Generating functionals for guided self-organization

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background: universality

homo sapiens sapiens

– cognitive information processing

– (diffusive) emotional control system

$\gg$ (essentially) unchanged since 30 000 years? $\ll$

$*$ functional in (extremly) diverse environments:
natural, social, virtual, ..

$*$ encoded by (only) $\sim 10^4$ genes

$\gg$ universality: abstracting from semantic content $\ll$

background: diffusive control

separation of time scales

fast	slow
$x \quad w$	$\gamma \quad \gamma'$

$\dot{x}$	:	cognitive information processing
$\dot{w}$	:	learning
$\dot{\gamma}$	:	metalearning
$\dot{\gamma}'$	=	0

* metalearning: adaption of slow variables
* diffusive control: metalearning indpendent of semantic content

guided self-organization

[Ay, Der, Prokopenko]

guided self-organization

$\equiv$ controlling self-organization diffusively

universal organizing principles

$\equiv$ abstracting from semantic content

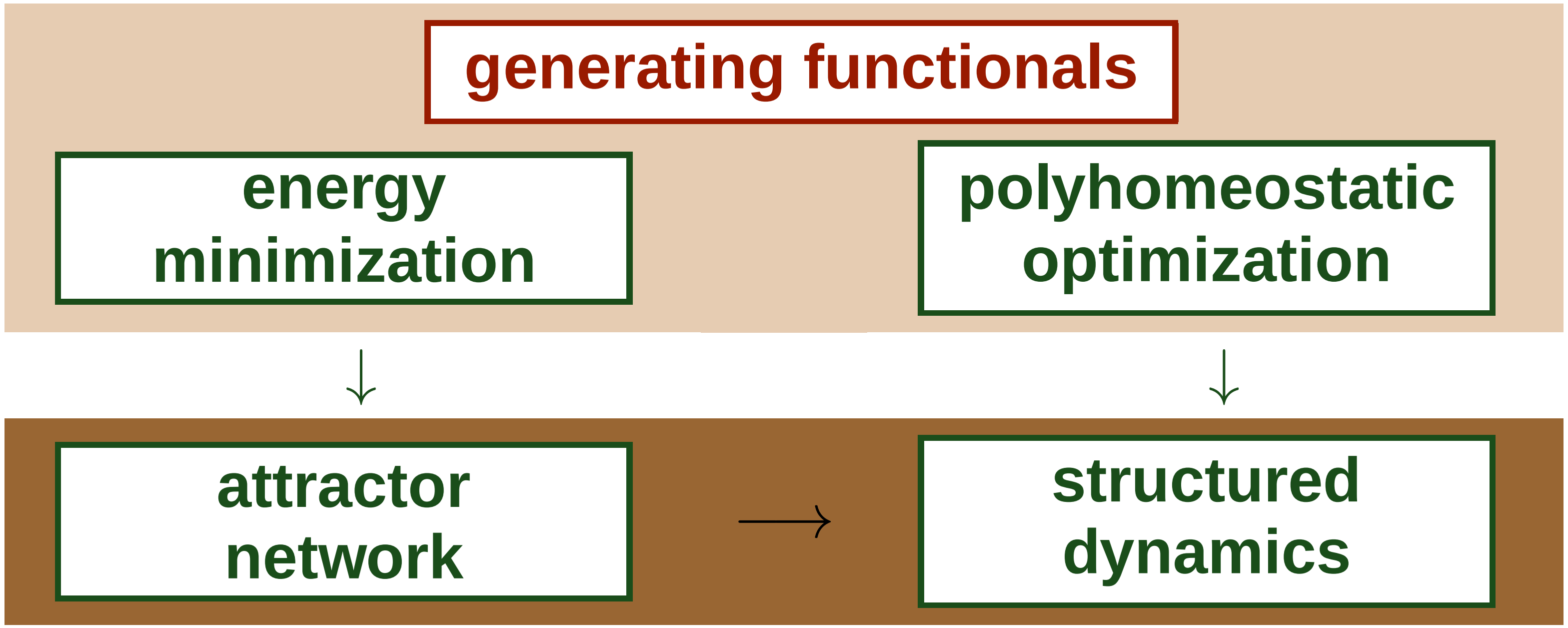
generating functionals

research goal

formulation of objective functions (generating functionals)
for complex, information processing, systems

modular and autonomous cognitive systems

* modularity, locality, self-adaption
* long-term goal: modules for autonomous cognitive systems
* here: interplay (stress) between objectives



overview

polyhomeostasis

- updating time-averaged distribution functions
- generation of structured dynamics
  - ▷ chaotic / intermittent bursting / synchronized

energy functionals

- transient state dynamics
- attractor relic networks

polyhomeostatic optimization

homeostasis

» keep in balance «

- a single scalar quantity

...
blood-sugar level
hormonal levels
body temperature
...
airplane velocity
furnace temperature
....

polyhomeostasis

- multiple scalar quantities

» keep in relative balance «

allocation & polyhomeostasis

time allocation

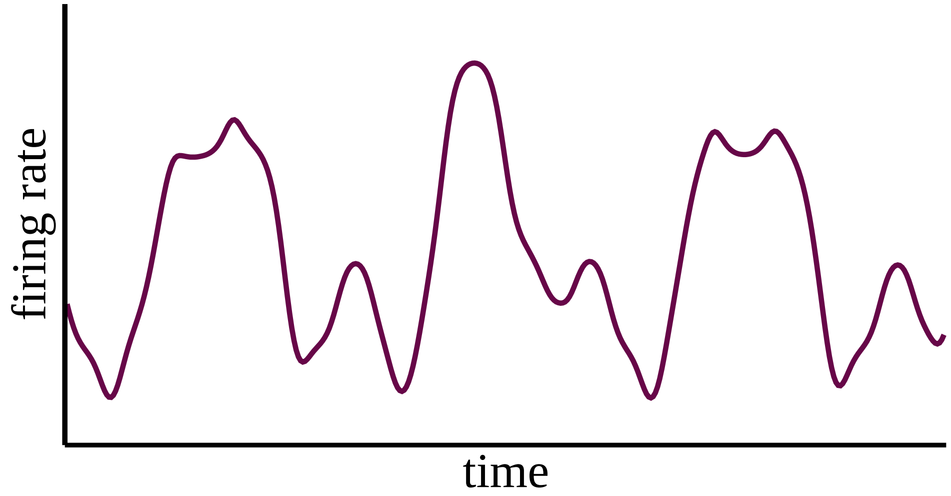
- individual target distribution functions
  - ▷ e.g. 60% working
    20% socializing
    20% eating / drinking

