Theory) unified?

Results for Quantum Games within different game classes

What are the main differences between classical and quantum game theory. Is the underlying Nash equilibrium structure of (2 player)-(2 strategy) games changed within a quantum game theory-based analysis?

Presentation of various applications

How can quantum game theory be applied to real game situations?

Focus of the cumulative phd-thesis

This talk summarizes the main outcomes of my cumulative phd-thesis, which will be submitted soon to the economic department of the *Johann Wolfgang Goethe University*.

Focus of the PHD-Thesis

The thesis summarizes the main results of classical and quantum game theory and focuses on the different game categories of (2 player)-(2 strategy) evolutionary games. Additionally, five different applications are discussed.

Structure of the cumulative phd-thesis

Structure of the cumulative phd-thesis

The thesis exists beside the "German Introduction" of six separate articles:

- Introductory Paper: Evolutionary Quantum Game Theory [3]
- Article 1: Quantum Game Theory and Open Access Publishing [4]
- Article 2: Evolutionary Quantum Game Theory and Scientific Communication [8]
- Article 3: Doves and hawks in economics revisited: *An evolutionary quantum game theory-based analysis of financial crises* [6]
- Article 4: Experimental Validation of Quantum Game Theory [5]
- Article 5: Evolutionary Game Theory and Complex Networks of Scientific Information[7]

Anfänge der Quantentheorie

Was ist Quantentheorie

Die Quantentheorie ist eine physikalische Theorie, die das Verhalten der Materie im atomaren und subatomaren Bereich beschreibt. Sie ist eine der Hauptsäulen der modernen Physik und bildet die Grundlage für viele ihrer Teilgebiete, so z.B. für die Atomphysik, die Festkörperphysik, die Kern- und Elementarteilchenphysik, ...
Die wesentlichen Konzepte der Quantentheorie wurden in den 20er Jahren des 20. Jahrhunderts erarbeitet.

Begründer der Quantentheorie

Begründer der Quantenmechanik waren Werner Heisenberg und Erwin Schrödinger. Weitere wichtige Beiträge wurden unter anderem von Max Born, Pascual Jordan, Wolfgang Pauli, Niels Bohr, Paul Dirac und John von Neumann geleistet.

Quantisierte Messgrößen

Beispiel: Das Wasserstoffatom

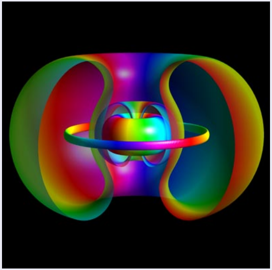


Figure: Aufenthaltswahrscheinlichkeit des Elektrons im Wasserstoffatom (n=4,l=2,m=2). Quelle: Bernd Thaller, Visual Quantum Mechanics

Der Zustand eines Elektrons im Wasserstoffatom wird mit Hilfe der stationären Schrödingergleichung berechnet. Die messbaren Eigenzustände des Elektrons ($\psi_{nlm}(\vec{r})$) sind durch ihre Quantenzahlen (n,l,m) quantisiert, d.h. Messgrößen wie z.B. die Energie können nur diskrete Werte annehmen. Der allgemeine Elektronenzustand ergibt sich durch Überlagerung (Superposition) der Eigenzustände ($a_{nlm} \in \mathbb{C}$).

$$\psi = \sum_{n=1}^{\infty} \sum_{l=0}^{n-1} \sum_{m=-l}^{l} a_{nlm} \, \psi_{nlm}$$

Welle-Teilchen-Dualismus

Das Doppelspaltexperiment

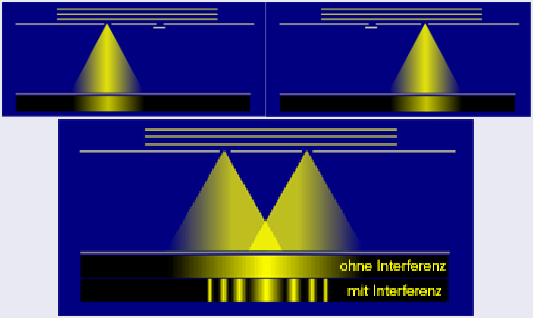


Figure: Beim Doppelspaltexperiment offenbaren Teilchen ihre Welleneigenschaften. Quelle: Michael Craiss

1961 wurde das Doppelspaltexperiment mit Elektronen durch Claus Jönsson durchgeführt und im September 2002 in einer Umfrage der englischen physikalischen Gesellschaft in der Zeitschrift 'Physics World' zum schönsten physikalischen Experiment aller Zeiten gewählt.

Superpositionen von Eigenzuständen

Schrödingers Katze

Figure: Theoretische Versuchsanordnung des Gedankenexperiments.

In einem geschlossenen Kiste befindet sich ein instabiler Atomkern, der innerhalb einer bestimmten Zeitspanne mit einer gewissen Wahrscheinlichkeit zerfällt. Im Falle eines Zerfalls werde Giftgas freigesetzt, was eine im Raum befindliche Katze tötet. Bevor ein Beobachter die Kiste öffnet, schwebt der Zustand ψ der Katze zwischen den Eigenzuständen '$\psi_1 :=$ Lebend' und '$\psi_2 :=$ Tot'.

$$\psi = \frac{1}{\sqrt{2}} (\psi_1 + \psi_2)$$

Mathematische Grundlagen der Quantentheorie (I)

Quantenzustände im Hilbertraum

Ein allgemeiner Quantenzustand $|\psi\rangle$ (Diracsche BraKet Formulierung) wird mathematisch als ein Vektor in einem komplexen Raum beschrieben, dem sogenannten Hilbertraum $\mathcal{H}$. Die orthonormalen Eigenzustände $|\psi_n\rangle$ bilden eine Basis dieses Hilbertraumes, so dass ein beliebiger Quantenzustand $|\psi\rangle$ stehts wie folgt dargestellt werden kann:

$$|\psi\rangle = \sum_n a_n \, |\psi_n\rangle \ , \quad a_n \in \mathbb{C}$$

Das Betragsquadrat der Gewichtungskoeffizienten a_n gibt die Wahrscheinlichkeit an, das Quantensystem im Eigenzustand $|\psi_n\rangle$ zu finden.

Mathematatische Grundlagen der Quantentheorie (II)

Operatoren, Eigenvektoren und Eigenwerte

Jede messbaren Eigenschaft a eines Quantensystems (z.B. Energie, Drehimpuls) entspricht einem linearen hermiteschen Operator $\hat{A}$. Die Messung einer physikalischen Größe wird durch das Anwenden des Operators $\hat{A}$ auf den Zustand $|\psi\rangle$ bewirkt. Dies verursacht den Kollaps der Zustandsfunktion $|\psi\rangle$ auf einen seiner Eigenvektoren $|\psi_a\rangle$. Der gemessene scharfe Wert der physikalischen Größe entspricht dem Eigenwert a des Operators $\hat{A}$.

$$\hat{A}\,|\psi_a\rangle = a\,|\psi_a\rangle \quad \text{bzw.} \quad \hat{A}\,|a\rangle = a\,|a\rangle$$

Der Durchschnittswert $\bar{A}$ einer messbaren Größe berechnet sich wie folgt:

$$\bar{A} \equiv \left\langle \hat{A} \right\rangle = \langle\psi|\,\hat{A}\,|\psi\rangle = \int \psi^* \hat{A} \psi dV$$

Das Einstein-Podolsky-Rosen Paradoxon

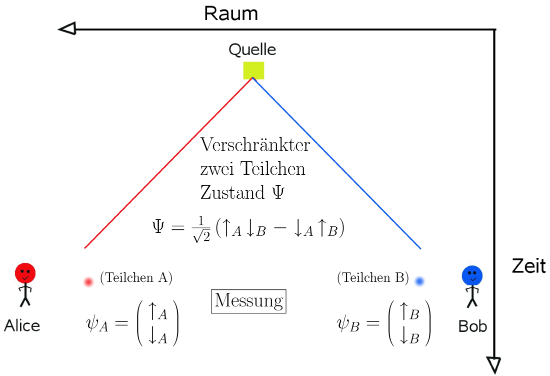


Figure: EPR Gedankenexperiment ('Bohm-Aharonov' Version). Der durch die Messung an Teilchen A verursachte instantane Kollaps der 2-Teilchen Wellenfunktion Ψ bestimmt den Zustand des Teilchens B.

Die Quantenverschränkung

Verschränkte 2-Niveau-Quantensysteme

Zwei Teilchen (A,B) haben die Möglichkeit zwischen zwei Zuständen ($\uparrow, \downarrow$) zu wählen. Die Basisvektoren der jeweiligen Hilberträume der Teilchen seien wie folgt definiert:

Zustand des Teilchens A: $\psi_A \in \mathcal{H}_A \equiv \mathbb{C}^2$, Basis: $\{\uparrow_A, \downarrow_A\}$

Zustand des Teilchens B: $\psi_B \in \mathcal{H}_B \equiv \mathbb{C}^2$, Basis: $\{\uparrow_B, \downarrow_B\}$

Der Hilbertraum des zusammengesetzten Systems ist ein komplexer vierdimensionaler Raum ($\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$). Der Gesamtzustand des 2-Teilchen Systems Ψ kann unter Umständen nicht in die jeweiligen Einzelzustände separiert werden.

