Vertex Routing Models

and

Polyhomeostatic Optimization

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Vertex Routing Models

– modelling conserved information flow –

[Markovic & Gros, NJP ‘09]

Polyhomeostatic Optimization

– a new paradigm for adaptive dynamical systems –

[Markovic & Gros, PRL ‘10]

vertex routing models

motivations

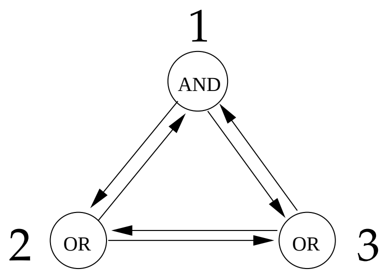
- criticality in dynamical systems
- information routing in networks
- cognitive processing via transient-state dynamics

criticality in dynamical systems

- $\exists$ conserved quantity
- polynomial scaling?
- $K = 2$ Kauffman network

random boolean networks

- Kauffman networks / NK-networks
  - ▷ N: network size
  - ▷ K: in-connectivity
  - ▷ random boolean functions



OR

1	2	3
0	0	0
0	1	1
1	0	1
1	1	1

OR

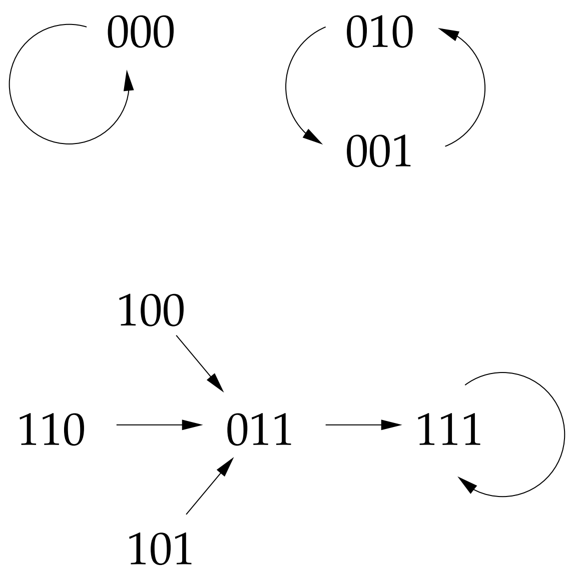
1	3	2
0	0	0
0	1	1
1	0	1
1	1	1

AND

2	3	1
0	0	0
0	1	0
1	0	0
1	1	1

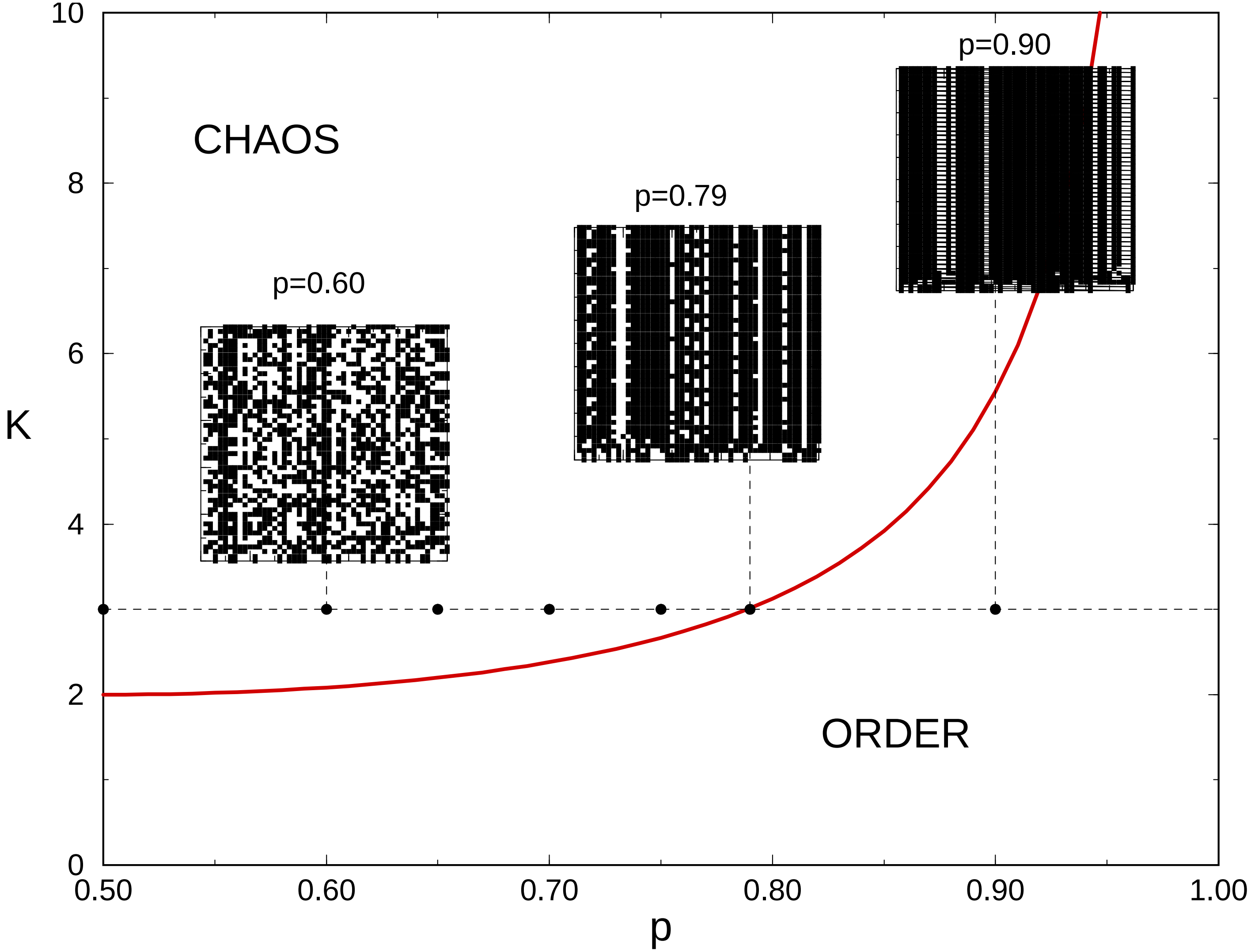
here: $N = 3$, $K = 2$

fixpoints and cycles



[Luque & Sole, ‘00]

phase transitions in boolean networks



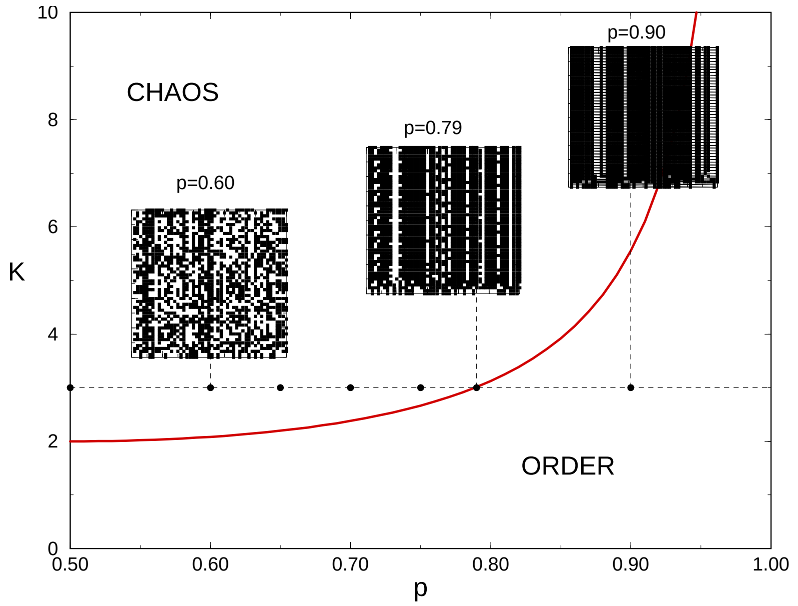
K: in-connectivity
p: magnetization

[Luque & Sole, ‘00]

life at the edge of chaos

competition: daily survival $\leftrightarrow$ evolutionary fitness

- gene regulation networks
  - ▷ basis of all living



frozen (regular) phase	chaotic phase
deterministic dynamics	irregular dynamics
• good for daily survival	• bad for daily survival
• bad for evolutionary adaption	• good for evolutionary adaption

[Kauffman '69]

critical boolean networks

scaling at criticality ($K = 2$)?

