Exercise Sheet #3

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Problem 1 (*Probability generating functions*) 6 Pts

(a) Prove that the variance σ_k^2 of a probability distribution p_k with a generating functional $G_0(x) = \sum_k p_k x^k$ and mean $\mu_k = \langle k \rangle$ is given by $\sigma^2 = G_0''(1) + \langle k \rangle - \langle k \rangle^2$. (3 Pts)

(b) Consider now a cumulative process generated by $G_C(x) = G_N(G_0(x))$, with $G_N(z) = \sum_n p_n z^n$. Calculate the mean μ_C and variance σ_C^2 for this cumulative process. (3 Pts)

Problem 2 (*Preferential Attachment & Network Growth*) 8 Pts

Suppose papers are published more or less at a constant rate, and that the number of references in a paper is essentially constant in time, at an average of m. Let's assume that the probability of a paper to be cited by a new paper (that is, that a new vertex j in the network of papers will link to an older node i) is proportional to the number of papers citing the paper, $\Pi_i \propto k_i + C$, for some constant C. Consider also that initially there were m_0 papers without references.

(a) Derive an equation for the evolution of the degree k_i and find the degree distribution of the network. (3 Pts)

(b) Is it a scale free network? If so, what is the exponent? (3 Pts)

(c) Phenomenologically, what causes the difference with the Barabási-Albert model? (2 Pts)

Problem 3 (*Giant Component*) 6 Pts

Above the percolation transition a network contains a giant connected component, which consists of a finite fraction $S \leq 1$ of all N vertices and can be expressed as a function of the coordination number z.

In the course this is done for a network with a Poissonian degree distribution. Here we want to examine networks with an exponential degree distribution,

$$p_k \propto \mathrm{e}^{-k/\kappa},$$

with a positive constant κ.

(a) Normalize the distribution. (2 Pts)

(b) Calculate the generating functional and the coordination number for this distribution. (2 Pts)

(c) Derive the size of the giant component $S(z)$ as a function of the coordination number. Note that $z \geq 0$. (2 Pts)