Verschränkter Zustand (z.B.): $\Psi = \frac{1}{\sqrt{2}} (\uparrow_A \downarrow_B - \downarrow_A \uparrow_B)$

Definition of a (2 player)-(2 strategy) game Γ

An unsymmetric (2×2) game Γ is defined as ...

(2×2) Game:	$\Gamma := \left(\{A, B\}, \mathcal{S}^A \times \mathcal{S}^B, \hat{\$}_A, \hat{\$}_B \right)$
Set of pure strategies of player A and B:	$\mathcal{S}^A = \left\{ s_1^A, s_2^A \right\}, \ \mathcal{S}^B = \left\{ s_1^B, s_2^B \right\}$
Set of mixed strategies of player A and B:	$\tilde{\mathcal{S}}^A = \left\{ \tilde{s}_1^A, \tilde{s}_2^A \right\}, \ \tilde{\mathcal{S}}^B = \left\{ \tilde{s}_1^B, \tilde{s}_2^B \right\}$
Payoff matrix for player A:	$\hat{\$}_A = \begin{pmatrix} \$_{11}^A & \$_{12}^A \\ \$_{21}^A & \$_{22}^A \end{pmatrix}$
Payoff matrix for player B:	$\hat{\$}_B = \begin{pmatrix} \$_{11}^B & \$_{12}^B \\ \$_{21}^B & \$_{22}^B \end{pmatrix}$

The mixed strategy payoff function $\tilde{\$}^\mu$ of player $\mu = A, B$

The mixed strategy payoff function $\tilde{\$}^\mu$ of player $\mu = A, B$ is defined as follows:

$$\tilde{\$}^A(\tilde{s}^A, \tilde{s}^B) = (\tilde{s}^A)^T \hat{\$}^A \tilde{s}^B \quad , \quad \tilde{\$}^B(\tilde{s}^A, \tilde{s}^B) = (\tilde{s}^A)^T \hat{\$}^B \tilde{s}^B$$

For two possible strategies the equation reduces to the following explicit formulation:

$$\begin{aligned} \tilde{\$}^A(\tilde{s}^A, \tilde{s}^B) &= \$^A_{11} \tilde{s}^B_1 \tilde{s}^A_1 + \$^A_{12} \tilde{s}^B_2 \tilde{s}^A_1 + \$^A_{21} \tilde{s}^B_1 \tilde{s}^A_2 + \$^A_{22} \tilde{s}^B_2 \tilde{s}^A_2 \\ \tilde{\$}^B(\tilde{s}^A, \tilde{s}^B) &= \$^B_{11} \tilde{s}^B_1 \tilde{s}^A_1 + \$^B_{12} \tilde{s}^B_2 \tilde{s}^A_1 + \$^B_{21} \tilde{s}^B_1 \tilde{s}^A_2 + \$^B_{22} \tilde{s}^B_2 \tilde{s}^A_2 \end{aligned}$$

The mixed strategy payoff function $\tilde{\$}^{\mu}$ of player $\mu = A, B$

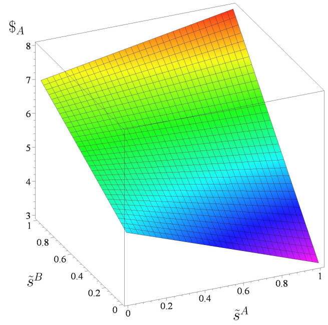
Due to the normalizing conditions
$\tilde{s}_1^{\mu} + \tilde{s}_2^{\mu} = 1 \; \forall \, \mu = A, B$
it is possible to simplify the functional dependence of the mixed strategy payoff function:

$$\begin{aligned} & \tilde{\$}^{\mu} : ([0,1] \times [0,1]) \to \mathbb{R} \\ \tilde{\$}^{\mu}(\tilde{s}^A, \tilde{s}^B) \; = \; & \$^{\mu}_{11} \tilde{s}^A \tilde{s}^B + \$^{\mu}_{12} \tilde{s}^A (1 - \tilde{s}^B) + \\ & + \$^{\mu}_{21} (1 - \tilde{s}^A) \tilde{s}^B + \$^{\mu}_{22} (1 - \tilde{s}^A)(1 - \tilde{s}^B) \end{aligned}$$

, where $\tilde{s}^A := \tilde{s}_1^A$, $\tilde{s}^B := \tilde{s}_1^B$, $\tilde{s}_2^A = 1 - \tilde{s}_1^A$ and $\tilde{s}_2^B = 1 - \tilde{s}_1^B$

The mixed strategy payoff function $\tilde{\$}^\mu$ of player $\mu = A, B$

Mixed strategy payoff function $\tilde{\$}^A\left(\tilde{s}^A, \tilde{s}^B\right)$ of player A
($\$^A_{11} = 8$, $\$^A_{12} = 5$, $\$^A_{21} = 7$, $\$^A_{22} = 3$)



Payoff $\tilde{\$}^\mu\left(\tilde{s}^A, \tilde{s}^B\right)$ as a function of $\tilde{s}^A, \tilde{s}^B \in [0,1]$:

$$\tilde{\$}^\mu : ([0,1] \times [0,1]) \rightarrow \mathbb{R}$$

$$\tilde{\$}^\mu(\tilde{s}^A, \tilde{s}^B) = \$^\mu_{11}\tilde{s}^A\tilde{s}^B + \$^\mu_{12}\tilde{s}^A(1-\tilde{s}^B) + $$
$$+\$^\mu_{21}(1-\tilde{s}^A)\tilde{s}^B + \$^\mu_{22}(1-\tilde{s}^A)(1-\tilde{s}^B)$$

, where $\tilde{s}^A := \tilde{s}^A_1$, $\tilde{s}^B := \tilde{s}^B_1$,

$\tilde{s}^A_2 = 1 - \tilde{s}^A_1$ and $\tilde{s}^B_2 = 1 - \tilde{s}^B_1$

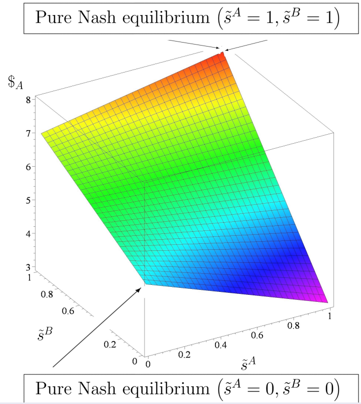
Payoff $\tilde{\$}^\mu\left(\tilde{\mathcal{S}}^A \times \tilde{\mathcal{S}}^B\right)$ as a function of the sets of mixed strategies for player A and B:

$$\tilde{\$}^\mu : \left(\tilde{\mathcal{S}}^A \times \tilde{\mathcal{S}}^B\right) \rightarrow \mathbb{R}$$

$$\tilde{\$}^\mu((\tilde{s}^A_1, \tilde{s}^A_2), (\tilde{s}^B_1, \tilde{s}^B_2)) = \$^\mu_{11}\tilde{s}^A_1\tilde{s}^B_1 + \$^\mu_{12}\tilde{s}^A_1\tilde{s}^B_2 + $$
$$+\$^\mu_{21}\tilde{s}^A_2\tilde{s}^B_1 + \$^\mu_{22}\tilde{s}^A_2\tilde{s}^B_2$$

Nash equilibria (NE)

Nash equilibria and $\tilde{\$}^{\mu}\left(\tilde{s}^A, \tilde{s}^B\right)$



A strategy combination $(\tilde{s}^{A*}, \tilde{s}^{B*})$ is called a Nash equilibrium, if:

$$\tilde{\$}^A(\tilde{s}^{A*}, \tilde{s}^{B*}) \geq \tilde{\$}^A(\tilde{s}^{A}, \tilde{s}^{B*}) \quad \forall \ \tilde{s}^A \in [0,1]$$
$$\tilde{\$}^B(\tilde{s}^{A*}, \tilde{s}^{B*}) \geq \tilde{\$}^B(\tilde{s}^{A*}, \tilde{s}^{B}) \quad \forall \ \tilde{s}^B \in [0,1]$$

A strategy combination $(\tilde{s}^{A\star}, \tilde{s}^{B\star})$ is called an interior (mixed strategy) Nash equilibrium, if:

$$\left.\frac{\partial \tilde{\$}^A(\tilde{s}^A, \tilde{s}^B)}{\partial \tilde{s}^A}\right|_{\tilde{s}^B = \tilde{s}^{B\star}} = 0 \quad \forall \ \tilde{s}^A \in [0,1] \ , \ \tilde{s}^{B\star} \in]0,1[$$
$$\left.\frac{\partial \tilde{\$}^B(\tilde{s}^A, \tilde{s}^B)}{\partial \tilde{s}^B}\right|_{\tilde{s}^A = \tilde{s}^{A\star}} = 0 \quad \forall \ \tilde{s}^B \in [0,1] \ , \ \tilde{s}^{A\star} \in]0,1[$$

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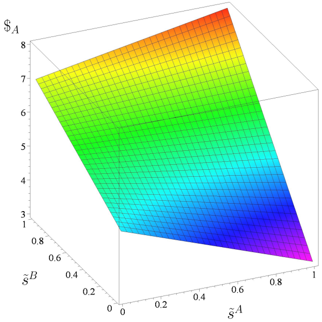