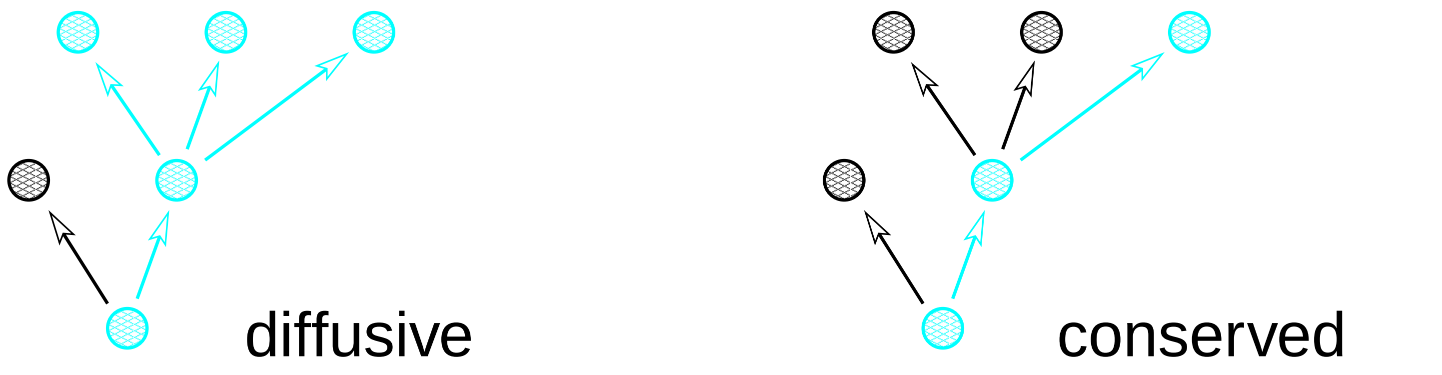
▷ number of attractors/cycles

gene regulations networks

Kauffman '69: $\sim \sqrt{N}$ $\Rightarrow$ cell differentiation

Samuelsson & Troein '03: $> O(N^p)$ (any p)

... and for other critical dynamical systems?



information routing vs. transmision

information transmission

- ▷ vertex $\Rightarrow$ vertex
- ▷ phase space: number of vertices

information routing

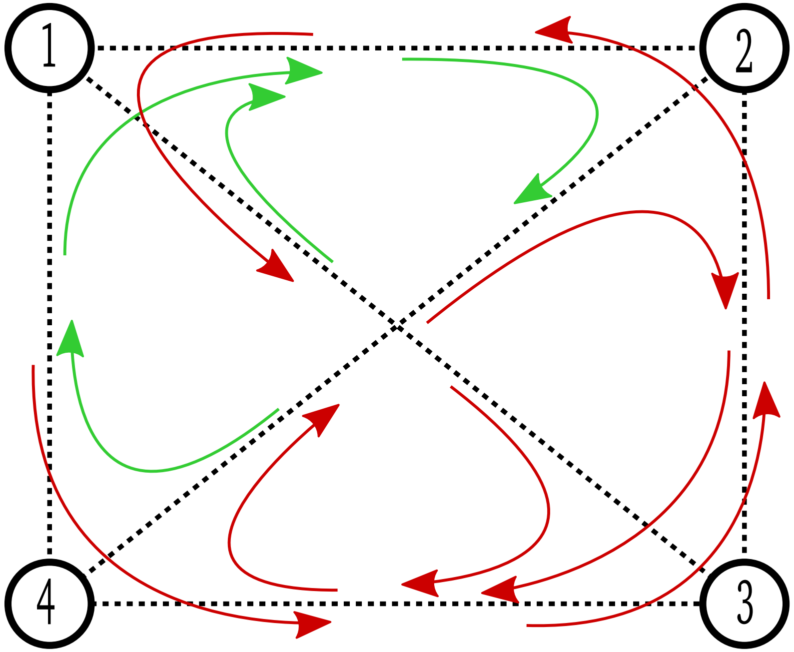
- ▷ link (incomming) $\Rightarrow$ link (outgoing)
- ▷ phase space: number of directed links

routing dynamics

- random routing tables at every vertex
  - ▷ quenched dynamics (fixed routing tables)