- dynamical process
  - $\hat{=}$ time allocation
  - $\hat{=}$ optimization of target distribution function

time allocation of neural activity

neural firing rate

- achieve maximal information content / transmission



- firing-rate distribution $$p(y) = \frac{1}{T}\int_0^T \delta\big(y - y(t-\tau)\big)\, d\tau$$

Shannon (information) entropy

$$H[p] \; = \; -\int dy\, p(y) \log p(y) \; \geq \; 0$$

maximal information distribution

maximal Shannon entropy $H[p]$

$$\text{no constraints} \quad \to \quad p(y) \sim \text{const.}$$

$$\text{given mean} \quad \to \quad p_\mu(y) \sim \exp(-y/\mu), \qquad \mu = \int y\, p(y)\, dy$$

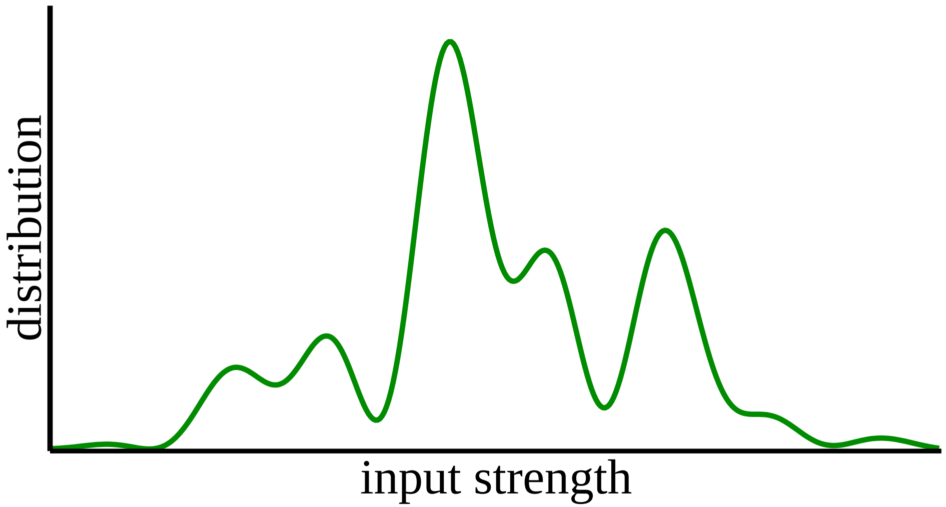
- target firing-rate distribution $\boxed{p_\mu(y)}$ (polyhomeostasis)

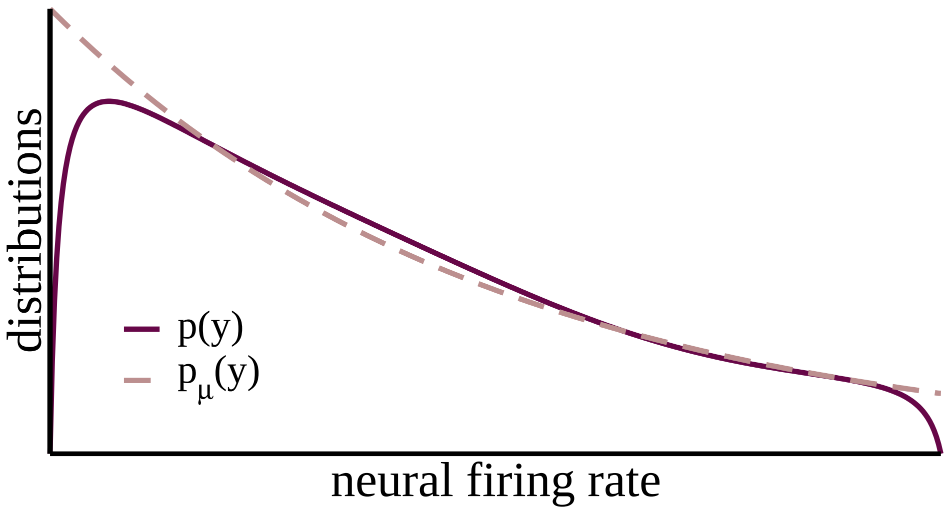
Kullback-Leibler divergence

$$D(p, p_\mu) \; = \; \int p(y) \log\left(\frac{p(y)}{p_\mu(y)}\right) dy \; \geq \; 0$$