A strategy combination $(\tilde{s}^{A*}, \tilde{s}^{B*})$ is called a Nash equilibrium, if:

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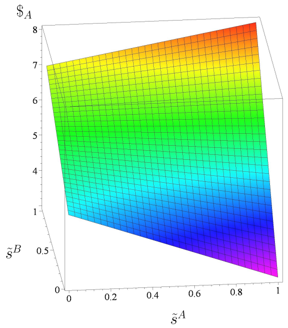

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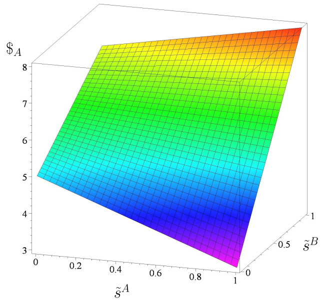

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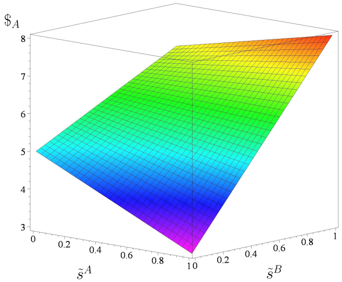

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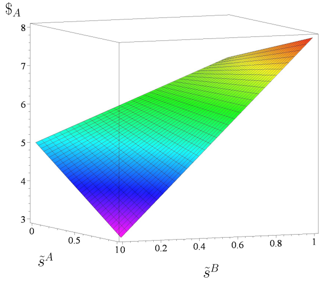

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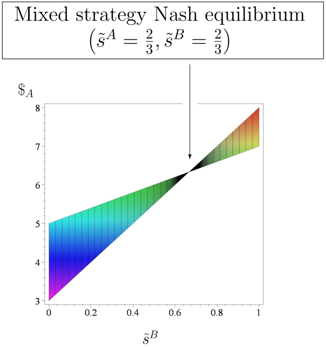
Nash equilibria and $\tilde{\$}^\mu\left(\tilde{s}^A, \tilde{s}^B\right)$



A strategy combination $(\tilde{s}^{A*}, \tilde{s}^{B*})$ is called a Nash equilibrium, if:

$$\tilde{\$}^A(\tilde{s}^{A*}, \tilde{s}^{B*}) \geq \tilde{\$}^A(\tilde{s}^{A}, \tilde{s}^{B*}) \quad \forall \; \tilde{s}^A \in [0,1]$$
$$\tilde{\$}^B(\tilde{s}^{A*}, \tilde{s}^{B*}) \geq \tilde{\$}^B(\tilde{s}^{A*}, \tilde{s}^{B}) \quad \forall \; \tilde{s}^B \in [0,1]$$

A strategy combination $(\tilde{s}^{A\star}, \tilde{s}^{B\star})$ is called an interior (mixed strategy) Nash equilibrium, if:

$$\left.\frac{\partial \tilde{\$}^A(\tilde{s}^A, \tilde{s}^B)}{\partial \tilde{s}^A}\right|_{\tilde{s}^B = \tilde{s}^{B\star}} = 0 \quad \forall \; \tilde{s}^A \in [0,1] \;\; , \;\; \tilde{s}^{B\star} \in \,]0,1[$$
$$\left.\frac{\partial \tilde{\$}^B(\tilde{s}^A, \tilde{s}^B)}{\partial \tilde{s}^B}\right|_{\tilde{s}^A = \tilde{s}^{A\star}} = 0 \quad \forall \; \tilde{s}^B \in [0,1] \;\; , \;\; \tilde{s}^{A\star} \in \,]0,1[$$

Replicatordynamics

Replicatordynamics: The dynamical behavior of a population of players

$$\frac{dx_i^A(t)}{dt} = x_i^A(t)\left[\sum_{l=1}^{2} \$_{il}^A \, x_l^B(t) - \sum_{l=1}^{2}\sum_{k=1}^{2} \$_{kl}^A \, x_k^A(t)\, x_l^B(t)\right]$$

$$\frac{dx_i^B(t)}{dt} = x_i^B(t)\left[\sum_{l=1}^{2} \$_{li}^B \, x_l^A(t) - \sum_{l=1}^{2}\sum_{k=1}^{2} \$_{lk}^B \, x_l^A(t)\, x_k^B(t)\right]$$

The two population vectors $\vec{x}^A$ and $\vec{x}^B$ have to fulfill the normalizing conditions of a unity vector

$$x_i^\mu(t) \geq 0 \quad \text{and} \quad \sum_{i=1}^{2} x_i^\mu(t) = 1 \quad \forall\, i = 1, 2\,,\ t \in \mathbb{R},\ \mu = A, B$$

Replicatordynamics of (2×2) games

Replicatordynamics of unsymmetric (2×2) games

$$\frac{dx(t)}{dt} = \left(\left(\$^A_{11} + \$^A_{22} - \$^A_{12} - \$^A_{21}\right)\left(x(t) - (x(t))^2\right)\right) y(t) + \left(\$^A_{12} - \$^A_{22}\right)\left(x(t) - (x(t))^2\right) =: g_A(x,y)$$

$$\frac{dy(t)}{dt} = \left(\left(\$^B_{11} + \$^B_{22} - \$^B_{12} - \$^B_{21}\right)\left(y(t) - (y(t))^2\right)\right) x(t) + \left(\$^B_{12} - \$^B_{22}\right)\left(y(t) - (y(t))^2\right) =: g_B(x,y)$$

Replicatordynamics of symmetric (2×2) games

$$\frac{dx}{dt} = x\left[\$_{11}(x - x^2) + \$_{12}(1 - 2x + x^2) + \$_{21}(x^2 - x) + \$_{22}(2x - x^2 - 1)\right]$$

$$= x\left[(\$_{11} - \$_{21})(x - x^2) + (\$_{12} - \$_{22})(1 - 2x + x^2)\right] =: g(x)$$

with: $x = x(t) := x_1(t) \quad \rightarrow \quad x_2(t) = (1 - x(t))$

Payoff transformation and Game classes

Nash equivalent games

The set of Nash equilibria, the dynamical behavior of evolutionary games and the existence of evolutionary stable strategies (ESS) are unaffected by positive affine payoff transformations and by additionally added constants, where the strategy choice of the other players are fixed (see e.g. Weibull(1995)[14]). In the following the second kind of payoff transformation will be used to transform the payoff matrices in order to classify the games into different categories.

Symmetric payoff matrix after payoff transformation

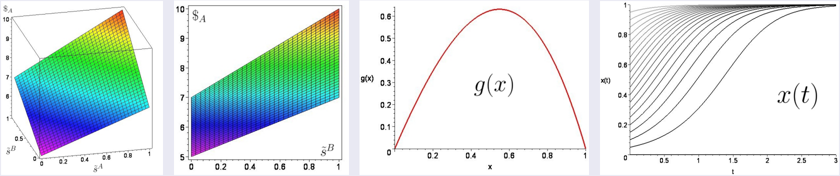
A\B	s_1^B	s_2^B
s_1^A	$(\$_{11},\$_{11})$	$(\$_{12},\$_{21})$
s_2^A	$(\$_{21},\$_{12})$	$(\$_{22},\$_{22})$

$\Longrightarrow$

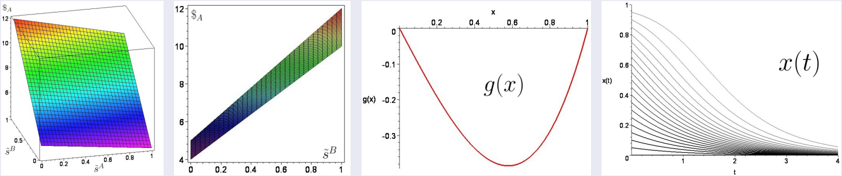
A\B	$Trafo\, s_1^B$	$Trafo\, s_2^B$
$Trafo\, s_1^A$	$(\underbrace{\$_{11} - \$_{21}}_{:=a},\underbrace{\$_{11} - \$_{21}}_{:=a})$	$(0,0)$
$Trafo\, s_2^A$	$(0,0)$	$(\underbrace{\$_{22} - \$_{12}}_{:=b},\underbrace{\$_{22} - \$_{12}}_{:=b})$

Symmetric (2×2) games: Dominant Class $(a < 0,\, b > 0)$ or $(b < 0,\, a > 0)$

Dominant Game: a=3, b=-2, one pure NE and one ESS (s_1^A, s_1^B)

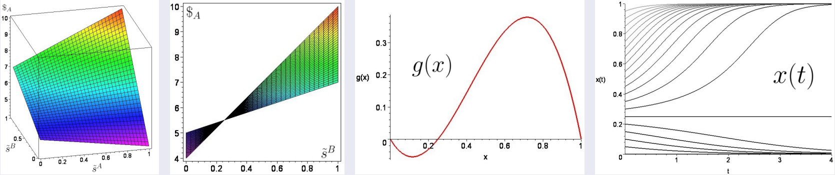


Prisoner's Dilemma: a=-2, b=1, one pure NE and one ESS (s_2^A, s_2^B)