$(1) \to (2) \to (4) \to \dots$

$(2) \to (3) \to (4) \to \dots$



information centrality

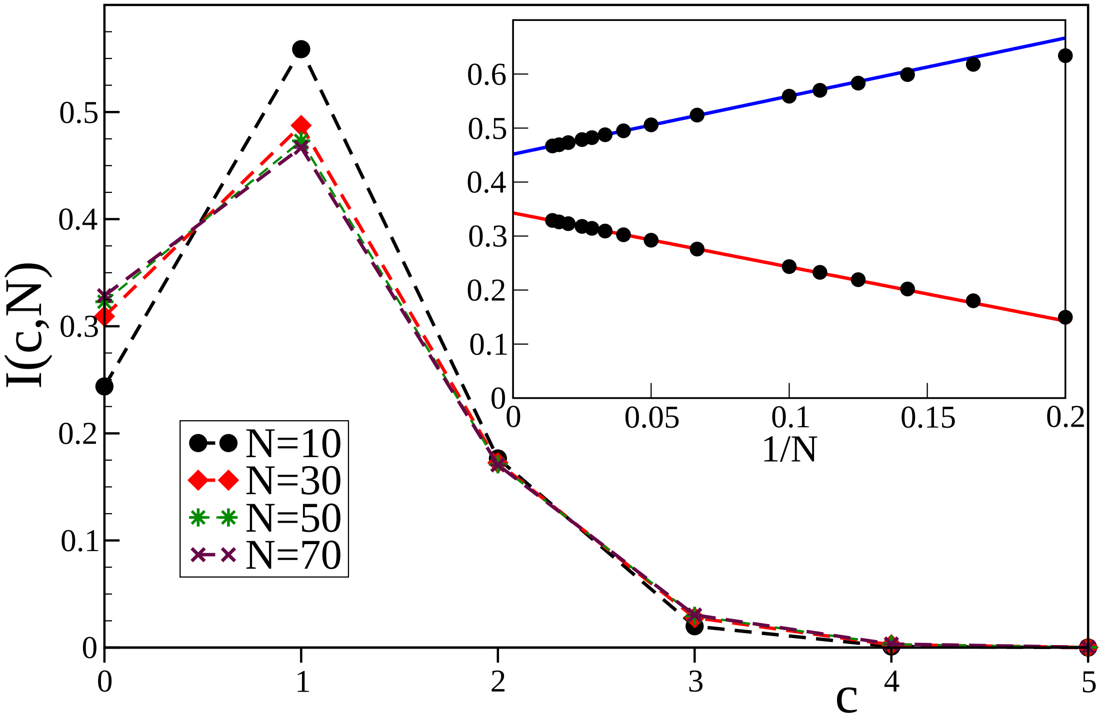
- ▷ overlapping cyclic attractors

number of attractors passing through a given vertex

information centrality

numerical simulations

fully connected graphs



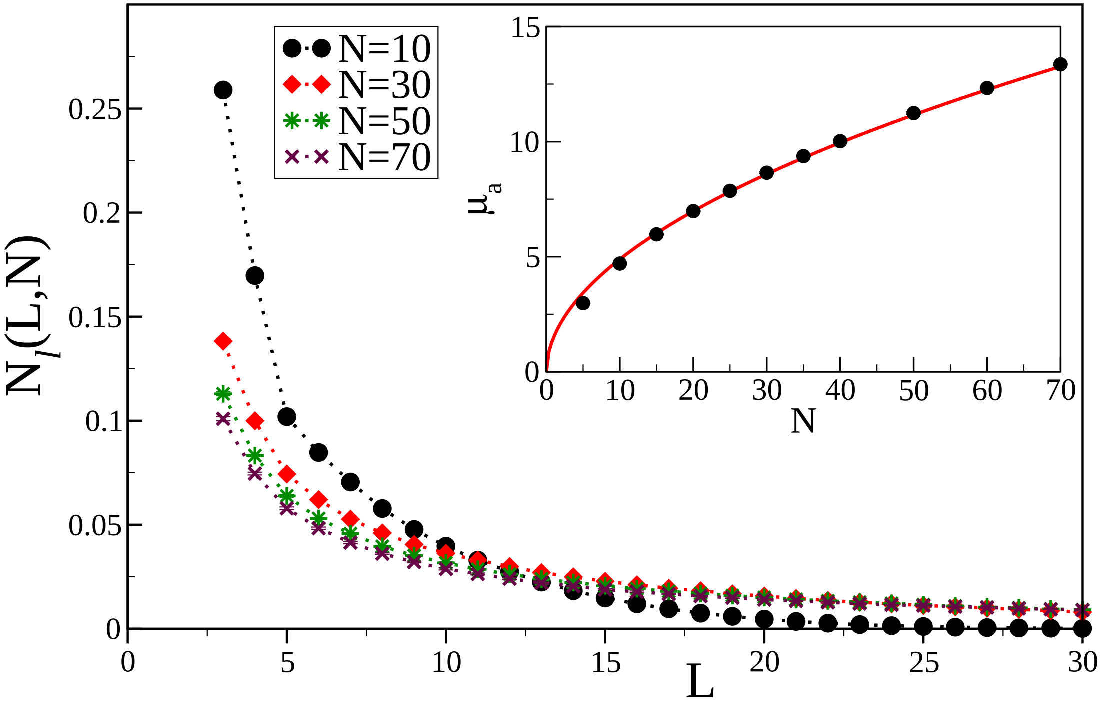
[Markovic & Gros, NJP '09]

democratic distribution of the information centrality

cycle-length distribution

numerical simulations

fully connected graphs



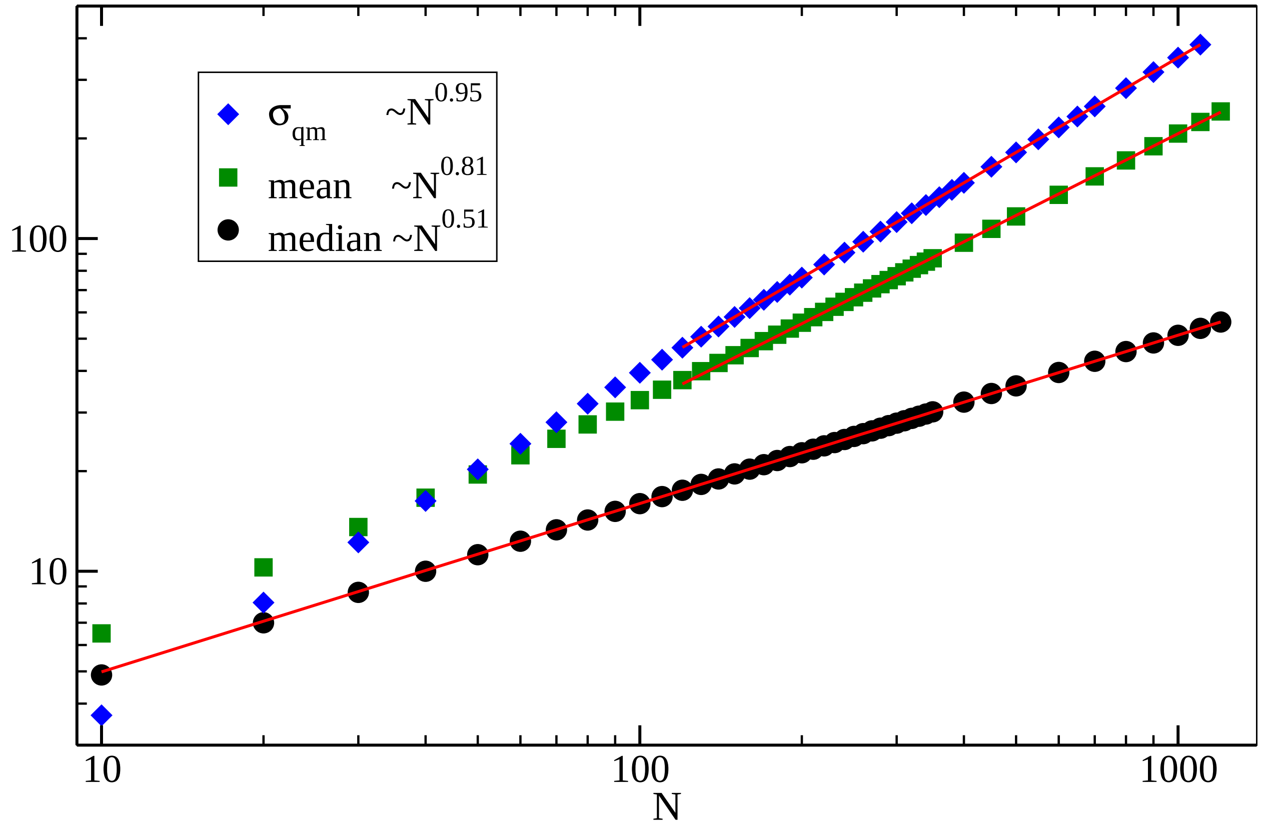
median $\mu_a(N)$ (half above/below)

$$\mu_a(N) \propto \sqrt{N}$$

scaling of mean cycle length

analytical & numerical

fully connected graphs



[Schuelein, Markovic & Gros, in prep]

non-trivial exponent

$$\langle L \rangle \propto N^{0.81}$$

$\langle L \rangle$: average cycle length

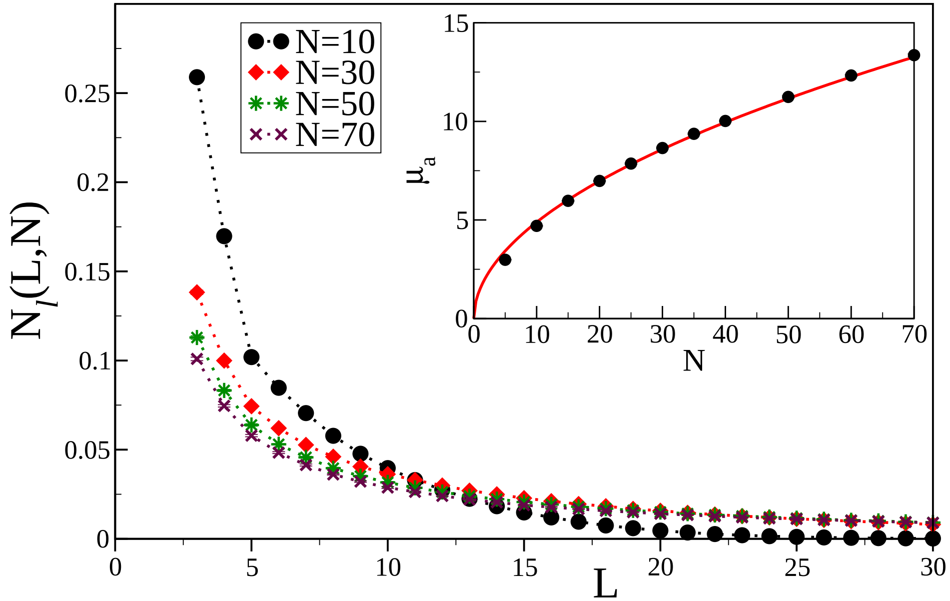
criticality in vertex routing models

non-trivial distribution of (cylic) attractors

$$\frac{\text{mean}}{\text{median}} \propto \frac{N^{0.81}}{N^{0.51}} = N^{0.3}$$

