

Exercise Sheet #9

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Problem 1 (*Van der Pol Oscillator*)

6 Pts

Consider the Van der Pol oscillator, governed by the equations

$$\begin{aligned}\dot{x} &= y \\ \dot{y} &= \epsilon(1 - x^2)y - x\end{aligned}$$

Estimate the period of the limit cycle in the limit $\epsilon \gg 1$.

To do this, follow the analysis made in the class using the Liénhard variables. Then, use the definition of the period T of a limit cycle

$$T = \int_0^T dt,$$

and approximate the differential dt by only taking into account the movement along the slow branches.

Hint: Change dt to a dx and integrate over x . You need to find suitable limits for this.

Problem 2 (*Markov chain of umbrellas*)

6 Pts

Lady Ann has four umbrellas which she uses whenever it rains to go from work to home, or vice versa. She takes only an umbrella with her whenever it rains, leaving the umbrellas otherwise in the office and at home respectively. It rains with probability $p \in [0, 1]$. How often does Lady Ann get wet?

Problem 3 (*Langevin Equation*)

8 Pts

Consider the

$$m\dot{v} = -\gamma mv + \eta(t)$$

Langevin equation, where the $\eta(t)$ term denotes white noise:

$$\begin{aligned}\langle \eta(t)\eta(t') \rangle &= Q\delta(t - t') \\ \langle \eta(t) \rangle &= 0.\end{aligned}$$

Show that in thermal equilibrium

$$\langle v(t)\eta(t) \rangle = 2k_B\gamma T,$$

using the equipartition theorem, $\frac{1}{2}m \langle v^2(t) \rangle = \frac{k_B T}{2}$.