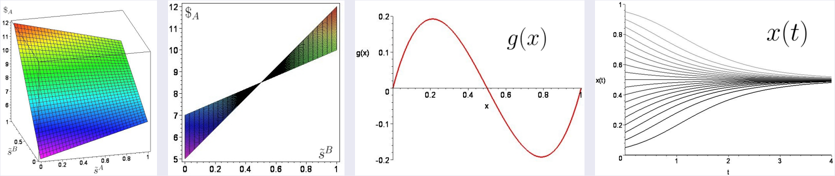


Coordination $(a, b > 0)$ and Anti-Coordination $(a, b < 0)$ Class

Coordination game: a=3, b=1, two pure and one interior NE at $\tilde{s}^\star = \frac{1}{4}$, two ESS $((s_1^A, s_1^B)$ and $(s_2^A, s_2^B))$



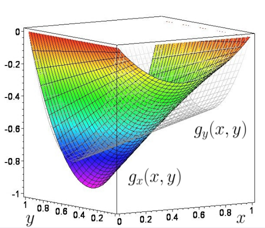
Anti-Coordination game: a=-2, b=-2, two pure asymmetric NE and one interior NE at $\tilde{s}^\star = \frac{1}{2}$, one ESS $(\tilde{s}^{A\star} = \frac{1}{2}, \tilde{s}^{B\star} = \frac{1}{2})$



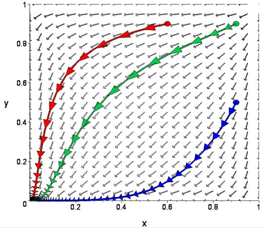
Game classes of unsymmetric (2 player)-(2 strategy) games

Corner Class (one ESS)

$g_x(x,y)$ (colored) and $g_y(x,y)$ (wired):

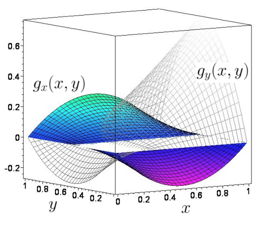


Phase diagram of xy-trajectories:

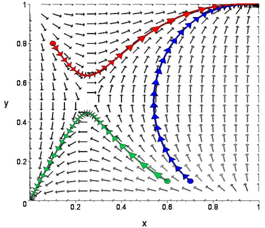


Saddle Class (two ESS)

$g_x(x,y)$ (colored) and $g_y(x,y)$ (wired):

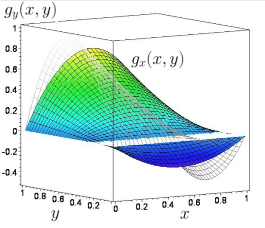


Phase diagram of xy-trajectories:

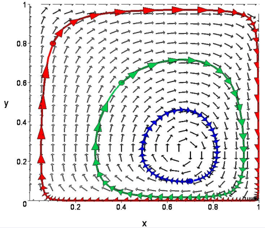


Center Class (no ESS)

$g_x(x,y)$ (colored) and $g_y(x,y)$ (wired):



Phase diagram of xy-trajectories:



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- The *Quantum* Coordination Game
  2003, B. A. Huberman and T. Hogg, *Quantum Solution of Coordination Problems*, Quantum Information Processing 2(6)
- The *Quantum* Ultimatum Game
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Related Literature (II)

- Economics and Quantum Game Theory

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  2002, Kay-Yut Chen, T. Hogg and R. Beaulsoleil *A Quantum Treatment of Public Goods Economics*, Quantum Information Processing 1(6)

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Related Literature (III)

- Quantum Computer and Quantum Game Theory
  2002, J. Du, H. Li, X. Xu, M. Shi, J. Wu, X. Zhou and R. Han *Experimental realization of quantum games on a quantum computer*, PRL 88 (137902)

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  2001, S. C. Benjamin and P. M. Hayden, *Multi-Player Quantum Games*, PRA 64 (030301) ...

Related Literature (IV)

- Experimental Economics and Quantum Game Theory
  2006, Kay-Yut Chen and Tad Hogg *How well do people play a quantum prisoner's dilemma?*, Quantum Information Processing 5(43)

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- and reviews in Physics World and Nature ...

A 2-Player–2-Strategy-Quantumgame

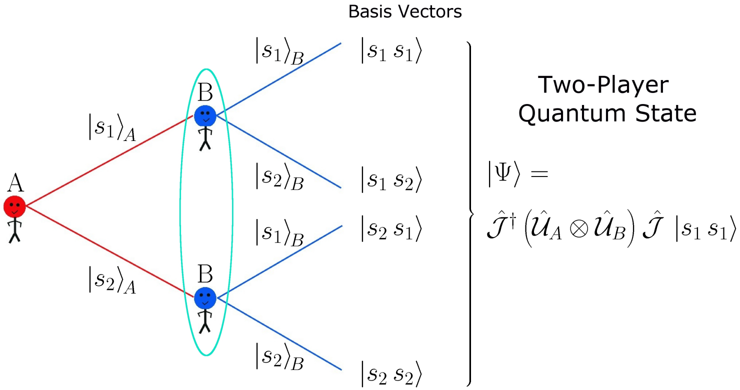


Figure: $|\Psi\rangle$: Two-Player State, $\hat{\mathcal{J}}(\gamma)$: Entangling Operator, γ: Strength of Entanglement, $\widehat{\mathcal{U}}_A, \widehat{\mathcal{U}}_B$: Strategy Decision Operator of Player A and B

The Two-Player Quantum Wavefunction $|\Psi\rangle$

The Two-Player Quantum State $|\Psi\rangle$

$$|\Psi\rangle = \hat{\mathcal{J}}^{\dagger} \left(\hat{\mathcal{U}}_A \otimes \hat{\mathcal{U}}_B\right) \hat{\mathcal{J}} \; |s_1 \, s_1\rangle$$

$\hat{\mathcal{U}}_A$: Decision Operator of Player A

$\hat{\mathcal{U}}_B$: Decision Operator of Player B

$\hat{\mathcal{J}}$: Entangling Operator

$\hat{\mathcal{J}}^{\dagger}$: Disentangling Operator

$\hat{\mathcal{J}} \; |s_1 \, s_1\rangle$: Two-Player Initial State ($|\Psi_0\rangle$)

In words ...

The setup of the quantum game begins with the choice of the initial state $|\Psi_0\rangle$. After the two players have chosen their individual quantum strategies ($\hat{\mathcal{U}}_A := \hat{\mathcal{U}}(\theta_A, \varphi_A)$ and $\hat{\mathcal{U}}_B := \hat{\mathcal{U}}(\theta_B, \varphi_B)$) the disentangling operator $\hat{\mathcal{J}}^{\dagger}$ is acting to prepare the measurement.

The quantum decision state $|\psi\rangle_\mu$ of player $\mu = A, B$

To illustrate the operator formalism of quantum game theory and the concept of quantum strategies, we want to focus at first on the real and imaginary values of the two spinor components ψ_1^A and ψ_2^A of the of the state $|\psi\rangle_A$ of player A:

$$|\psi\rangle_A = \psi_1^A \left| s_1^A \right\rangle + \psi_2^A \left| s_2^A \right\rangle = \begin{pmatrix} \psi_1^A \\ -\psi_2^A \end{pmatrix} \in \mathcal{H}_A$$

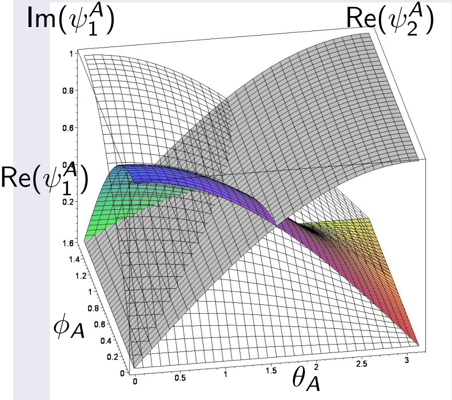
$$\left| s_1^A \right\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} , \left| s_2^A \right\rangle = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$|\psi\rangle_A = \widehat{\mathcal{U}}(\theta_A, \varphi_A) \left| s_1^A \right\rangle = \begin{pmatrix} e^{i\,\varphi_A} \cos(\frac{\theta_A}{2}) \\ -\sin(\frac{\theta_A}{2}) \end{pmatrix}$$

$$\widehat{\mathcal{U}}(\theta, \varphi) := \begin{pmatrix} e^{i\,\varphi} \cos(\frac{\theta}{2}) & \sin(\frac{\theta}{2}) \\ -\sin(\frac{\theta}{2}) & e^{-i\,\varphi} \cos(\frac{\theta}{2}) \end{pmatrix} \;\forall\; \theta \in [0, \pi] \wedge \varphi \in [0, \frac{\pi}{2}]$$