$$\frac{\sigma}{\text{mean}} \propto \frac{N^{0.95}}{N^{0.81}} = N^{0.14}$$

▷ fat tails

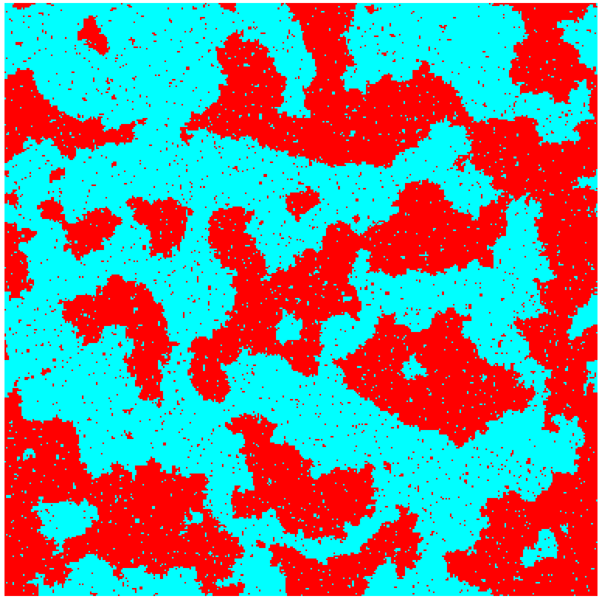


scale invariance

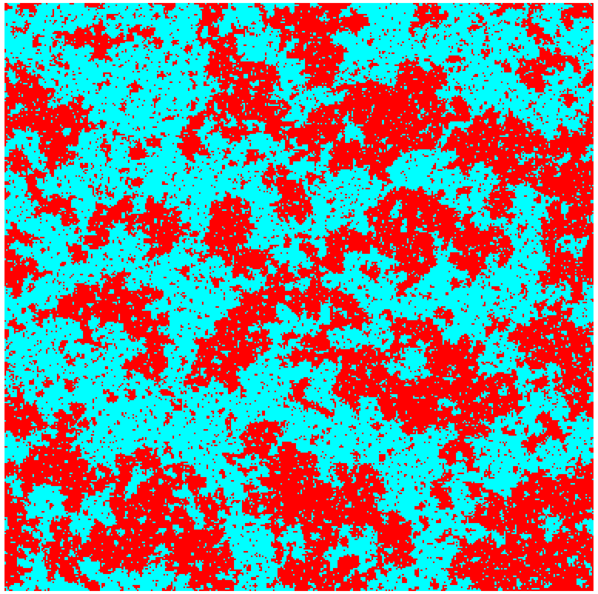
- fully connected graphs: scale-invariant
- Erdös-Rényi graphs: work in progrress

criticality in complex systems

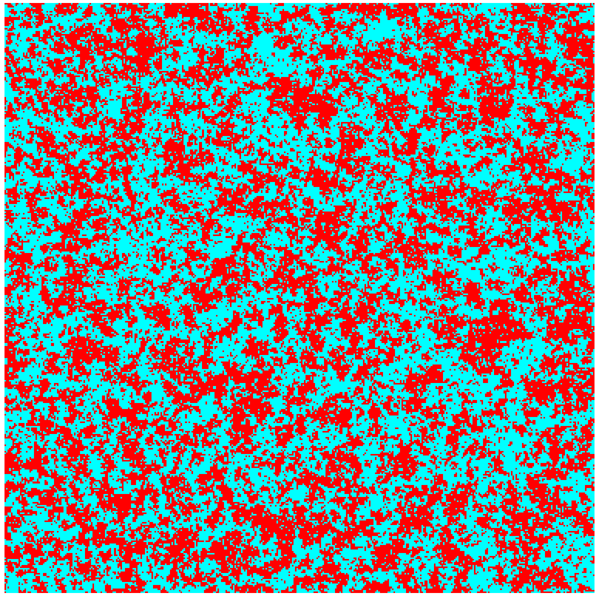
thermodynamic systems – 2D Ising



frozen



critical



chaotic

- critical thermodynamic systems are scale invariant
- and critical dynamical systems?

  ▷ NK-networks $\to$ no? $\qquad \Omega \sim 2^N$

  ▷ vertex routing models $\to$ yes? $\qquad \Omega \sim N(N-1)$

polyhomeostatic optimization

homeostasis

» keep in balance «

- a single scalar quantity

...
blood-sugar level
hormonal levels
body temperature
...
airplane velocity
furnace temperature
....

polyhomeostasis

- multiple scalar quantities

» keep in relative balance «

allocation problems

time allocation

- individual target distribution functions
  - ▷ e.g. 80% working
    20% socializing

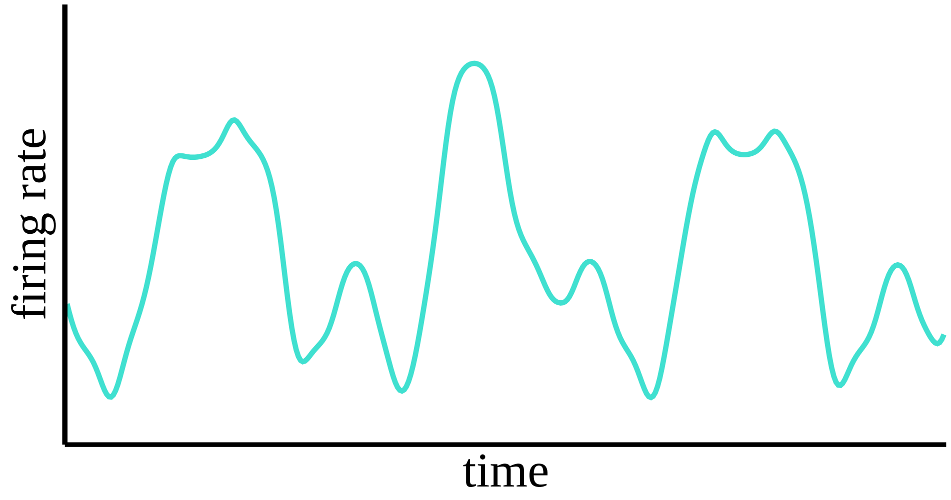
polyhomeostatic games

- goal

  achieve target distribution function

firing-rate distributions

allocation of neural activities



▷ target

» maximal information transmission «

Shannon (information) entropy

$$H[p] = -\int dy\, p(y) \log p(y) \geq 0$$

- firing-rate distribution $p(y)$, $\int dy\, p(y) = 1$

maximal information distribution

maximal Shannon entropy $H[p]$

no contraints $\rightarrow$ $p(y) \sim$ const.

given mean $\rightarrow$ $p_\mu(y) \sim \exp(-y/\mu)$, $\mu = \int y\, p(y)\, dy$

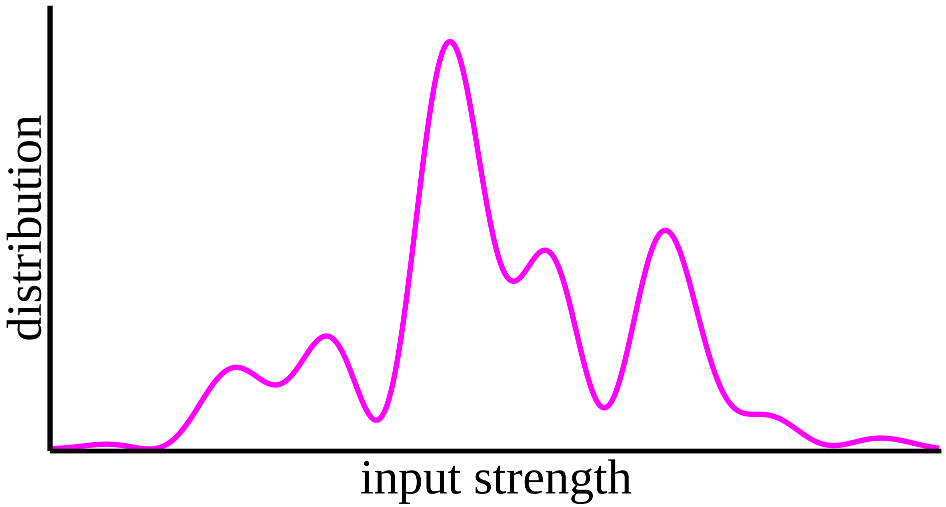
- energy constraints
  - ▷ target firing-rate distribution $p_\mu(y)$ (polyhomeostasis)

