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About this Book

From Evolving Networks to Cognitive Systems Theory This textbook covers a wide range of concepts, notions and phenomena of a truly interdisciplinary subject of rapidly growing importance. Complex system theory deals with dynamical systems containing a very large number of variables, showing a plethora of emergent features, arising in a broad range of contexts. A central focus of these notes is the notion of complexity arising within evolving and dynamical network structures, such as the gene expression networks at the basis of all living, adaptive ecological networks or neural networks for cognitive information processing.

Complex systems theory ultimately forms the basis of our long-standing quest for exploring and understanding cognitive systems in general and our brain in particular – the mammalian brain is probably the most complex of all adaptive networks known to humanity.

Readership and Preconditions This primer is intended for students and scientists from the natural sciences, engineering, informatics or neuroscience. Technically, the reader should have a basic knowledge of mathematics as it is used in the natural sciences or engineering. This textbook is suitable both for studies in conjunction with teaching courses as well as for the individual reader.

Course Material and the Modular Approach When used for teaching, this primer is suitable for a course running over 40–60 lecture hours, depending on the pace and on the number of chapters covered. Essentially all mathematical transformations are performed on a step-by-step basis and in general the reader should have no problem following the respective derivations.

Individual chapters, apart from the first three, having mostly introductory character, may be skipped whenever time considerations demand it. I have followed a basic modular approach and the individual chapters are, as far as possible, independent of each other. Notwithstanding, cross references between the different chapters are included throughout the text, since interrelations between distinct topics are helpful for a thorough understanding.

Style This interdisciplinary primer sets a high value on conveying concepts and notions within their respective mathematical settings. Believing that a concise style helps the reader to go through the material I mostly abstained from long text passages with general background explanations or philosophical considerations. Widespread use has been made of paragraph headings, with the intention to facilitate scientific reading in this way.

A Primer to Scientific Common-Sense Knowledge To a certain extent one can regard this textbook as a primer to a wide range of scientific common-sense knowledge regarding complex systems. Basic knowledge about life's organizational principles, to give an example, such as the notion of "life at the edge of chaos", is important in today's world to an educated scientist. Other areas of scientific common-sense knowledge discussed in this primer include network theory, which has applications ranging from social networks to gene expression networks, the fundamentals of evolution, cognitive systems theory, as well as the basic principles of dynamical systems and information theory.

Content All of the chapters making up this book deal with a subject central to the modern theory of complex systems. I have tried to present succinct expositions of the fundamental notions and concepts, suitable for the casual reader, retaining all the while the level of mathematical rigor necessary for a study course.

1. *Graph Theory and Small-World Networks*
   Networks, ranging from neural networks, social networks, ecological networks to gene expression networks, are at the basis of many complex systems. Networks tend to be adaptive and evolving. Network theory is a prerequisite for a thorough understanding of complex systems and adaption.
2. *Chaos, Bifurcations and Diffusion*
   This chapter introduces basic notions of dynamical systems theory, such as attractors, bifurcations, deterministic chaos, diffusion and stochastic resonances, many of which are used throughout these notes. Complexity emergent from dynamical systems containing many variables, the central theme of this textbook, is ultimately based on the concepts of classical dynamical systems theory, treated in this chapter, which deals with differential equations involving a handful of variables.
3. *Complexity and Information Theory*
   Information theory provides the basis for statistical analysis and time-series characterization, providing powerful concepts likes the Shannon entropy and mutual information. This chapter also provides a critical assessment of measures proposed to quantify the degree of complexity of dynamical systems.
4. *Random Boolean Networks*
   A prime model for complex systems with an infinite number of variables

are random graphs with Boolean variables. It allows for the characterization of typical dynamical behaviors, e.g. "frozen" vs. "chaotic", which are of relevance in many contexts. Of especial importance are random Boolean networks for the fundamentals in the realm of life, leading to the notion of "life at the edge of chaos".

5. *Cellular Automata and Self-Organized Criticality*
   Regular dynamical systems on lattices, the cellular automata, allow detailed studies of the dynamics of complex systems, a key issue being the organizational principle necessary for a dynamical system to show the emergent phenomenon of "self-organized criticality".
6. *Darwinian Evolution, Hypercycles and Game Theory*
   Evolution of living organisms is, without a doubt, the paradigm for an adaptive and complex dynamical system, that of interacting species. Key concepts such as the "error catastrophe" and "hypercycles" for the prebiotic evolution are discussed within the standard statistical approach, together with the foundations of game theory and macroecology.
7. *Synchronization Phenomena*
   When many distinct computational units interact, which is a typical situation in complex systems, they might evolve synchronously, in phase, or rather independently. Synchronization is an issue of wide ranging importance, from the outbreak of epidemics to the definition of objects in cortical circuits.
8. *Elements of Cognitive Systems Theory*
   The most complex of any known dynamical systems, and probably also the least understood of all, is the brain. It constitutes the biological support for the human cognitive system, supposedly the most evolved cognitive system known to date. Basic principles and important concepts of cognitive systems theory are developed in this chapter.

The basic material and mathematical notions for the course are developed in the first three chapters. The scientific investigations of complex systems are just beginning and the subjects chosen in Chaps. 4–8 are of exemplary importance for this rapidly developing field.

Exercises and Suggestions for Individual Studies Towards the end of each individual chapter a selection of exercises is presented. Some of them deal with simple extensions of the material, such as a proof of a specific formula or the application of a method discussed in the main text to a different or related problem. Other exercises are of the form of small work studies, such as the numerical implementation via a C++ or Maple code of a basic model, with the objective to obtain a hands-on experience of an interesting phenomenon from the investigation of the results obtained from the simulation runs.

This interdisciplinary field is very suitable for making an inroad with a basic research project. The suggestions for work studies presented in the respective exercise sections therefore also serve as guides and motivations for a first step towards scientific research in this field, which in the end may possibly lead to research goals developed by the individual reader. It is a highly satisfying experience and is truly recommended.

References and Literature The section "Further Reading" at the end of each individual chapter contains references to standard introductory textbooks and review articles, and to some articles for further in-depth studies dealing with selected issues treated within the respective chapter. Certain original research literature containing some of the first investigations of phenomena discussed in the respective chapter is also selectively listed whenever of scientific or historical interest.

Evolving with the Third Edition This book has seen, since its first printing in 2008, several extensions. I have incorporated many feedbacks and suggestions, which are more than welcome. The third edition comes with a substantial number of new subsections, like the extended discussion of bifurcation theory in Chap. 2, or the new introduction to Bayesian statistics in Chap. 3. You will find, in addition, many new figures and exercises. The field of complex systems is continuously evolving and, with it, this book.

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