The quantum decision state $|\psi\rangle_\mu$ of player $\mu = A, B$

Real and imaginary parts of $|\psi\rangle_A$



Quantum state of player A:

$$|\psi\rangle_A = \psi_1^A \left| s_1^A \right\rangle + \psi_2^A \left| s_2^A \right\rangle = \begin{pmatrix} \psi_1^A \\ -\psi_2^A \end{pmatrix} \in \mathcal{H}_A$$

$$\text{with:} \quad \left| s_1^A \right\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} , \quad \left| s_2^A \right\rangle = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

s_1-quantum strategies and the decision operator $\widehat{\mathcal{U}}(\theta, \varphi)$:

$$|\psi\rangle_A = \widehat{\mathcal{U}}(\theta_A, \varphi_A) \left| s_1^A \right\rangle = \begin{pmatrix} e^{i\,\varphi_A} \cos(\frac{\theta_A}{2}) \\ -\sin(\frac{\theta_A}{2}) \end{pmatrix}$$

$$\widehat{\mathcal{U}}(\theta, \varphi) := \begin{pmatrix} e^{i\,\varphi} \cos(\frac{\theta}{2}) & \sin(\frac{\theta}{2}) \\ -\sin(\frac{\theta}{2}) & e^{-i\,\varphi} \cos(\frac{\theta}{2}) \end{pmatrix}$$

$$\forall \quad \theta \in [0, \pi] \ \wedge \ \varphi \in [0, \frac{\pi}{2}]$$

Interpretation

Die quantentheoretische Beschreibung des Entscheidungszustandes des Spielers A kurz vor der definitiven Auswahl und Bekundung der reinen Strategie besitzt demnach im Allgemeinen neben den reelwertigen auch imaginäre Anteile. Bei s_1-Quantenstrategien kann sich der Spieler nur im imaginären Raum der ersten Strategie gedanklich bewegen. Eine grundlegende Eigenschaft der gesamten Quantentheorie ist die prinzipielle Unbeobachtbarkeit des Quantenzustandes. Diese Eigenschaft spiegelt sich in der Quanten Spieltheorie in der Unbeobachtbarkeit des Gedankenprozesses wider. Die einzelnen Gedankenwege, die wärend des Entscheidungs-prozesses im Gehirn des Spielers (bewust oder unterbewust) ablaufenden, können nicht direkt gemessen werden.
s_1-Quantenstrategien können als der gedankliche Weg während des Entscheidungsprozesses interpretiert werden, welcher vom gedanklichen Ursprung her von der klassischen Strategie s_1 startet und hypotetisch, gebunden an die Wünsche und Ängste des Spielers, den Gedankenweg weiterbildet.

The 2-player state $|\Psi\rangle$ and the entangling operator $\widehat{\mathcal{J}}(\gamma)$

$$|\Psi\rangle = \hat{\mathcal{J}}^{\dagger} \left(\hat{\mathcal{U}}_A \otimes \hat{\mathcal{U}}_B\right) \hat{\mathcal{J}} \left|s_1^A s_1^B\right\rangle$$

$$\widehat{\mathcal{J}} := e^{i\,\frac{\gamma}{2}(\widehat{s_1}\otimes\widehat{s_1})} = \begin{pmatrix} \cos\left(\frac{\gamma}{2}\right) & 0 & 0 & i\sin\left(\frac{\gamma}{2}\right) \\ 0 & \cos\left(\frac{\gamma}{2}\right) & -i\sin\left(\frac{\gamma}{2}\right) & 0 \\ 0 & -i\sin\left(\frac{\gamma}{2}\right) & \cos\left(\frac{\gamma}{2}\right) & 0 \\ i\sin\left(\frac{\gamma}{2}\right) & 0 & 0 & \cos\left(\frac{\gamma}{2}\right) \end{pmatrix}$$

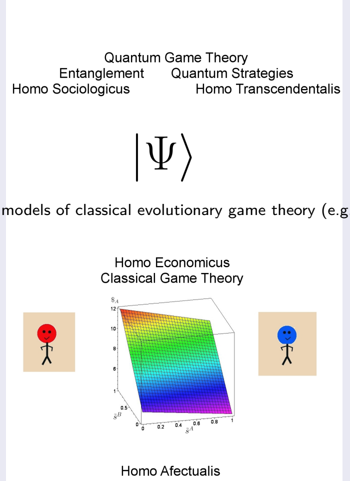
$$\gamma \in [0, \frac{\pi}{2}] \,, \quad \left|s_1^A s_1^B\right\rangle := \left|s_1^A\right\rangle \otimes \left|s_1^B\right\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Interpretation

The most important, but also most difficult mathematical concept in QGT is the two player quantum state $|\Psi\rangle$. It is formally constructed with the use of the decision operators $\hat{\mathcal{U}}_A$ and $\hat{\mathcal{U}}_B$ of player A and B and the entangling and disentangling operator $\hat{\mathcal{J}}$ and $\hat{\mathcal{J}}^\dagger$. $|\Psi\rangle$ is an spinor in a complex valued, 4-dimensional, abstract mathematical space called the 2-player "Hilbertspace" $\mathcal{H}$. The space of all conceivable decision paths is extended from the purely rational, measurable space in the Hilbertspace of complex numbers. Trough the concept of a potential entanglement of the imaginary quantum strategy parts, it is possible to include corporate decision path, caused by cultural or moral standards. QGT is therefore a model which goes beyond *Homo Economicus* and the parameter γ, which is a measure for the strength of entanglement and fellow feeling, describes how strongly the players behave as *Homo Sociologicus* or *Homo Transzendentalis*.

The 2-player state $|\Psi\rangle$ and the entangling operator $\widehat{\mathcal{J}}(\gamma)$

Beyond Homo Economicus



Extended models of classical evolutionary game theory (e.g. [10, 9])

The final 2-player quantum state:

$$|\Psi\rangle = \widehat{\mathcal{J}}^{\dagger} \left(\widehat{\mathcal{U}}_A \otimes \widehat{\mathcal{U}}_B \right) \widehat{\mathcal{J}} \, |s_1^A s_1^B\rangle$$

$\widehat{\mathcal{J}}(\gamma)$: Entangling operator

$\widehat{\mathcal{J}}^{\dagger}(\gamma)$: Disentangling operator

$\gamma \in [0, \pi]$: Strength of entanglement

$\widehat{\mathcal{U}}_A$: Decision Operator for player A

$\widehat{\mathcal{U}}_B$: Decision Operator for player B

The extended payoff $\$_\mu(\theta_A, \varphi_A, \theta_B, \varphi_B)$ of player $\mu = A, B$

The extended payoff $\$_\mu(\tau_A, \tau_B)$ of player $\mu = A, B$ is an amplification of the classical mixed strategy payoff function $\tilde{\$}^\mu(\tilde{s}^A, \tilde{s}^B)$:

$$\begin{aligned} \$_A &= \$^A_{11}\, P_{11} + \$^A_{12}\, P_{12} + \$^A_{21}\, P_{21} + \$^A_{22}\, P_{22} \\ \$_B &= \$^B_{11}\, P_{11} + \$^B_{12}\, P_{12} + \$^B_{21}\, P_{21} + \$^B_{22}\, P_{22} \\ \text{with:} \quad & P_{\sigma\sigma'} = \left| \langle \sigma\sigma' | \Psi \rangle \right|^2 \;, \quad \sigma = \left\{ s^A_1, s^A_2 \right\} \text{ and } \sigma' = \left\{ s^B_1, s^B_2 \right\} \end{aligned}$$

$P_{\sigma\sigma'}$ are the real valued probabilities of finding the two player state $|\Psi\rangle$ in the pure strategy Eigenstate $|\sigma\sigma'\rangle$, e.g.

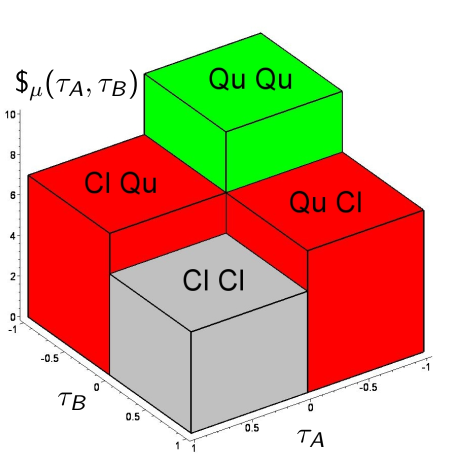
$$P_{12} := P_{s^A_1 s^B_2} = \left| \left\langle s^A_1 s^B_2 | \Psi \right\rangle \right|^2$$

The extended payoff $\$_\mu(\tau_A, \tau_B)$ of player $\mu = A, B$

In contrast to the classical mixed payoff functions ($\tilde{\$}^A(\tilde{s}^A, \tilde{s}^B)$ and $\tilde{\$}^B(\tilde{s}^A, \tilde{s}^B)$), which depend only on the two parameters $\tilde{s}^A$ and $\tilde{s}^B$, the quantum version of the mixed strategy payoff function depends in general on five parameters; namely the four decision angles of player A and player B ($\theta_A, \varphi_A, \theta_B$ and φ_B) and the entangling parameter γ. In order to visualize the payoff function as a surface in a three dimensional space it is necessary to reduce the set of parameters in the final state:
$|\Psi\rangle = |\Psi_f(\theta_A, \varphi_A, \theta_B, \varphi_B)\rangle \rightarrow |\Psi(\tau_A, \tau_B)\rangle$. The two strategy angles θ and φ depend only on a single parameter $\tau \in [-1, 1]$. Positive τ-values represent pure and mixed classical strategies, whereas negative τ-values correspond to quantum strategies, where $\theta = 0$ and $\varphi > 0$. The whole strategy space is separated into four regions, namely the absolute classical region (*ClCl*: $\tau_A, \tau_B \geq 0$), the absolute quantum region (*QuQu*: $\tau_A, \tau_B < 0$) and the two partially classical-quantum regions (*ClQu*: $\tau_A \geq 0 \wedge \tau_B < 0$ and *QuCl*: $\tau_A < 0 \wedge \tau_B \geq 0$).