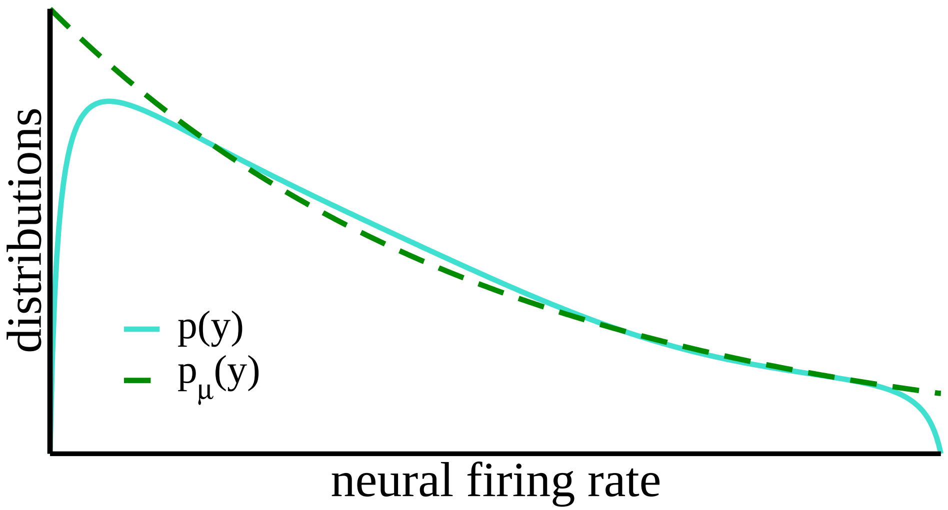
Kullback-Leibler divergence

$$D(p, p_\mu) = \int p(y) \log\left(\frac{p(y)}{p_\mu(y)}\right) dy \geq 0$$

- asymmetric measure for the distance of two probability distribution functions

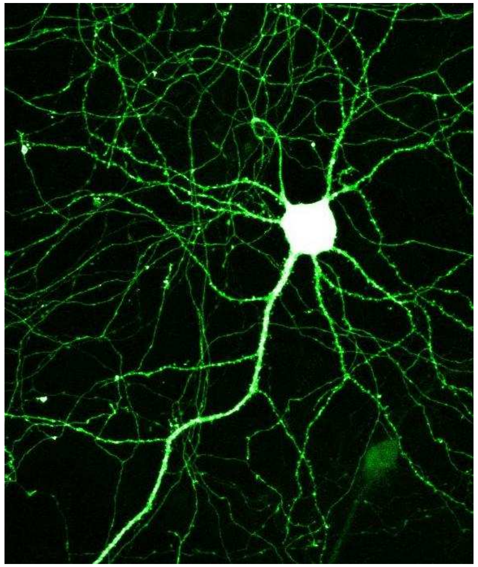
intrinsic plasticity





adaption of internal neural parameters

input



output

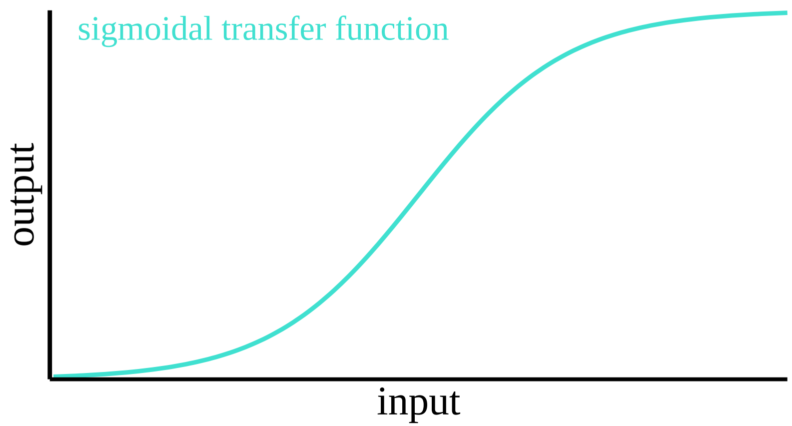
via non-linear neural transfer function

stochastic adaption

minimization of Kullback-Leibler divergence

$$D_{a,b}(p, p_\mu) = \int p(y) \log\left(\frac{p(y)}{p_\mu(y)}\right) dy \qquad y(t) = \frac{1}{e^{-ax(t-1)-b} + 1}$$

