Exercise Sheet #1

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Problem 1  (Random graphs 1)  (0 P.)

For the random graph given in Fig.1 evaluate:

(a) coordination number $z$,
(b) connection probability $p$,
(c) average distance $l$,
(d) and clustering coefficient $C$.

Problem 2  (Random graphs 2)  (0 P.)

Use a random number generator (dice, coin, roulette wheel, software...) to generate five random graphs with $N = 5$ vertices and connection probability $p = 2/3$.

(a) Evaluate the degree distribution for each graph and the degree distribution of the ensemble average.

(b) Derive a formula for the probability that a node has k edges.

Compare the simulation results (a) with the prediction via (b).

Figure 1: Random graph with N=8 vertices
Problem 3  (Driven harmonic oscillator)  (0 P.)

Investigate the dynamics of the damped driven harmonic oscillator

\[ \ddot{x}(t) = -\omega_0^2 x(t) - \alpha \dot{x}(t) + f_0 \cos(\Omega t + \phi_0), \]  

where \( \omega_0 \) is the natural frequency of the oscillator and \( \alpha \) is the damping factor.

The oscillator is driven by an external harmonic force with amplitude \( f_0 \), driving frequency \( \Omega \) and initial phase \( \phi_0 \).

Solve Eq.(1) for some initial conditions \( x(t_0) = x_0, \dot{x}(t_0) = v_0 \) analytically.

Then sketch the system’s long-time evolution, i.e. \( x(t) \) for \( t \gg t_0 \), for each of the following cases:

(a) weak damping without driving: \( \frac{\alpha}{2} < \omega_0, f_0 = 0 \)

(b) strong damping without driving: \( \frac{\alpha}{2} > \omega_0, f_0 = 0 \)

(c) very weak damping in presence of ext. force: \( \frac{\alpha}{2} \ll \omega_0, f_0 \neq 0 \)

(d) very weak damping, i.e. energy up-take, with or without driving: \( \alpha < 0 \)

Compare these four qualitatively different long-term dynamics!