Exercise Sheet #5

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Problem 1 (*SIRS Model With Political Reaction, 20pts*)

Consider the discrete SIRS model:

$$s_t = 1 - \sum_{k=0}^{\tau_R} x_{t-k}$$
$$x_{t+1} = g_0 x_t s_t$$

With s_t, x_t being the fraction of susceptible and infected individuals at time t respectively, and g_0 the infection factor. We would like to see how the behavior of this model changes when a reaction term is added corresponding to political forces pushing towards a desired infection factor.

a) Write a numerical simulation of the discrete SIRS model in a programming language of your choice. Use $\tau_R = 3$ and start the simulation with a number of infected people close to zero. Pay attention to common numerical errors: s and x should always be within the range $[0, 1]$. In case the number of new infections is too large and causes s to become negative, set s to zero instead.

b) Experiment with g_0 and see what state the system converges to. For which values does the system converge to a stable fixpoint? When does it converge to a limit cycle? When does the cycle become unstable? Plot the dynamics for values below and above the transition from fixpoint to limit cycle.

c) Now add the reaction term α from equation (2.42) in the lecture notes:

$$x_{t+1} = g_t x_t s_t$$
$$g_t = \frac{g_0}{1 + \alpha_t x_t}$$
$$\alpha_{t+1} = \alpha_t + \epsilon(g_t - g_\infty)$$

Where g_∞ is the politically desired infection term and the term α_t represents the effects of preventive measures on the infection factor, which now changes in time. Set $\epsilon = 0.1$ and $\alpha_0 = 0$

d) In an effort to combat COVID-19, some countries attempt to keep the infection factor below 1. Set the base infection factor to a value resembling that of COVID-19, $g_0 = 1.4$, and g_∞ to a value below 1. What happens? Plot the behavior with and without the reaction term and compare them.

e) We want to see if the reaction term simply maps after enough time to the original SIRS system, or has a different phase space. With $g_0 = 1.4$, look at the behavior of the system for varying g_∞ values. Which value ranges lead to a fixpoint, and which to a limit cycle? is the point of transition to a limit cycle the same as in (b)? When does the limit cycle lose stability? Plot the system before and after the appearance of the limit cycle.

Please attach your code together with all the plots to your submission.