- rate-encoding neurons
  - ▷ gain a
  - ▷ threshold $-b/a$

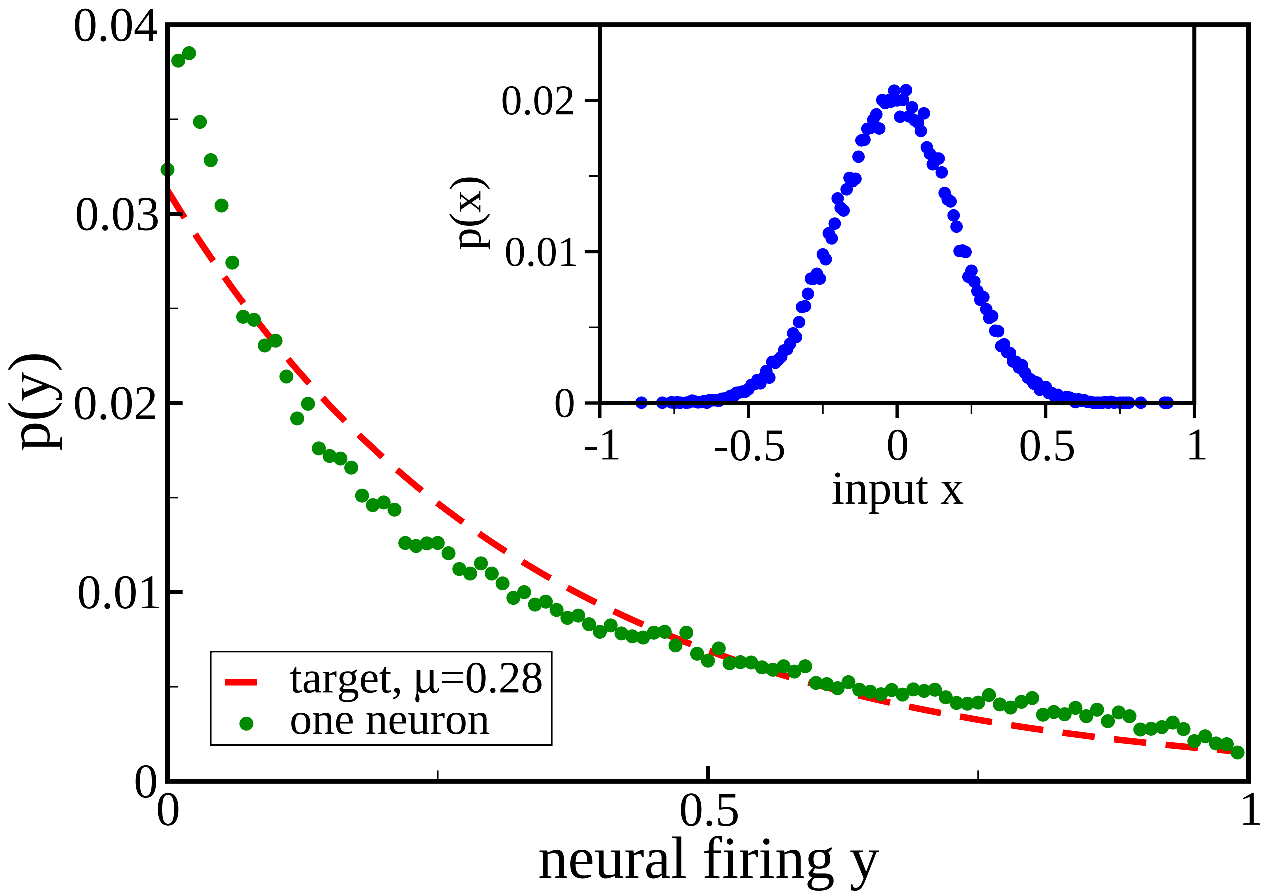


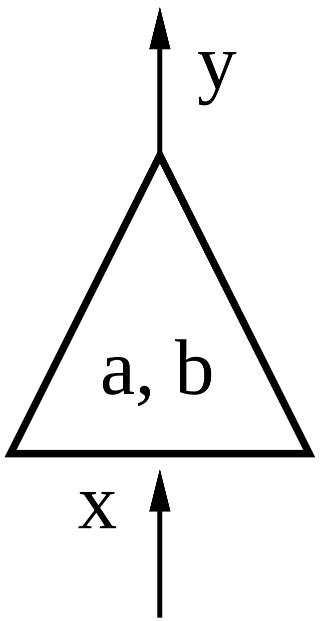
stochastic adaption rules

$$\begin{aligned}\Delta a &\propto 1/a + x\left(1 - (2+\lambda)y + \lambda y^2\right)\\ \Delta b &\propto 1 - (2+\lambda)y + \lambda y^2\end{aligned}$$

[Triesch, '05]

feed-forward polyhomeostasis

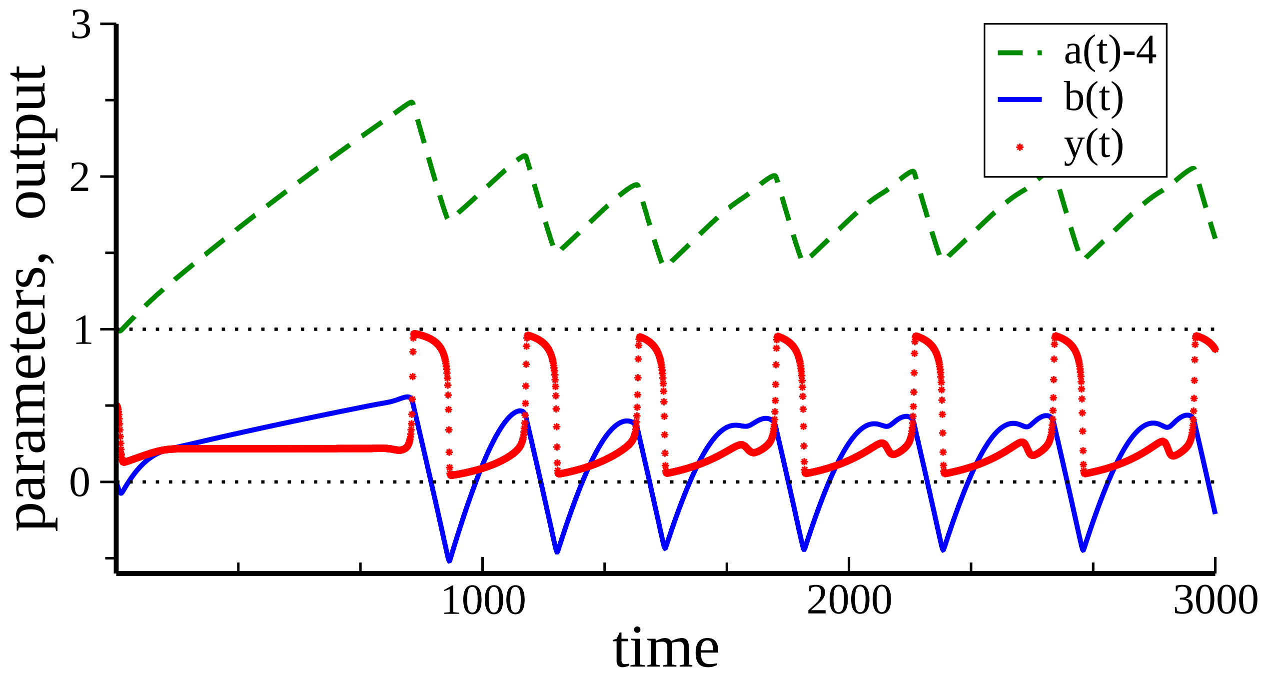


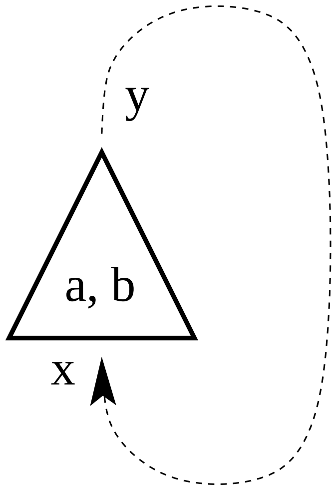


$p(x)$: given input distribution

$p(y)$: output distribution

autapse: self-coupled neuron





[Markovic & Gros, PRL '10]

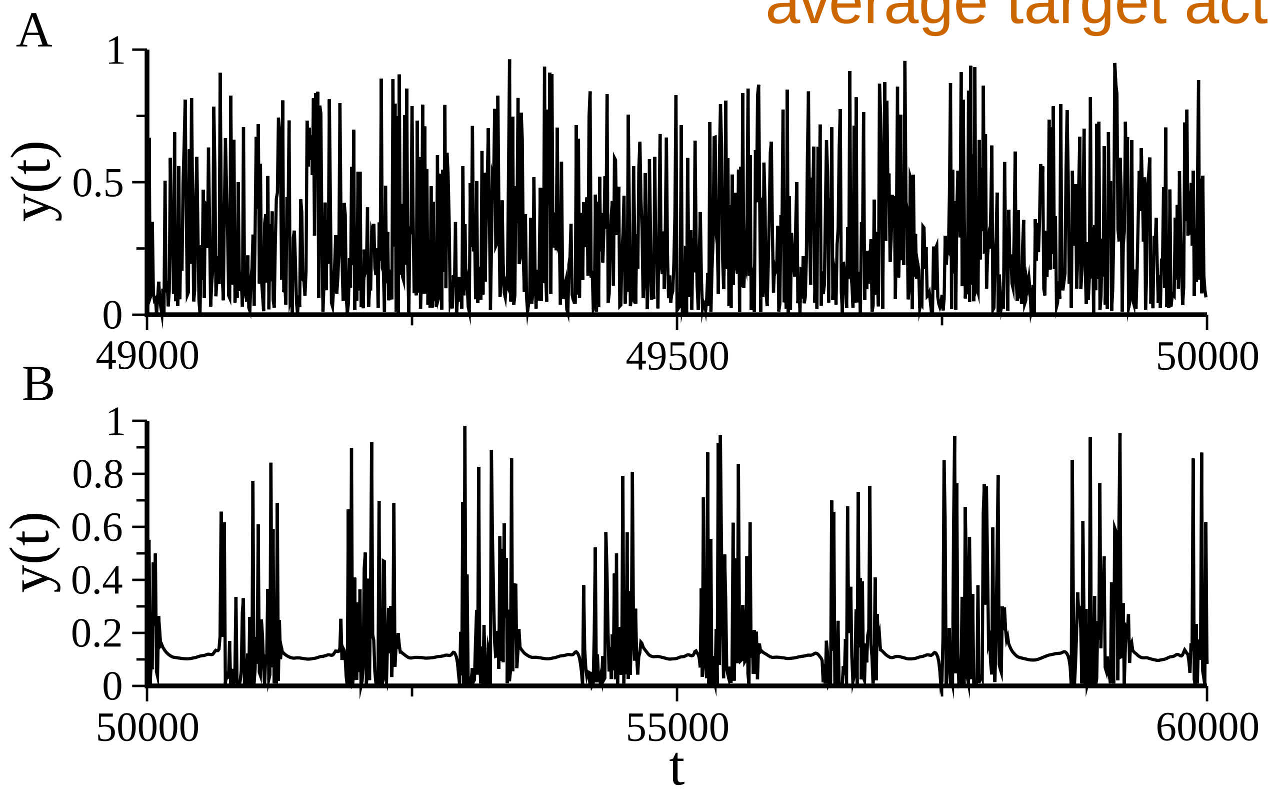
polyhomeostatic optimization induces continuous, self-contained neural activity

