Contents

1 Graph Theory and Small-World Networks 1
1.1 Graph Theory and Real-World Networks 1
1.1.1 The Small-World Effect 1
1.1.2 Basic Graph-Theoretical Concepts 3
1.1.3 Graph Spectra and Degree Distributions 9
1.1.4 Graph Laplacian 13
1.2 Percolation in Generalized Random Graphs 15
1.2.1 Graphs with Arbitrary Degree Distributions 15
1.2.2 Probability Generating Function Formalism 20
1.2.3 Distribution of Component Sizes 23
1.3 Robustness of Random Networks 26
1.4 Small-World Models 30
1.5 Scale-Free Graphs 33
Exercises 38
Further Reading 39
References 39

2 Bifurcations and Chaos in Dynamical Systems 41
2.1 Basic Concepts of Dynamical Systems Theory 41
2.2 Fixpoints, Bifurcations and Stability 47
2.2.1 Fixpoints Classification and Jacobian 49
2.2.2 Bifurcations and Normal Forms 51
2.2.3 Hopf Bifurcations and Limit Cycles 53
2.3 Global Bifurcations 56
2.3.1 Catastrophe Theory 60
2.4 The Logistic Map and Deterministic Chaos 64
2.5 Dynamical Systems with Time Delays 69
Exercises 72
Further Reading 74
References 74

3 Dissipation, Noise and Adaptive Systems 75
3.1 Dissipation and Adaption 75
3.1.1 Dissipative Systems and Phase Space Contraction 75
3.1.2 Strange Attractors and Dissipative Chaos 79
3.1.3 Adaptive Systems 81
3.1.4 Conserving Adaptive Systems 86
3.2 Diffusion and Transport 89
3.2.1 Random Walks, Diffusion and Lévy Flights 89
3.2.2 Markov chains 92
3.2.3 The Langevin Equation and Diffusion 94
3.3 Noise-Controlled Dynamics 96
3.3.1 Stochastic Escape 97
3.3.2 Stochastic Resonance 100
Exercises 103
Further Reading 104
References 105

4 Self Organization and Pattern Formation 107
4.1 Interplay between Diffusion and Reaction 107
4.1.1 Travelling Wavefronts in the Fisher Equation 109
4.1.2 Sum rule for the Shape of the Wavefront 112
4.1.3 Self-Stabilization of Travelling Wavefronts 113
4.2 Interplay between Activation and Inhibition 115
4.2.1 Turing Instability 115
4.2.2 Pattern Formation 117
4.2.3 The Gray-Scott reaction diffusion system 118
4.3 Collective Phenomena and Swarm Intelligence 122
4.3.1 Phase Transitions in Social Systems 123
4.3.2 Collective Decision Making and Stigmergy 125
4.3.3 Collective Behavior and Swarms 127
4.3.4 Opinion Dynamics 129
4.4 Car Following Models 131
4.4.1 Linear Flow and Carrying Capacity 131
4.4.2 Self-Organized Traffic Congestions 133
Exercises 135
Further Reading 136
References 136

5 Complexity and Information Theory 137
5.1 Probability Distribution Functions 137
5.1.1 Bayesian Statistics 140
5.1.2 The Law of Large Numbers 143
5.1.3 Time Series Characterization 145
5.2 Entropy and Information 149
5.2.1 Information Content of a Real-World Time Series 155

5.2.2 Mutual Information . . . 157
5.2.3 Kullback-Leibler Divergence . . . 161
5.3 Complexity Measures . . . 164
5.3.1 Complexity and Predictability . . . 166
5.3.2 Algorithmic and Generative Complexity . . . 169
Exercises . . . 170
Further Reading . . . 172
References . . . 173

6 Cellular Automata and Self-Organized Criticality . . . 175
6.1 The Landau Theory of Phase Transitions . . . 175
6.2 Criticality in Dynamical Systems . . . 181
6.2.1 1/f Noise . . . 184
6.3 Cellular Automata . . . 185
6.3.1 Conway's Game of Life . . . 186
6.3.2 The Forest Fire Model . . . 188
6.4 The Sandpile Model and Self-Organized Criticality . . . 189
6.4.1 Absorbing Phase Transitions . . . 192
6.5 Random Branching Theory . . . 193
6.5.1 Branching Theory of Self-Organized Criticality . . . 193
6.5.2 Galton-Watson Processes . . . 198
6.6 Application to Long-Term Evolution . . . 200
Exercises . . . 207
Further Reading . . . 208
References . . . 208