- asymmetric measure for the distance of two probability distribution functions

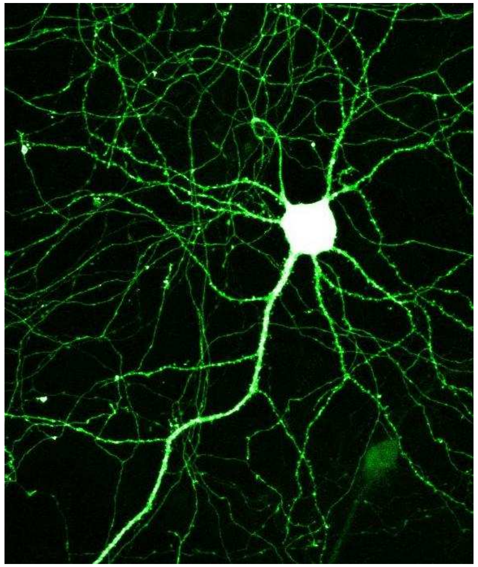
intrinsic plasticity





adaption of internal neural parameters

input



output

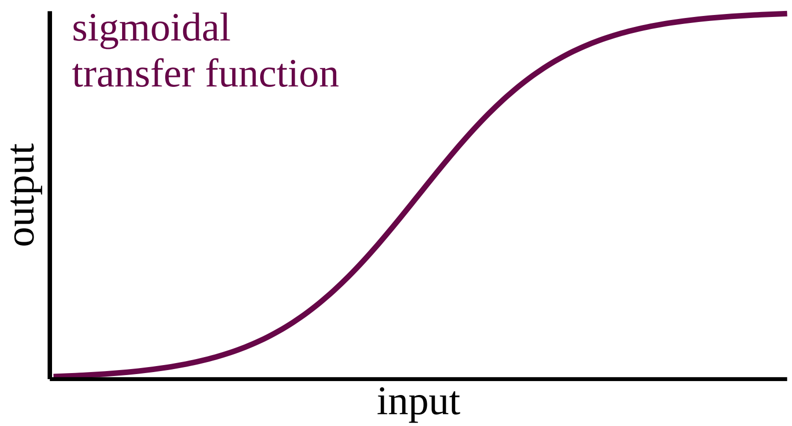
via non-linear neural transfer function

generating functional for polyhomeostasis

minimization of Kullback-Leibler divergence

$$D_{a,b}(p,p_\mu) \;=\; \int p(y)\log\left(\frac{p(y)}{p_\mu(y)}\right)dy \qquad\qquad y \;=\; \frac{1}{e^{-a(x-b)}+1}$$

- rate-encoding neurons
  - ▷ gain a
  - ▷ threshold b



- D mimimal with respect to $a,\; b$

generating stochastic adaption rules

- distributions of input / output $p(x)$ / $p(y)$

$$D = \int p(y) \log\left(\frac{p(y)}{p_y(y)}\right) dy \equiv \int p(x)\, d(x)\, dx$$

with

$$p(y)\, dy = p(x)\, dx, \qquad d(x) \equiv \log(p) - \log(\partial y/\partial x) - \log(p_\lambda)$$

adaption rules for all input statistics

$$\left[\, \delta D = 0, \quad \forall p(x) \,\right] \quad \Longrightarrow \quad \delta d = 0$$

stochastic adaption rules

instantaneous adaption rates

$$\frac{d}{dt}a \;=\; -\epsilon_a \frac{\partial d(x)}{\partial a}$$

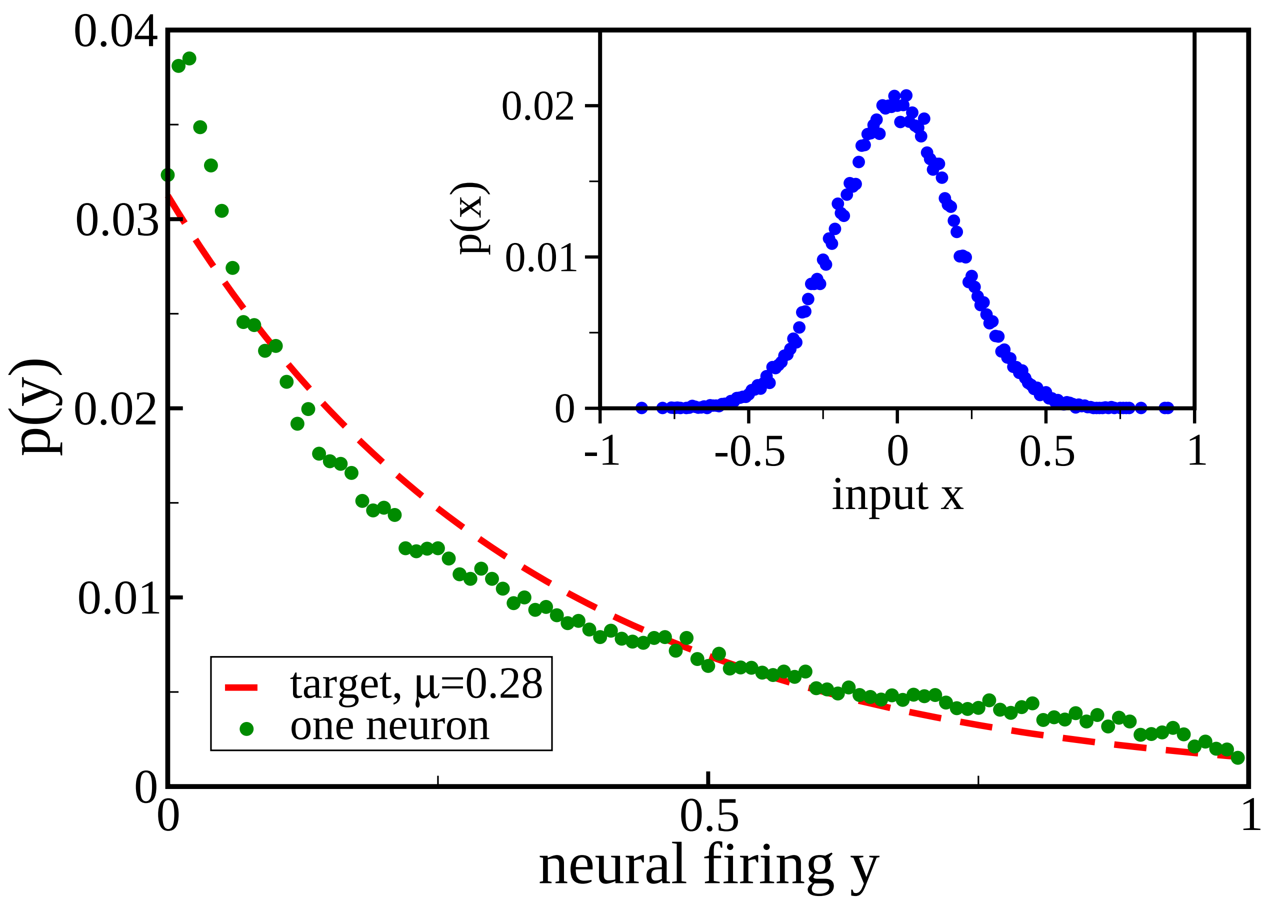
- average over time $\hat{=}$ average over $p(x)$
- adpation rate ϵ_a