▷ limiting cycle

network of polyhomeostatic neurons

$x_i(t) = \sum_{j\neq i} w_{ij} y_j(t)$ $\qquad$ $w_{ij} = \pm 1/\sqrt{N-1}$ randomly

average target activities $\mu = 0.28$
0.15



$N = 100$

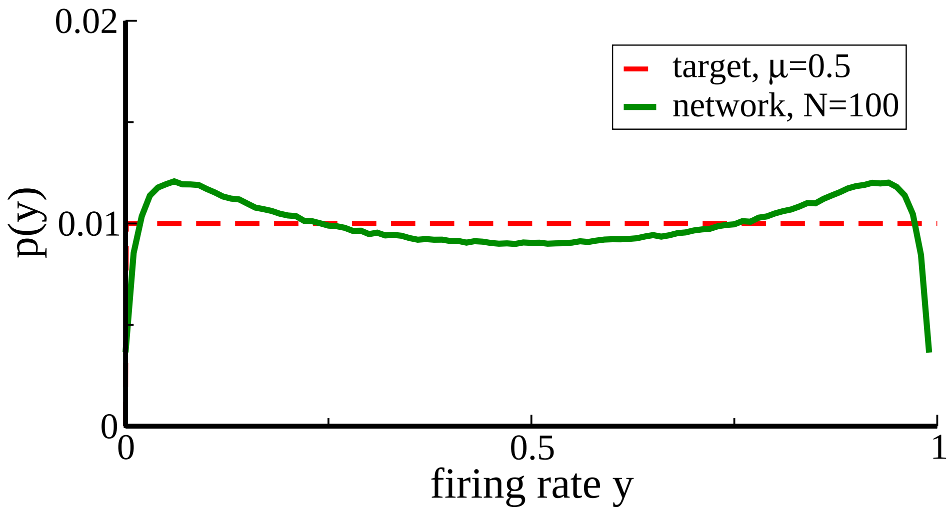
[Markovic & Gros, PRL ‘10]

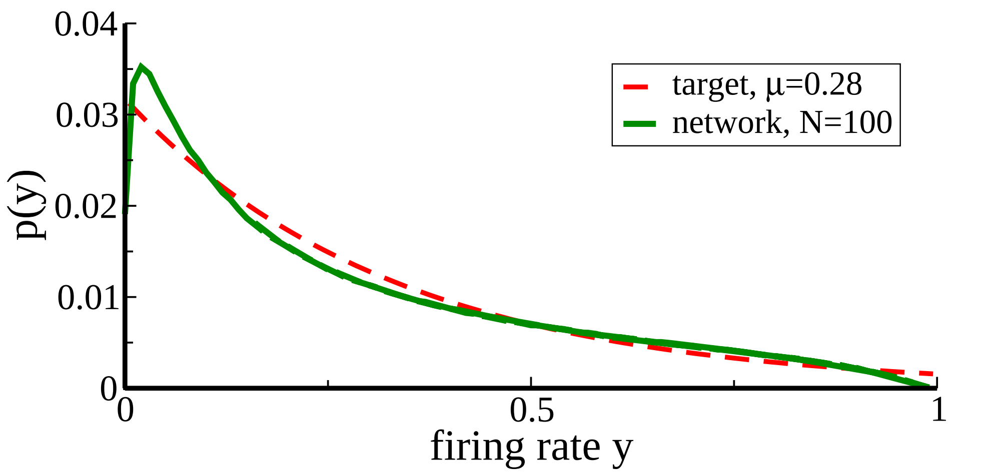
- self-organized chaos
- spontaneous intermittent bursting

polyhomeostatic optimization

distribution of averaged neural activities

[Markovic & Gros, PRL ‘10]

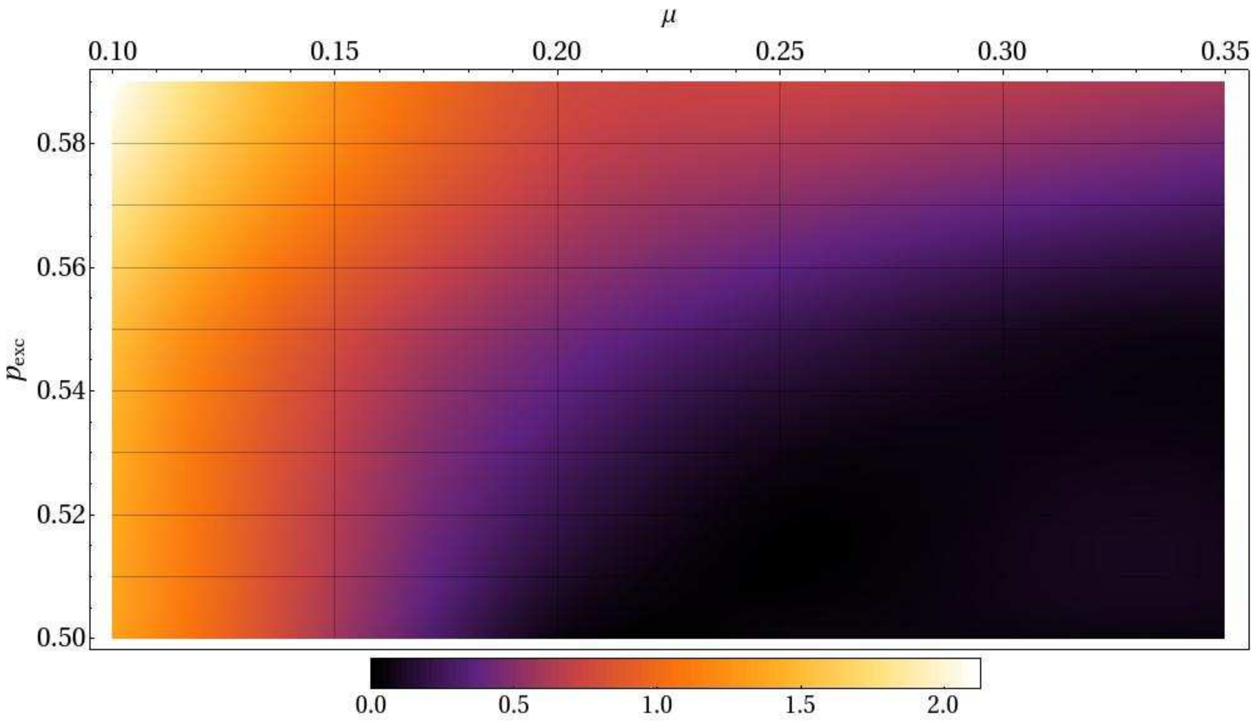




polyhomeostatic adaption

- dynamical system with local adaption rules
- adapting time-averaged statistics of local activities
  - ▷ non-trivial phase diagram