The extended payoff $\$_\mu(\tau_A, \tau_B)$ of player $\mu = A, B$

Visualisationspace of $\$_\mu(\tau_A, \tau_B)$



The expected payoff within a quantum version of a general 2-player game:

$$\begin{aligned}
\$_A &= \$^A_{11}\, P_{11} + \$^A_{12}\, P_{12} + \$^A_{21}\, P_{21} + \$^A_{22}\, P_{22} \\
\$_B &= \$^B_{11}\, P_{11} + \$^B_{12}\, P_{12} + \$^B_{21}\, P_{21} + \$^B_{22}\, P_{22} \\
\text{with:} \quad P_{\sigma\sigma'} &= |\langle \sigma\sigma' | \Psi \rangle|^2 \ , \quad \sigma, \sigma' = \{s_1, s_2\}
\end{aligned}$$

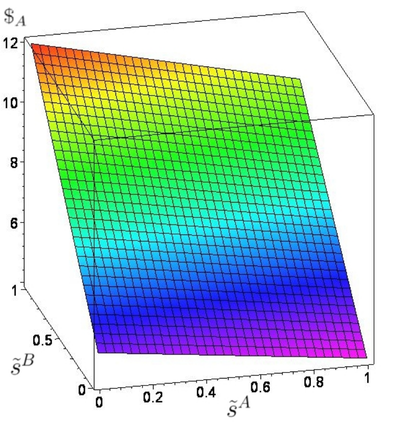
Reduction of quantum strategies:

$|\Psi\rangle = |\Psi(\theta_A, \varphi_A, \theta_B, \varphi_B)\rangle \rightarrow |\Psi(\tau_A, \tau_B)\rangle$

$$\underbrace{\{(\tau\,\pi, 0) \mid \tau \in [0,1]\}}_{\text{classical region } Cl} \wedge \underbrace{\{(0, \tau\,\frac{\pi}{2}) \mid \tau \in [-1, 0[\}}_{\text{quantum region } Qu}$$

Quantum extension of dominant class games

Classical payoff for player A



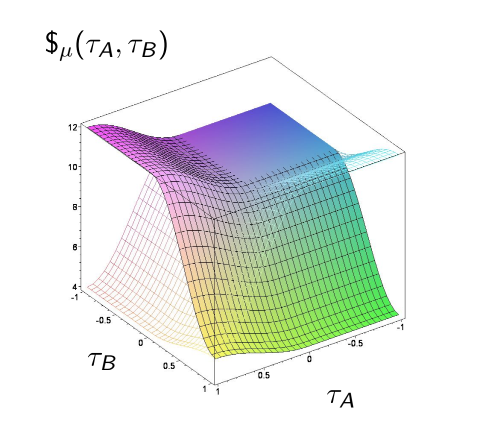
A\B	s_1^B	s_2^B
s_1^A	(10,10)	(4,12)
s_2^A	(12,4)	(5,5)

Table: Payoffmatrix of a dominant, prisoners dilemma like game.

This dominant, prisoners dilemma like game has only one pure, symmetric Nash equilibrium (s_2^A, s_2^B) which is the only ESS of the evolutionary game.

Quantum extension of dominant class games

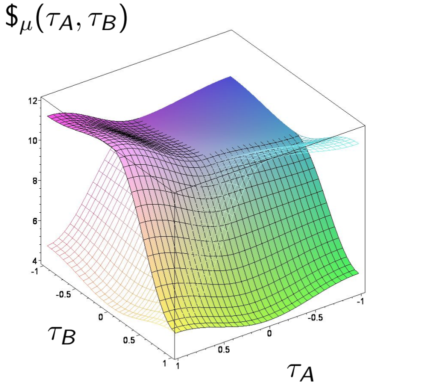
Payoff of player A (colored) and player B (wired) for $\gamma = 0$ (no entanglement)



The diagram clearly exhibits that the non-entangled quantum game simply describes the classical version of the prisoner's dilemma game. For the case, that both players decide to play a quantum strategy ($\tau_A < 0 \wedge \tau_B < 0$) their payoff is equal to the case where both players choose the classical pure strategy s_1 ($\$_A(\tau_A = 0, \tau_B = 0) = 10$). The classical Nash equilibrium ((s_2^A, s_2^B), the dominant strategy) corresponds to the following τ-values:$(s_2^A, s_2^B) \hat{=} (\tau_A = 1, \tau_B = 1)$.

Quantum extension of dominant class games

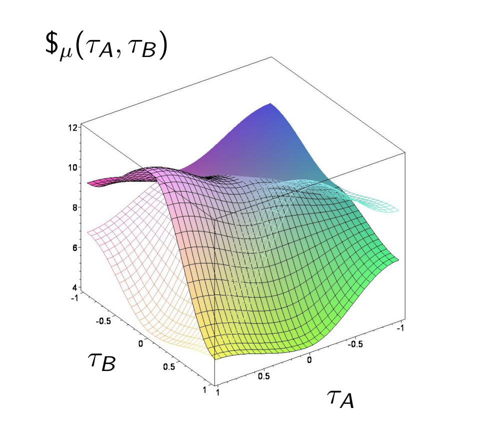
Payoff of player A (colored) and player B (wired) for $\gamma = \frac{\pi}{10} \approx 0.31$



For the absolute classical region *ClCl* the shape of the surfaces does not change, whereas for the partially classical-quantum (*ClQu* and *QuCl*) and absolute quantum region regions *QuQu* the payoff structure changes due to a possible interference of quantum strategies within Hilbertspace. The structure of Nash-equilibria does not change for the left picture, whereas for the following pictures the previously present dominant strategy of the prisoner's dilemma game has disappeared and a new, advisable quantum Nash-equilibrium will appear at $(\widehat{Q}, \widehat{Q} \hat{=} (\tau_A = -1, \tau_B = -1))$. During the transition from this figure to the next picture two separate phenomena occur. At first, for an entanglement value $\gamma_1 \approx 0.37$, the best response for player A to the strategy $s_2^B \hat{=} \tau_B = 1$ is no longer the strategy $s_2^A \hat{=} \tau_A = 1$, as $\$_A(\tau_A = -1, \tau_B = 1) \approx 5.05$ is now higher than $\$_A(\tau_A = 1, \tau_B = 1) = 5$. Secondly, for an entanglement value $\gamma_2 \approx 0.53$, the best response for player A to the strategy $\widehat{Q}_B \hat{=} \tau_B = -1$ is no longer the strategy $s_2^A \hat{=} \tau_A = 1$, as $\$_A(\tau_A = 1, \tau_B = -1) \approx 9.96$ is for $\gamma_2 = 0.53$ lower than $\$_A(\tau_A = -1, \tau_B = -1) = 10$.

Quantum extension of dominant class games

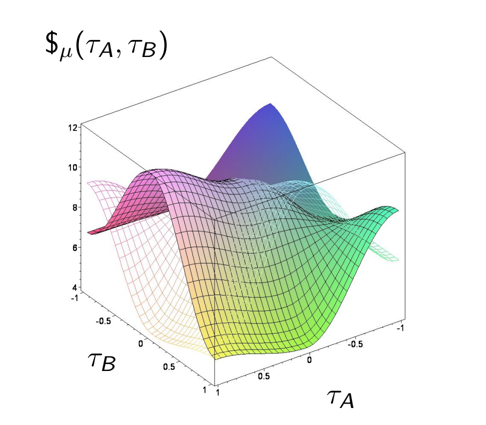
Payoff of player A (colored) and player B (wired) for $\gamma = \frac{\pi}{8} \approx 0.52$



For the absolute classical region *ClCl* the shape of the surfaces does not change, whereas for the partially classical-quantum (*ClQu* and *QuCl*) and absolute quantum region regions *QuQu* the payoff structure changes due to a possible interference of quantum strategies within Hilbertspace. The structure of Nash-equilibria did not change for the last figure, whereas for this and thee following pictures the previously present dominant strategy of the prisoner's dilemma game has disappeared and a new, advisable quantum Nash-equilibrium has appeared $(\widehat{Q}, \widehat{Q} \hat{=} (\tau_A = -1, \tau_B = -1))$. During the transition from the last picture to this figure two separate phenomena occurred. At first, for an entanglement value $\gamma_1 \approx 0.37$, the best response for player A to the strategy $s_2^B \hat{=} \tau_B = 1$ is no longer the strategy $s_2^A \hat{=} \tau_A = 1$, as $\$_A(\tau_A = -1, \tau_B = 1) \approx 5.05$ is now higher than $\$_A(\tau_A = 1, \tau_B = 1) = 5$. Secondly, for an entanglement value $\gamma_2 \approx 0.53$, the best response for player A to the strategy $\widehat{Q}_B \hat{=} \tau_B = -1$ is no longer the strategy $s_2^A \hat{=} \tau_A = 1$, as $\$_A(\tau_A = 1, \tau_B = -1) \approx 9.96$ is for $\gamma_2 = 0.53$ lower than $\$_A(\tau_A = -1, \tau_B = -1) = 10$.