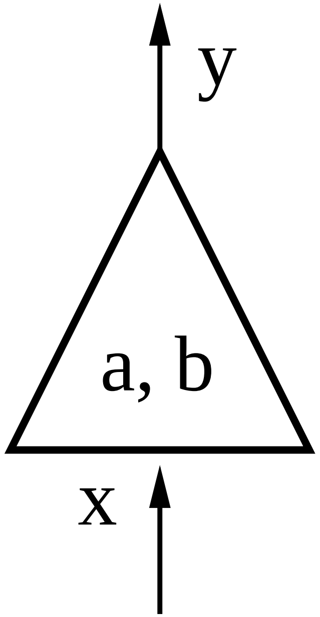
stochastic adaption rules

$$\begin{aligned}\frac{da}{dt} &\propto \Big(1-2y+y(1-y)\lambda\Big)(x-b)+\frac{1}{a}\\ \frac{db}{dt} &\propto \Big(1-2y+y(1-y)\lambda\Big)(-a)\end{aligned}$$

[Triesch, ‘05]

feed-forward polyhomeostasis

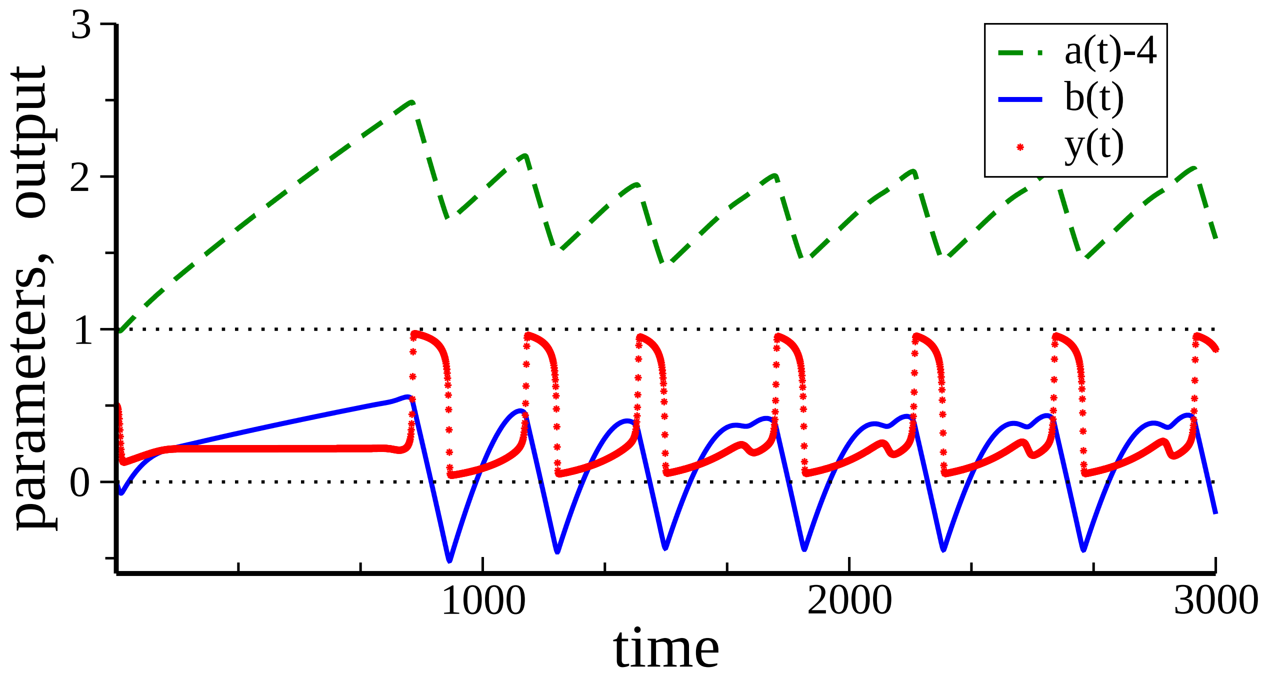


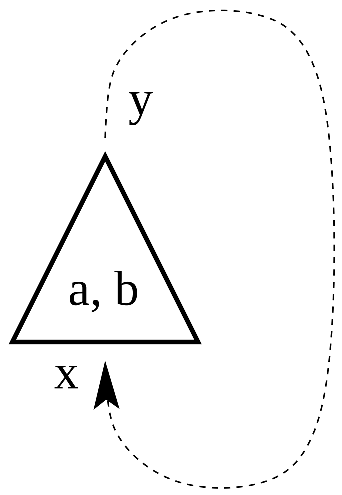


$p(x)$: given input distribution

$p(y)$: output distribution,
after adapting the transfer function polyhomeostatically

autapse: self-coupled neuron





[Marković & Gros, PRL ‘10]