magnitude of average Kullback-Leibler divergence D



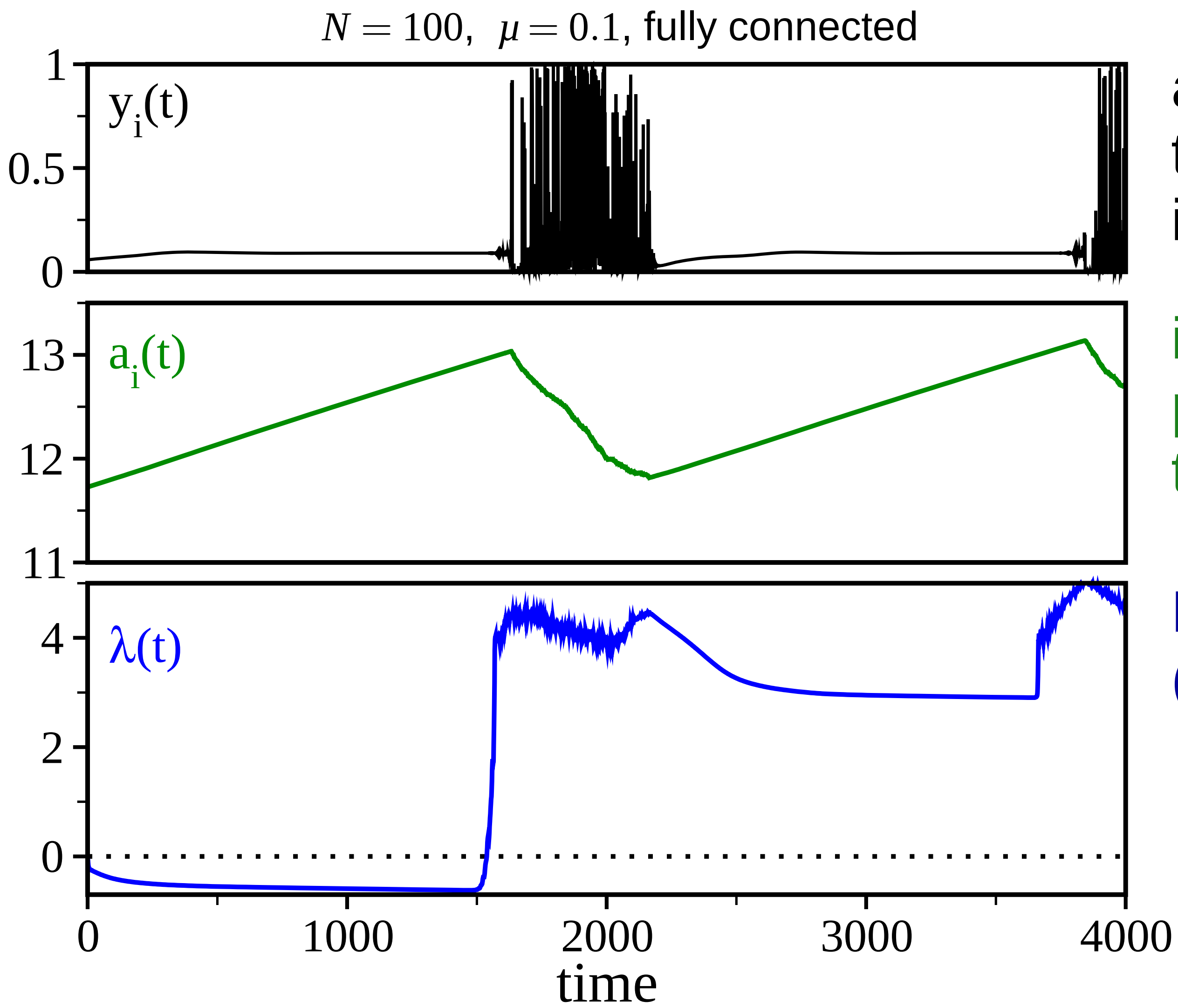
fraction of excitatory links

$N = 1000$
$K = 100$

target mean activity

D(intermittent bursting) $>$ D(chaotic phase)

intermittent route to chaos



activity:
transient attractors
intermittent bursting

internal parameters:
polyhomeostatic adaption
towards threshold

Lyapunov exonent
(global, maximal)

intermittency $\hat{=}$ punctuated equilibrium (evolution)

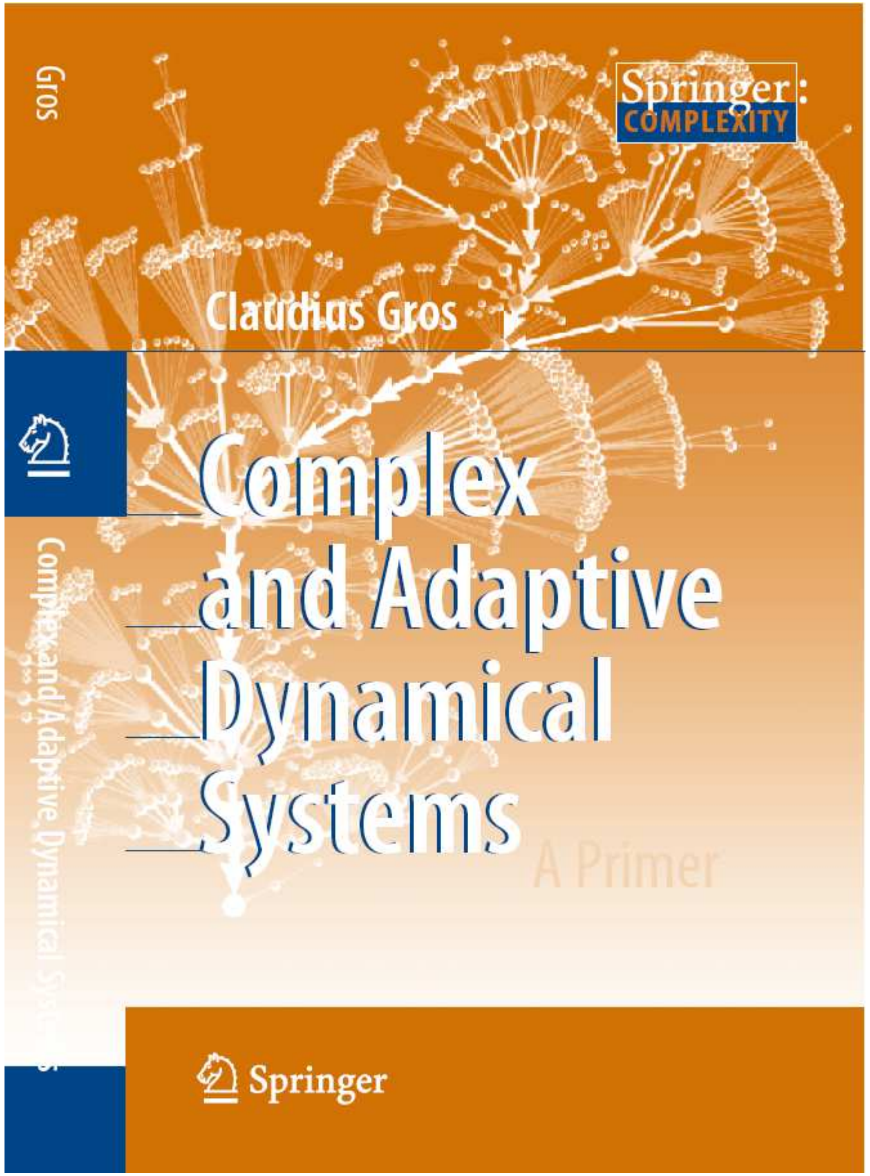
concepts and models in complex system theory

complex system theory – still an emergent field

- ▷ many models and paradigms yet to be formulated

- vertex routing models
  - ▷ critical dynamical network
  - ▷ democratic information centrality
  - ▷ ...
- polyhomeostatic optimization
  - ▷ neural network / game theory / allocation problems
  - ▷ non-trivial autonomous dynamics
  - ▷ ...

graduate level textbook



- Information theory and complexity
- Phase transitions and self-organized criticality
- Life at the edge of chaos and punctuated equilibrium
- Cognitive system theory and diffusive emotional control

second edition fall 2010