7 Random Boolean Networks . . . 211
7.1 Introduction . . . 211
7.2 Random Variables and Networks . . . 213
7.2.1 Boolean Variables and Graph Topologies . . . 213
7.2.2 Coupling Functions . . . 215
7.2.3 Dynamics . . . 217
7.3 The Dynamics of Boolean Networks . . . 218
7.3.1 The Flow of Information Through the Network . . . 219
7.3.2 The Mean-Field Phase Diagram . . . 221
7.3.3 The Bifurcation Phase Diagram . . . 223
7.3.4 Scale-Free Boolean Networks . . . 227
7.4 Cycles and Attractors . . . 229
7.4.1 Quenched Boolean Dynamics . . . 229
7.4.2 The K = 1 Kauffman Network . . . 232
7.4.3 The K = 2 Kauffman Network . . . 234
7.4.4 The K = N Kauffman Network . . . 235
7.5 Applications . . . 238
7.5.1 Living at the Edge of Chaos . . . 238
7.5.2 The Yeast Cell Cycle . . . 239

7.5.3 Application to Neural Networks 241
Exercises 243
Further Reading 244
References 245

8 Darwinian Evolution, Hypercycles and Game Theory 247
8.1 Introduction 247
8.2 Mutations and Fitness in a Static Environment 250
8.3 Deterministic Evolution 253
8.3.1 Evolution Equations 254
8.3.2 Beanbag Genetics: Evolutions Without Epistasis 257
8.3.3 Epistatic Interactions and the Error Catastrophe 259
8.4 Finite Populations and Stochastic Escape 262
8.4.1 Strong Selective Pressure and Adaptive Climbing 263
8.4.2 Adaptive Climbing Versus Stochastic Escape 267
8.5 Prebiotic Evolution 268
8.5.1 Quasispecies Theory 269
8.5.2 Hypercycles and Autocatalytic Networks 270
8.6 Macroecology and Species Competition 274
8.7 Coevolution and Game Theory 276
Exercises 282
Further Reading 283
References 284

9 Synchronization Phenomena 287
9.1 Frequency Locking 287
9.2 Coupled Oscillators and the Kuramoto Model 288
9.3 Synchronization in the Presence of Time Delays 295
9.4 Synchronization Mechanisms 297
9.4.1 Aggregate Averaging 297
9.4.2 Causal Signaling 301
9.5 Synchronization and Object Recognition in Neural Networks . 305
9.6 Synchronization Phenomena in Epidemics 309
Exercises 312
Further Reading 313
References 313

10 Elements of Cognitive Systems Theory 315
10.1 Introduction 315
10.2 Foundations of Cognitive Systems Theory 318
10.2.1 Basic Requirements for the Dynamics 318
10.2.2 Cognitive Information Processing vs. Diffusive Control 321
10.2.3 Basic Layout Principles 324
10.2.4 Learning and Memory Representations 326
10.3 Generating Functionals and Diffusive Emotional Control 330

10.3.1 Metalearning Through Polyhomeostasis 330
10.3.2 Emotional and Neutral Diffusive Control 332
10.4 Competitive Dynamics and Winning Coalitions 335
10.4.1 General Considerations 335
10.4.2 Associative Thought Processes 339
10.4.3 Autonomous Online Learning 343
10.5 Environmental Model Building 346
10.5.1 The Elman Simple Recurrent Network 346
10.5.2 Universal Prediction Tasks 350
Exercises 353
Further Reading 354
References 355

11 Solutions 357
11.1 Solutions to the Exercises of Chapter 1, Graph Theory and Small-World Networks 358
11.2 Solutions to the Exercises of Chapter 2, Bifurcations and Chaos in Dynamical Systems 363
11.3 Solutions to the Exercises of Chapter 3, Dissipation, Noise and Adaptive Systems 369
11.4 Solutions to the Exercises of Chapter 4, Self Organization and Pattern Formation 373
11.5 Solutions to the Exercises of Chapter 5, Complexity and Information Theory 375
11.6 Solutions to the Exercises of Chapter 6, Cellular Automata and Self-Organized Criticality 382
11.7 Solutions to the Exercises of Chapter 7, Random Boolean Networks 388
11.8 Solutions to the Exercises of Chapter 8, Darwinian Evolution, Hypercycles and Game Theory 390
11.9 Solutions to the Exercises of Chapter 9, Synchronization Phenomena 394
11.10 Solutions to the Exercises of Chapter 10, Elements of Cognitive Systems Theory 398

Index 403