polyhomeostatic optimization induces continuous, self-contained neural activity

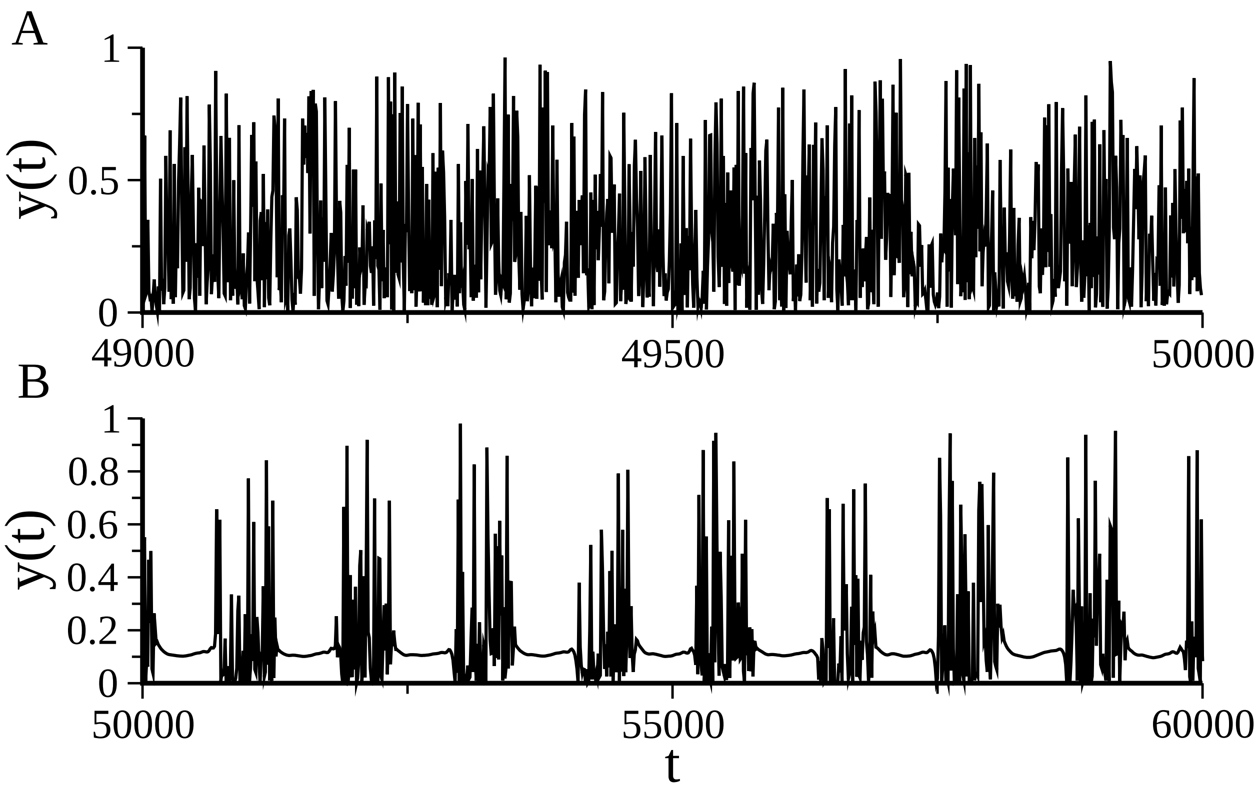
▷ limiting cycle

network of polyhomeostatic neurons

$$x_i(t) \;=\; \sum_{j\neq i} w_{ij} y_j(t) \qquad\qquad w_{ij} = \pm 1/\sqrt{N-1}$$ randomly