Quantum extension of dominant class games

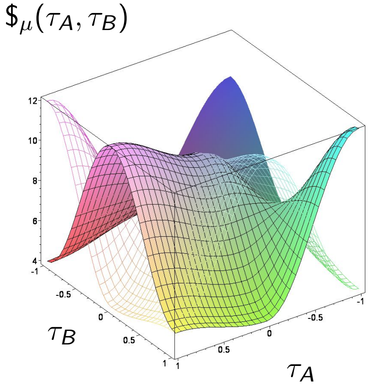
Payoff of player A (colored) and player B (wired) for $\gamma = \frac{\pi}{6} \approx 0.94$



The results show, that a quantum extension of a classical prisoner's dilemma game is able to change the structure of Nash-equilibria, and even previously present dominant strategies could become nonexistent, if the value of entanglement increases further than a defined γ-threshold. Players with a higher strategic entanglement value γ escape the dilemma as they see the advantage of the quantum strategy combination $(\widehat{Q}_A, \widehat{Q}_B)$, which is measured as if both are playing the classical strategy s_2.

Quantum extension of dominant class games

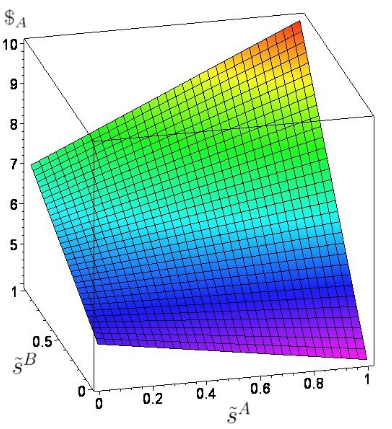
Payoff of player A (colored) and player B (wired) for $\gamma = \frac{\pi}{2} \approx 1.57$



The results show, that a quantum extension of a classical prisoner's dilemma game is able to change the structure of Nash-equilibria, and even previously present dominant strategies could become nonexistent, if the value of entanglement increases further than a defined γ-threshold. Players with a higher strategic entanglement value γ escape the dilemma as they see the advantage of the quantum strategy combination $(\widehat{Q}_A, \widehat{Q}_B)$, which is measured as if both are playing the classical strategy s_2.

Quantum extension of coordination class games

Classical payoff for player A



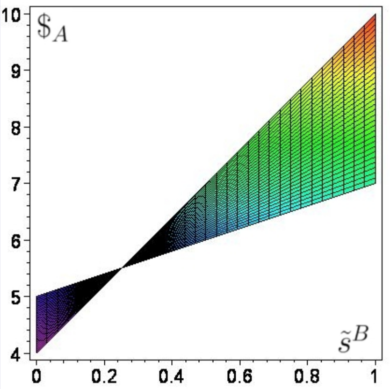
A\B	s_1^B	s_2^B
s_1^A	(10,10)	(4,7)
s_2^A	(7,4)	(5,5)

Table: Payoffmatrix of a coordination game.

This coordination game has two pure, symmetric Nash equilibria and one interior NE at $s^\star = \frac{1}{4}$. The evolutionary game game has two ESSs.

Quantum extension of coordination class games

Classical payoff for player A (projected)



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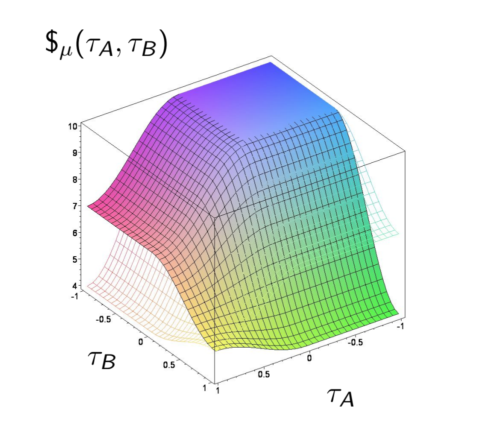
This coordination game has two pure, symmetric Nash equilibria and one interior NE at $s^\star = \frac{1}{4}$. The evolutionary game game has two ESSs.

Quantum extension of coordination class games

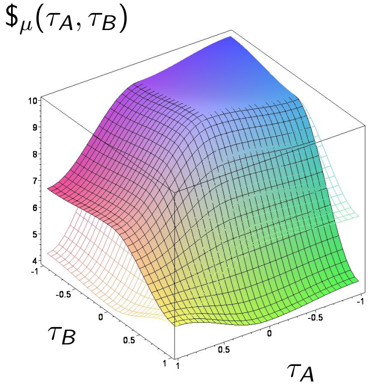
Payoff of player A (colored) and player B (wired) for $\gamma = 0$ (no entanglement)



Again, the diagram clearly indicates that the non-entangled quantum game is identical to the classical version of the underlying coordination game. For the case, that both players decide to play a quantum strategy $(\tau_A < 0 \wedge \tau_B < 0)$ their payoff is equal to the case where both players choose the classical pure strategy s_1 $(\$_A(\tau_A = 0, \tau_B = 0) = 10)$, with the overall highest possible payoff. The classical pure Nash equilibria correspond to the following τ-values: $(s_1^A, s_1^B) \hat{=} (\tau_A = 0, \tau_B = 0)$ and $(s_2^A, s_2^B) \hat{=} (\tau_A = 1, \tau_B = 1)$, whereas the classical mixed strategy equilibrium is at: $\tau^\star = \frac{2}{\pi} \arccos(\sqrt{\frac{1}{4}}) = \frac{2}{3}$.

Quantum extension of coordination class games

Payoff of player A (colored) and player B (wired) for $\gamma = \frac{\pi}{10} \approx 0.31$



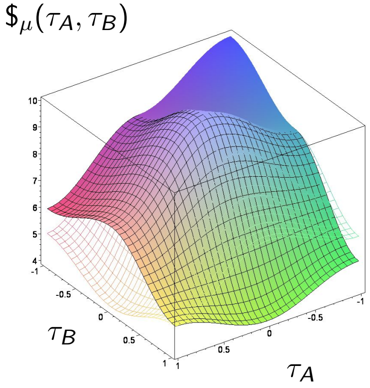
Even for tiny values of γ a new quantum Nash-equilibrium appears $(\tau_A = -1, \tau_B = -1)$.

At moderate values of γ the low payoff evolutionary stable strategy $(\tau_A = 1, \tau_B = 1)$ disappears.

The specific γ-value at which this disappearance happens, depends on the whole set of payoff parameters and not only on a and b.

Quantum extension of coordination class games

Payoff of player A (colored) and player B (wired) for $\gamma = \frac{\pi}{8} \approx 0.52$



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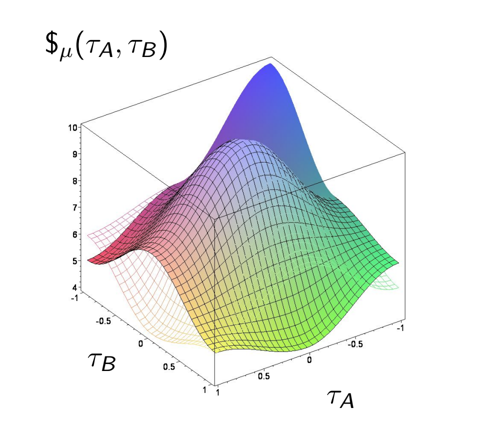
Quantum extension of coordination class games

Payoff of player A (colored) and player B (wired) for $\gamma = \frac{\pi}{6} \approx 0.94$



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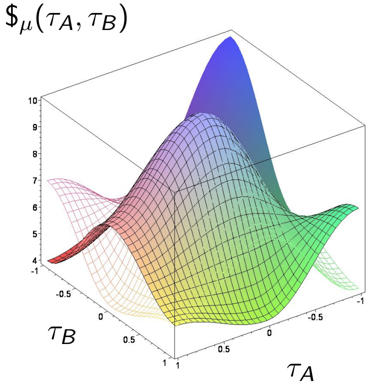

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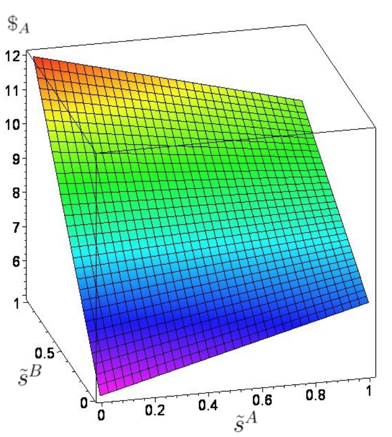
Quantum extension of anti-coordination class games

Classical payoff for player A



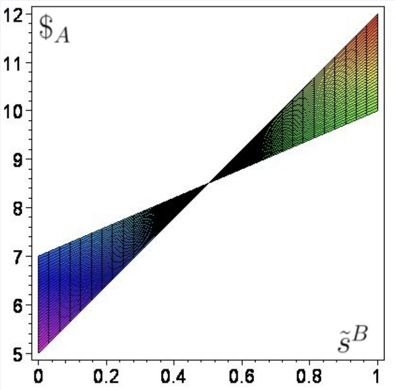
A\B	s_1^B	s_2^B
s_1^A	(10,10)	(7,12)
s_2^A	(12,7)	(5,5)

Table: Payoffmatrix of a coordination game.