$\mu = 0.28$

$\mu = 0.15$



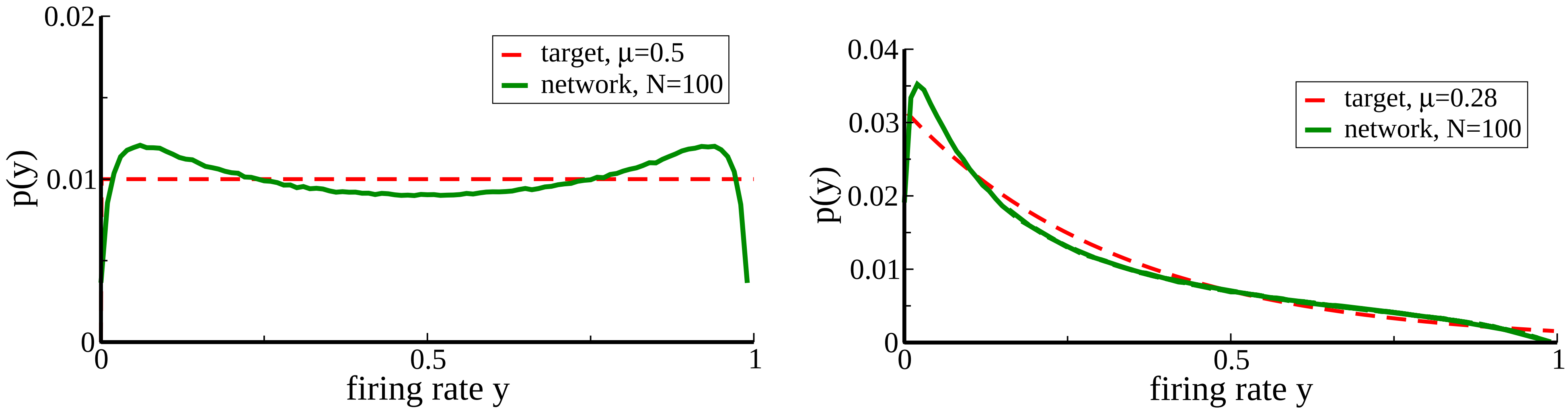
$N = 100$

- self-organized chaos
- spontaneous intermittent bursting

} **diffusively controlled**

polyhomeostatic optimization

distribution of averaged neural activities



first step

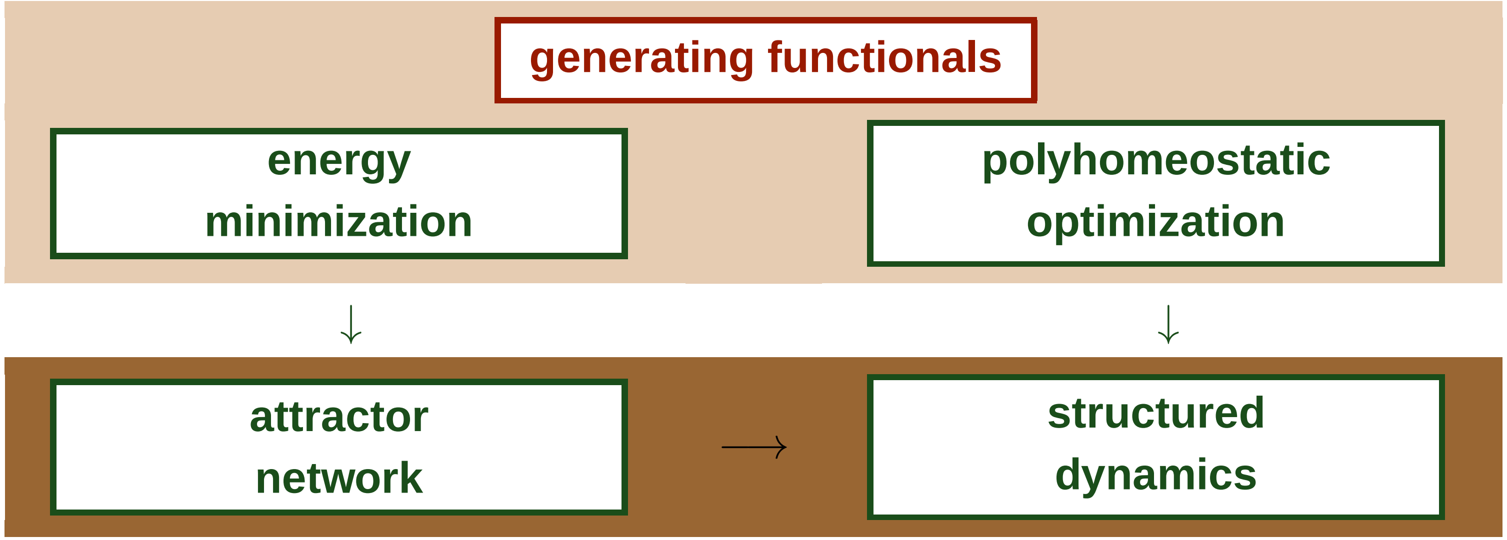
generating functional for instrinsic plasticity

next step

generating functional for neural dynamics

⋮

competing generating functionals



- energy functional: Hopfield

$$E(\{x_k\}) = \frac{1}{2}\sum_{kl}\Big(y_k w_{kl} y_l\Big)^2 - \frac{\Gamma}{2}\sum_k x_k^2$$

leaky Integrator

attractor network - continuous time

$$E(\{x_k\}) \;=\; \frac{1}{2}\sum_{kl}\left(y_k w_{kl} y_l\right)^2 - \frac{\Gamma}{2}\sum_k x_k^2$$

* membrane potential $x_i(t)$
* firing rate $y_i(t)$

$$\frac{d}{dt}x_i \;\equiv\; \frac{d}{dx_i}E(\{x_k\}) \;=\; -\Gamma x_i + a_i y_i(1-y_i)\sum_j w_{ij} y_j$$

normally neglected: $a_i y_i(1-y_i)$

* transfer function: $y(x) \;=\; \dfrac{1}{1+e^{a(b-x)}}$
* adapted polyhomeostatically: $a,\ b$

transient state dynamics

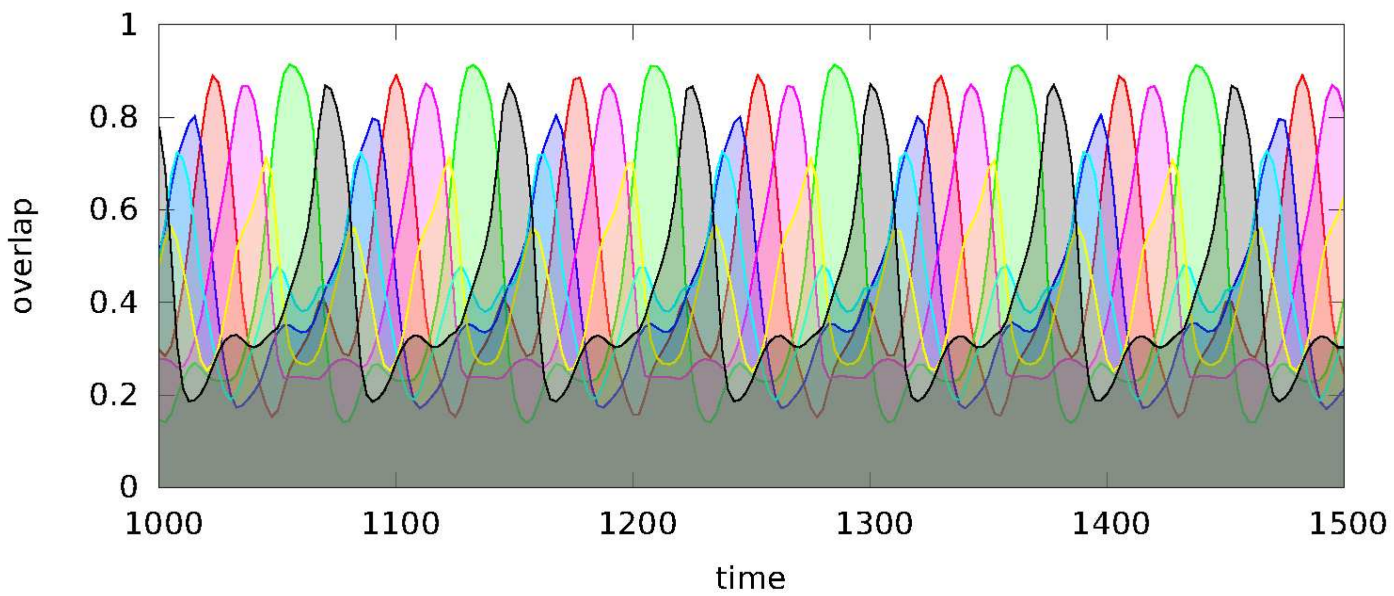
$$w_{ij} = \frac{1}{N_p} \sum_{\alpha} \xi_i^{(\alpha)} x_j^{(\alpha)}$$