This anti-coordination game has two pure, unsymmetric Nash equilibria and one interior NE at $s^\star = \frac{1}{2}$. The evolutionary game game has one mixed strategy ESS.

Quantum extension of anti-coordination class games

Classical payoff for player A (projected)



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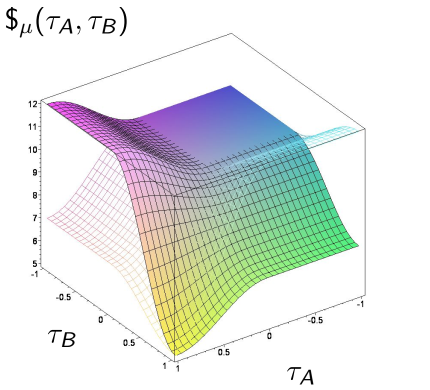
Quantum extension of anti-coordination class games

Payoff of player A (colored) and player B (wired) for $\gamma = 0$



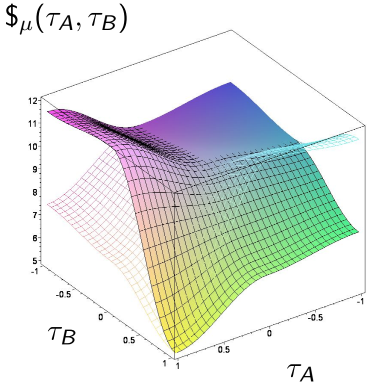
Beside the mixed strategy evolutionary stable strategy, a new quantum ESS appears at a specific γ-value.

For details see:

- M. Hanauske, *Advances in Evolutionary Game Theory*, 2009, *Lecture at the 'Université Lumière Lyon 2' in Lyon, France (MINERVE Exchange Program); Slides and additional material*
- *M. Hanauske, J. Kunz, S. Bernius, and W. König. ,Doves and hawks in economics revisited: An evolutionary quantum game theory-based analysis of financial crises. , 2009, to appear in Physica A, arXiv:0904.2113, RePEc:pra:mprapa:14680 and SSRN$_{id}$:1597735 .*

Quantum extension of anti-coordination class games

Payoff of player A (colored) and player B (wired) for $\gamma = \frac{\pi}{10} \approx 0.31$



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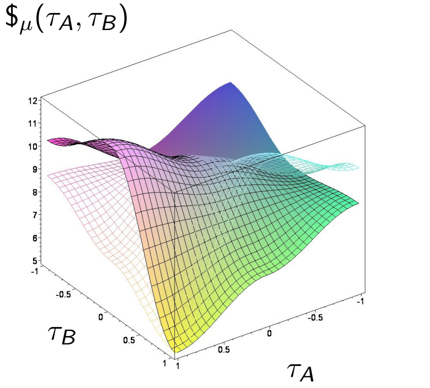

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Quantum extension of anti-coordination class games

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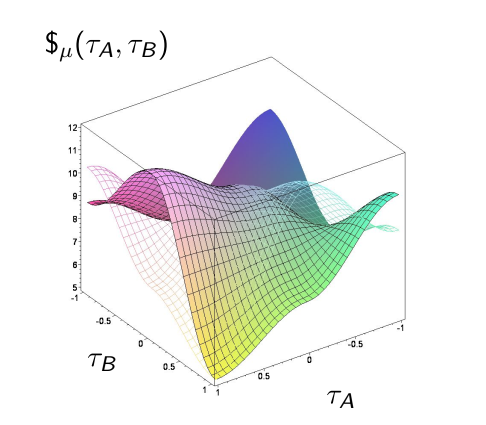

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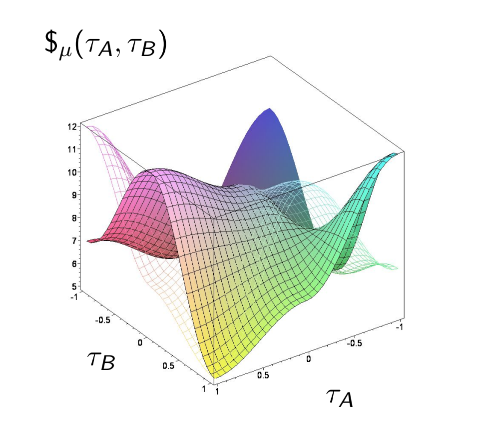

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Quantum Game Theory and Scientific Communication

M. Hanauske, S. Bernius, and B. Dugall, *Quantum Game Theory and Open Access Publishing*, *Physica A*, 382(2):650–664, 2007, arXiv:physics/0612234

Matthias Hanauske, Wolfgang König, and Berndt Dugall, *Evolutionary Quantum Game Theory and Scientific Communication*, 2010, Accepted article of the "Second Brasilian Workshop of the Game Theory Society", Internet-Link

Matthias Hanauske, *Evolutionary Game Theory and Complex Networks of Scientific Information*, Article is going to be published as a chapter in the book *"Models of science dynamics - Encounters between complexity theory and information science", Spinger book in the Complexity series, Editors: Andrea Scharnhorst, Katy Börner and Peter van den Besselaar*, 2011

Quantum Game Theory and Scientific Communication

In recent years the market of scientific publishing faces several forces that may cause a major change of traditional market mechanisms. In the following we understand open access publishing as the electronic publication of scientific information on a platform that provides access to this information for all potential users, without financial or other barriers. In contrast most other scientific disciplines do not make use of open access publishing, even though they support this model if asked for [2]. Instead, they submit research papers to traditional journals that do not provide free access to their articles. Considering that the majority of scientists regard open access publishing as superior to the traditional system, the question arises, why it is only adopted by few disciplines.

Quantum Game Theory and Scientific Communication

Based on the assumption, that the main goal of scientists is the maximization of their reputation, we try to answer this question from the perspective of the producers of scientific information by using a game theoretical approach. To describe the underlying open access game we use a normal-form representation of a two-player game Γ where each player (Player 1 $\hat{=}$ A, Player 2 $\hat{=}$ B) can choose between two strategies ($\mathcal{S}^A = \{s_1^A, s_2^A\}$, $\mathcal{S}^B = \{s_1^B, s_2^B\}$). In our case the two strategies represent the authors' choice between publishing open access (o) or not (ø).

Quantum Game Theory and Scientific Communication

The payoff structure of this game is modeled by the following payoff matrix:

A\B	o	ø
o	$(r+\delta, r+\delta)$	$(r-\alpha, r+\beta)$
ø	$(r+\beta, r-\alpha)$	(r, r)

Table: Researchers open access payoff matrix.

The actual reputation of the two scientists is represented by a single parameter r. If only one of the two players chooses the open access strategy ((ø,o) or (o,ø)) the parameters α and β ($\alpha, \beta \geq 0$) describe the decrease and the increase of the scientists' reputation, depending on the selected strategy. The parameter δ represents the potential benefit in the case that both players choose the open access strategy (o,o).

Quantum Game Theory and Scientific Communication

As the presented open access game is a symmetric game and the parameter $b = \alpha$ is positive, the underlying game class depends only on the sign of the parameter $a = \delta - \beta$. For $\delta > \beta$ the game belongs to the class of coordination games, whereas for $\delta < \beta$ the game has the structure of a dominant game with a dilemma.

By extending the model using the quantum game theory approach it can be shown, that if the strength of entanglement exceeds a certain value, the scientists will overcome the dilemma and terminate to publish only traditionally in all settings.

Doves and hawks in economics revisited: An evolutionary quantum game theory-based analysis of financial crises

M. Hanauske, J. Kunz, S. Bernius, and W. König *Doves and hawks in economics revisited: An evolutionary quantum game theory-based analysis of financial crises*, *Physica A*, November 2010, arXiv:0904.2113, RePEc:pra:mprapa:14680, and SSRN_{id}:1597735.

The last financial and economic crisis demonstrated the dysfunctional long-term effects of aggressive behavior in financial markets. Yet, evolutionary game theory predicts that under the condition of strategic dependence a certain degree of aggressive behavior remains within a given population of agents. Through the extension of the well-known hawk-dove game by a quantum approach, we can show that dependent on entanglement, evolutionary stable strategies also can emerge, which are not predicted by the classical evolutionary game theory and where the total economic population uses a non-aggressive quantum strategy.

Summary

Summary of the talk

Quantum game theory is a mathematical and conceptual amplification of classical game theory. The space of all conceivable decision paths is extended from the purely rational, measurable space in the Hilbertspace of complex numbers. Trough the concept of a potential entanglement of the imaginary quantum strategy parts, it is possible to include corporate decision path, caused by cultural or moral standards. If this strategy entanglement is large enough, then, additional Nash-equilibria can occur, previously present dominant strategies could become nonexistent and new evolutionary stable strategies can appear.

Within this talk the framework of Quantum Game Theory was described in detail. The formal mathematical model, the different concepts of equilibria and the various classes of quantum games have been defined, explained and visualized to understand the main ideas of Quantum Game Theory. Additionally, two applications where discussed at the end of the talk.

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Matthias Hanauske.

Evolutionary Game Theory and Complex Networks of Scientific Information.

Article is going to be published as a chapter in the book *"Models of science dynamics - Encounters between complexity theory and information science", Spinger book in the Complexity series, Editors: Andrea Scharnhorst, Katy Börner and Peter van den Besselaar*, year = 2011.

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