for convenience

Hopfield patterns: $\xi_i^{(\alpha)}$

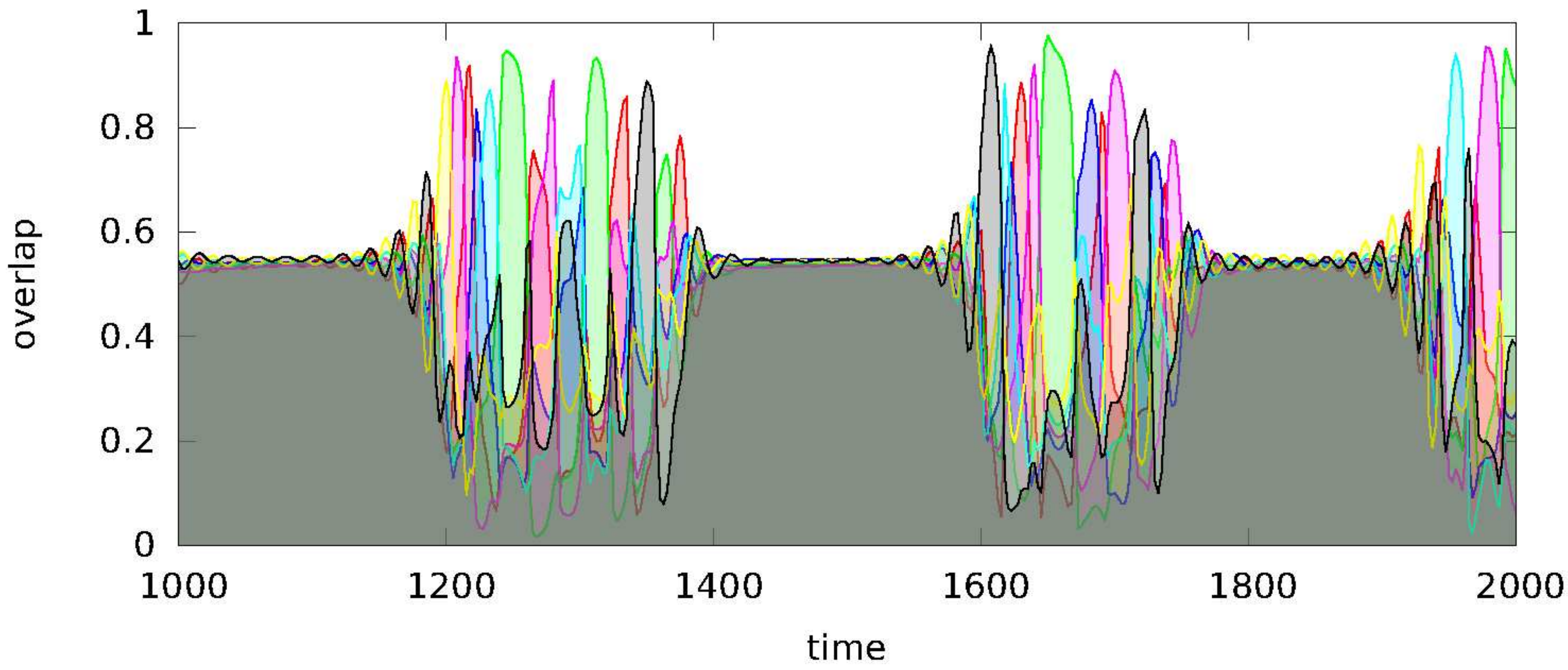
overlap firing $y_i - x_i^{(\alpha)}$ **patterns**

[Linkerhand & Gros]



competing objective functions

bursting transient state dynamics

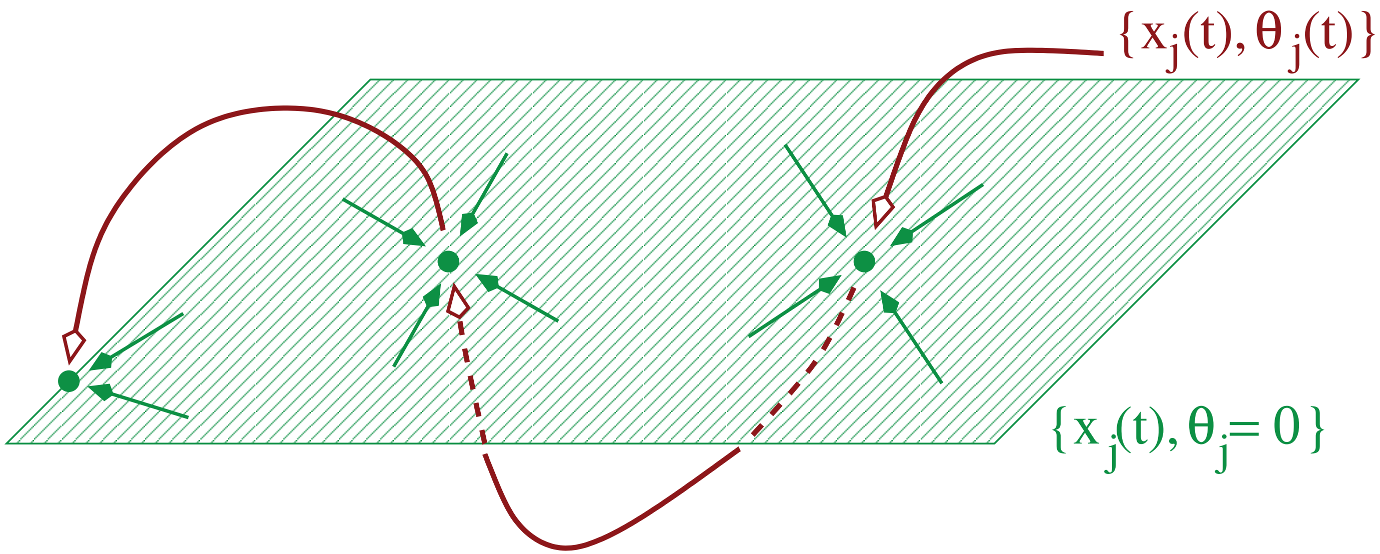


target activtiy $\mu = 0.15$

$\langle \xi_i^{(\alpha)} \rangle = 0.3$ mean activity of attractors

$\Rightarrow$ guiding self-organization

attractor relic networks



fixpoints $\not\leftrightarrow$ polyhomeostasis

constant activities — distribution of activities

attractors turn into attractor relics

perspective

generating functionals define modules

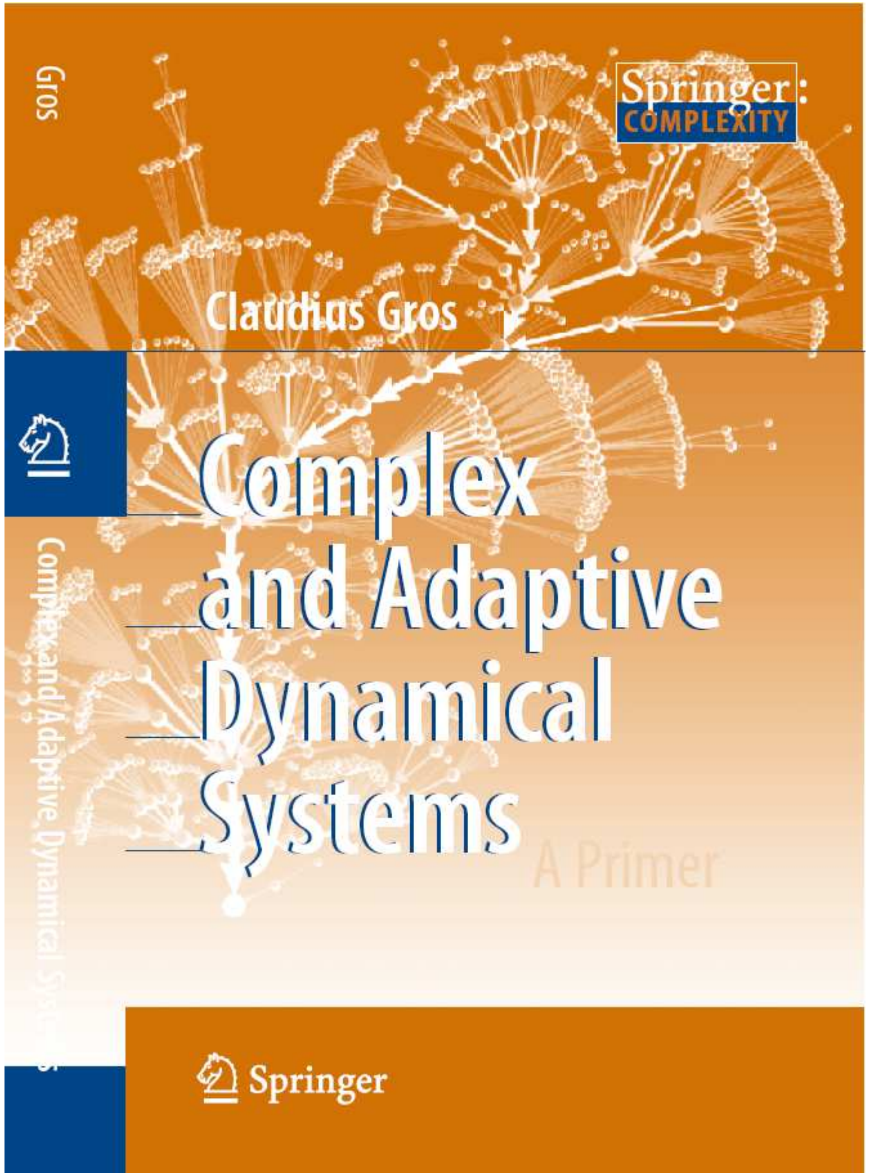
- guiding self organisation
- intermodular competition

modules for cognitive systems

- self adapting internally
- interfaces fully adapting
- for: sensory-motor loop
  internal dynamics
  synaptic plasticity
  sensory stream analysis
  . . .

... generating complex systems

graduate level textbook



Complex and Adaptive Dynamical Systems, a Primer

- The small world phenomenon in social and scale-free networks
- Phase transitions and self-organized criticality
- Life at the edge of chaos and coevolutionary avalanches
- Living dynamical systems and emotional diffusive control within cognitive system theory

(Springer, 2008, second edition 2010)